School Mathematics for Reconciliation:

From a 19th to a 21st Century Curriculum

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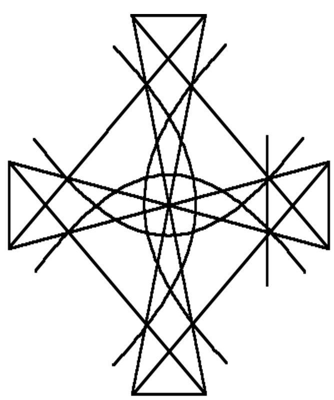
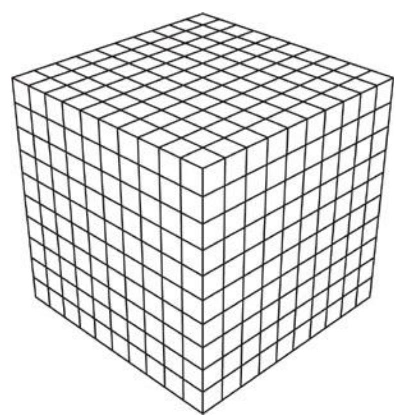
Chaos

Indigenous
Papua New Guinea
Mathematizing



Indigenous Polynesian
Mathematizing



Euclidian-Based
Mathematizing

TABLE OF CONTENTS

Preface

This monograph is intended to serve as a policy discussion document. School mathematics substantially influences high school graduation rates; improving them for some students, depressing them for most Indigenous students and others. That being the case, this document delves into the various domains of school mathematics to critically challenge its validity in the 21st century era of reconciliation between non-Indigenous and Indigenous citizens.

Substantial efforts by some mathematics researchers and teachers have led to innovations that generally replace a 19th century understanding of school mathematics with a 21st century perspective that transforms school mathematics into a largely valid, ethically defensible enterprise. This is one of the monograph's three main emphases.

Culturally responsive or place-based education that focuses on Indigenous students has established a presence in the research literature. This culture-based innovation represents an historical shift in mathematics education. Moreover, it demonstratively advances student academic achievement for both Indigenous and non-Indigenous students.

Its success has exposed deep fault lines in conventional school mathematics. And many innovative researchers and teachers have not fully divested themselves of all problematic, taken-for-granted notions that characterize conventional school mathematics. Consequently, student academic success can be further improved beyond present levels.

These innovative researchers and teachers generally adapt or develop appropriate teaching materials for classrooms. As a result, their words, plans, and actions become amenable to self-reflection and self-critical analysis; a second major emphasis of this monograph.

The seismic shift from conventional to culture-based school mathematics will be experienced by some students as shifting from a formal algorithmic rite of passage, towards a community-based lived experience. Other students will perceive their school mathematics changing from emotionless calculations to relevant cultural practices.

Canada's Federal Government apologized in 2008 for having kidnapped Indigenous children over many generations and for often holding them in faraway residential schools in a cruel attempt to rid them of their Indigenous self-identities. The Government also established the Truth and Reconciliation Commission whose final report was issued in January 2016.

A third main emphasis of this monograph is a response to the Commission's 10th Call to Action, which includes: "improving education attainment levels and success rates" and "developing culturally appropriate curricula" (Truth and Reconciliation Commission, 2016a, p. 165).

Accordingly, I have two main goals: (1) to *illustrate* how critical analysis can help educators decide which taken-for-granted notions about school mathematics should continue to be embraced, and which ones should be updated because they interfere with the engagement and achievement of most Indigenous students and a large majority of non-Indigenous students; and (2) to *identify* concrete ways in which mathematics educators, teachers, researchers, and curriculum writers can help enhance both Indigenous and non-Indigenous students' achievement through culture-based school mathematics in a way that promotes reconciliation. These goals lead to the following questions addressed in this monograph:

1. What conventional taken-for-granted notions impede student achievement?
2. Which of these conventional notions continue to be held by many innovators who have enhanced school mathematics in a culture-based or cross-cultural way?
3. Which innovative taken-for-granted notions improve student achievement?
4. Exactly how do researchers or teachers "see" school mathematics content "embedded" in an Indigenous artifact or everyday activity?
5. Which notions found in conventional school mathematics continue to serve students' interests?
6. How can mathematics curricula mitigate systemic racism and support reconciliation?
7. What specific actions can mathematics teachers, researchers, teacher educators, and curriculum writers take to regenerate what is essentially an ancient-Greek-based Victorian era 19th century elitist curriculum (particularly Grades 5 and higher) into a 21st century curriculum in harmony with today's realities?

In other words, the monograph critiques the domain of both conventional and innovative school mathematics, in an era of reconciliation. Examples of innovative teachers and researchers fulfilling a professional role in reconciliation appear throughout the monograph.

1. Introduction

Respect is more than tolerance and inclusion – it requires dialogue and collaboration.
8Ways, Bangamalanha Centre, Australia (2012, p. 4)

In many countries with Indigenous citizens, an increasing number of mathematics educators and Ministries of Education understand the merits of enhancing school mathematics with Indigenous ways of knowing, doing, living, and being – Indigenous perspectives (Alberta Education, 2006; Fyhn, Sara Eira, & Sriraman, 2011; Greer, Mukhopadhyay, Powell & Nelson-Barber, 2009; Meaney, Trinick & Fairhall, 2012; Lipka, Wong & Andrew-Ihrke, 2013). These merits have motivated research and development (R&D) projects that draw upon Indigenous perspectives to promote greater academic achievement by Indigenous students.

The strategy has "increased student mastery of science and math concepts, deeper levels of student engagement in science and math and increased student achievement in math and science" (U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education, 2008, p. 13) (see also: Banks & Banks, 2012; Fowler, 2012; Lipka, Webster & Yanez, 2005; Keane, 2008; Meaney et al., 2012; Nichol & Robinson, 2000; Perso, 2012; Richards, Hove & Afolabi, 2008; Sakiestewa-Gilbert, 2011). Greater achievement is also found for non-Indigenous students in those same classrooms (Adams, Shehenaz Adam & Opbroek, 2005; Beatty & Blair, 2015; Davison, 2002; Lipka et al., 2013; Nelson-Barber & Lipka, 2008; Richards et al., 2008; Rickard, 2005).

To improve Indigenous students' wellbeing in school mathematics, researchers have developed a number of major R&D agendas: to explore culturally responsive teaching (e.g., Enyedy, Danish, & Fields, 2011; LOCUMS, 2016; Nicol, Archibald & Baker, 2013) or place-based teaching (e.g., Boyer, 2006; Sterenberg, 2013b); to interrogate school curricula (Lipka et al., 2005; Jannok Nutti, 2013); to investigate teacher professional development (e.g. Dawson, 2013; Donald, Glanfield & Sterenberg, 2011; Furuto, 2014); and to produce culturally specific teaching materials required by all of the above major R&D agendas (section 8 of this monograph). This last item defines the focus and scope of this monograph; a vehicle for addressing the issues and questions laid out in the Preface.

My purpose is to illustrate how a critical analysis of researchers' and teachers' words, plans, and actions can be helpful to mathematics educators, especially to those of non-Indigenous ancestry, who aim to nurture Indigenous students in their academic achievement while strengthening their cultural self-identities (Nasir, 2002). The school mathematics literature reports on the processes and products of R&D projects that aim to do this. Drawing on this evidence and on personal experience in Indigenous cross-cultural science education, I offer suggestions about what to consider and what to avoid in future R&D projects. My purpose is not a comprehensive review of the literature.

Notably, cultural perspectives and identities within any identifiable Indigenous group are far from homogeneous. By referring to a group's singular name, I do not suggest the group is homogeneous, even though the group may embrace some fundamental ideas unanimously.

Some projects risk accusations of tokenism, insensitivity, marginalization, appropriation, or neo-colonization, while other projects showcase how to collaboratively develop teaching materials that have made a difference. In all cases, we need to be vigilantly aware of not inadvertently recolonizing the people we intend to serve.

Māori researcher and educator McKinley (2001) warned us that effective teaching of Indigenous students requires educators to deeply understand past political-social events that have caused current political-social power imbalances between, for example, school curricula and Indigenous communities. This colonial-initiated imbalance inevitably favours non-Indigenous researchers or teachers. Accordingly, this political-social issue pervades this monograph's text and subtext. Mathematics educators who are unaware of, or ignore this political-social power imbalance will be blind to some important interpersonal dynamics with Indigenous adults and students. McKinley's article, "Masking Power with Innocence," alludes to what could be called "privilege-blindness." Researchers' privilege-blindness will disrupt teaching or a project aimed at creating teaching materials for Indigenous students. It will also disrupt teachers engaged in culturally responsive or place-based mathematics instruction (Belczewski, 2009).

For example (to paraphrase St. Denis [2004] quoted in Donald et al., 2011, p. 76): if a researcher or teacher subscribes to the popular notion that many Indigenous students have "*lost their culture*," the researcher or teacher implicitly blames the Indigenous community for the loss – being "reckless caretakers of their culture." This attitude will certainly raise barriers and impede progress in a R&D project or during instruction. However, if a privilege-savvy educator (i.e., a person politically and socially informed deeply in terms of privilege-blindness) subscribes to the notion that European colonizers *almost obliterated* the students' Indigenous culture, then blame rests on the colonizers' shoulders for creating a deficit in Indigenous students' understanding their own culture. This important shift of emphasis, from blaming an Indigenous group to blaming the colonizer, pivoted around a critical analysis of the word "lost."

My intent is to promote *privilege-savvy* by discussing positive exemplars in the literature and by critiquing instances similar to "lost their culture" but in the context of school mathematics. In response to various types of privilege blindness found in the mathematics education research literature, I seek to: expose instances of masking power with innocence, critically examine their consequences, explore their political racialized roots, and propose features for an evidence-based alternative curriculum for the benefit of all students.

I invite the reader on a journey from *tolerance* and *inclusion* of Indigenous perspectives in school mathematics to an in-depth *dialogue* and *collaboration* with Indigenous communities.

Our journey takes a rather cyclical spiral route through topics and issues that prepare us to appreciate exemplary innovations (section 8) and to recognize directions for their improvement (section 9) as a basis for a 21st century mathematics curriculum. This journey entails:

- sections 2 and 3 that describe significant contexts, both political-social and educational, in which many mathematics teachers and researchers find themselves when involved with Indigenous students. Issues include: contemporary consequences of colonization, privilege blindness, the mathematics curriculum, the high status of school mathematics, and culture-based alienation of students.
- section 4 that provides an in-depth analysis of the nature of mathematics. Topics deal with generally held presuppositions (i.e., taken-for-granted notions) concerning what school mathematics content is and what it means. It also addresses the historical manipulations that have rendered school mathematics unnecessarily challenging for so many students.
- section 5 that recognizes early innovators who pioneered the historical shift to culture-based school mathematics.
- section 6 that explores how appropriation and marginalization of Indigenous cultures can occur in spite of the best of intentions. It offers ways to avoid these problems by paying attention to what gets lost in translation between Indigenous and Euro-American cultures.
- section 7 that identifies a wide diversity of innovative sites and content for the renewal of school mathematics (e.g., Indigenous mathematizing, mathematics-in-action in the everyday world, and the culture of mathematics), along with a critical analysis of the popular ethnomathematics innovation.

- section 8 that describes innovative presuppositions about teaching mathematics that improve student achievement, and describes conventional presuppositions that continue to serve students. This section synthesizes 10 notable innovative projects published in the research literature.
- section 9 that deals with a few unhelpful notions found in conventional school mathematics that continue to be held by many innovators who have enhanced school mathematics culturally. Issues include: the hegemony of mathematics, myth blindness, misinterpretations of quantified indicators of student success, and researchers' and teachers' belief in being able to "see" school mathematics content "embedded" in an Indigenous artifact or everyday activity.
- section 10 that revisits the mathematics curriculum by summarizing ideas and implications that have accumulated throughout the monograph. The section: investigates how professional educational organizations can stifle innovation; suggests concrete action for mathematics educators to take in their own political-social context; broadly interprets opposition to culture-based mathematics as professing a search for certainty in political-social and educational contexts that, in reality, comprise degrees of *uncertainty*; and identifies how mathematics curricula are related to Canadian reconciliation with Indigenous people.
- section 11 that synthesizes major implications for all stakeholders of school mathematics.

2. Political-Social Contexts

If Indigenous knowledge and pedagogy are to be integrated effectively into the national and provincial curricula, educators must be made aware of the existing interpretative monopoly of Eurocentric education and learn how the fundamental political processes of Canada have been laced with racism.

Marie Battiste (2002, pp. 9-10)

2.1. Consequences of Colonial Genocide

Many Indigenous peoples worldwide have faced colonial genocide (Woolford, Benvenuto & Hinton, 2014), "the destruction of those structures and practices that allow the group to continue as a group" (Truth and Reconciliation Commission, 2016a, p. 3). It takes the form of violence, starvation, cultural erosion, racism, oppression, and marginalization (Daschuk, 2013). As a direct result, with a few notable exceptions, Indigenous people currently suffer degrees of deprivation in social assistance, housing, health care, education, employment, and criminal justice.

In Canada, a country of three founding nations (Indigenous, French, and English; in chronological order), Indigenous peoples comprise three very different groups: First Nations, Métis, and Inuit (in order of decreasing population), together known as Indigenous peoples[i]. About one-half of the First Nations population currently lives on reserves, originally established to force them onto small tracks of mostly non-productive land to make room for European settlers to build a nation. At the same time, First Nations people were prohibited by law from participating in that economic-political nation-building development. Since the late 17th century, First Nations have signed treaties with the British Crown, but Canada's Federal Government has only honoured them partially, as it pleases.

The First Nations population in western Canada plummeted in the late 19th century due to engineered epidemics and orchestrated starvation (Daschuk, 2013). The Federal Government stole children from their families for long periods of time and forced the children into residential schools. "The policy behind the government funded, church-run schools attempted to 'kill the Indian in the child'" (Truth and Reconciliation Commission, 2016b, website quotation). The mortality rate for children in western Canadian residential schools was known to be 30 to 50 percent in 1910, yet residential schools were continued unchanged for decades. The Deputy Superintendent of Ottawa's Department of Indian Affairs, Duncan Scott, dismissed these data thusly: "This alone does not justify a change in the policy of this Department, which is geared towards *the final solution of our Indian Problem*" (quoted in King, 2012, p. 114, emphasis added).

Since then, government discriminatory policies towards Indigenous people continued even more covertly, thus provoking an enormous accumulation of negative consequences. For example, infants were targeted by taking them from their mothers for adoption by, or placed in foster homes of, "White families," in numbers surpassing residential school numbers. The numbers spiked in the 1960s, now known as "The 60's Scoop." Discriminatory funding for education continues to the present, which ensures that a minority of Indigenous students graduate from high school. Currently, reserve schools are funded at about 60 percent of what non-Indigenous schools are given, per pupil (Cuthand, 2012).

Due to many such policies, living conditions on First Nations reserves and in the far north among the Inuit population ranked 72nd and 63rd, respectively, on the 2013 United Nations' Human Development Index, "an international measure of quality of life" (McMahon, 2014, website quotation). This compares to Canada's mainstream ranking of 8th, nestled between the U.S. and Japan. This enormous standard-of-living gap precipitates a human rights issue. The gap has been maintained over the years by the government's Department of Indian and Northern Affairs (a.k.a. "the Colonial Office").

Although many Canadian Indigenous nations have survived colonial genocide and its aftermath, a testament to their cultural strength and resiliency (Kinew, 2015), there are many individual casualties; notably intergenerational family dysfunctions principally caused by residential schooling (Truth and Reconciliation Commission, 2016a). "We have normalized racial discrimination against [Indigenous] children as a legitimate fiscal restraint measure" (Adams, 2016; quoting Cindy Blackstock, president of First Nations Child and Family Caring Society, which successfully sued the federal government recently in a human rights court over the government's discriminatory low funding of child social services).

These individual casualties are sources of malaise among many Indigenous students entering mathematics classes today. The casualties imply that all teachers and researchers need to become privilege-savvy and consistently express it through their words, plans, and actions (McKinley, 2001).

2.2. Looking Forward

Although the consequences of colonial genocide are brutal, times are now beginning to change. For instance, an Indigenous, grass-roots, social movement called "Idle No More" took Canada by storm beginning in December 2012 (The Kino-nda-niimi Collective, 2014). It drew attention to anti-environment legislation proposed in a Federal Government bill. Peaceful protests across the country usually took the form of flash mobs dancing to Indigenous drumming and singing in city malls. Idle No More will effectively rise up again as needed, for example, gaining equal access to education, health care, social services, and the justice system.

The Prime Minister of Canada apologized in 2008 for the cruel and oppressive treatment many Indigenous people suffered in residential schools. As a result, Canada has now been challenged by its Truth and Reconciliation Commission's (2016a) final report, which concluded with a directive to all Canadians to reinvent our relationships between Indigenous and non-Indigenous citizens. In this context, educators in Canada and elsewhere are beginning to do their part towards such reconciliation by committing themselves to a professional journey into enhancing school subjects with Indigenous perspectives, mathematics included.

Worldwide, a renaissance by Indigenous peoples towards sovereignty has been taking place over the past five decades at least (Battiste, 2002). One indicator of success is the United Nation's Declaration on the Rights of Indigenous Peoples. Related to school mathematics, Article 14 states, "Indigenous peoples have the right to establish and control their educational systems and institutions providing education in their own languages, in a manner appropriate to their cultural methods of teaching and learning" (United Nations, 2007, p. 6). Norway, among most countries, signed the convention with its Indigenous Sámi population in mind (Fyhn et al., 2011). Originally, Canada, United States, Australia, and Aotearoa New Zealand refused, but have now reversed, or intend to reverse, their positions.

2.3. A Postcolonial Response

What are the implications for developing Indigenous-relevant teaching materials for mathematics in any country with a history of colonialism? A large majority of non-Indigenous citizens tend to be blind to the many privileges and power they enjoy; blind to the historical facts that account for those privileges and power; and unconcerned with Indigenous people's current oppressive circumstances that places their reserves at the UN's 72nd ranking. This blindness includes mathematics teachers who may very honestly be perplexed by the idea that they benefit from many social and cultural privileges unavailable to many, but not all, Indigenous families. This is understandable. When a person lives within only one cultural milieu, it can define for them what normal is; and normal soon becomes invisible, thereby creating privilege-blindness.

Escape from privilege blindness into a pluralistic inclusive world is made possible if we engage in a cultural immersion or some cultural enlightening experience (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, 2017; Fyhn et al., 2011; Lunney Borden, Wagner & Johnson, 2017; Michell, Vizina, Augustus, & Sawyer, 2008), where we have postcolonial conversations with colleagues and perhaps even a postcolonial relationship with Indigenous neighbours. The term "postcolonial" does not mean that colonialism has ended. Instead, it means that colonialism is explicitly recognized and efforts are made to diminish and extinguish its power, a process of decolonization (Battiste, 2013; Belczewski, 2009). This is the current political-social context in which school mathematics educators and researchers find themselves.

Decolonizing present day school mathematics happens at many levels. There are major changes such as explicitly including Indigenous perspectives in curricula beyond window-dressing (sections 2.4 & 10).

There are also very minor changes such as writers showing respect and extending privileges to Indigenous cultures. One example of this is the capitalization of the words "Indigenous" and "Aboriginal." (In Canada, the terms are interchangeable.) The mass media and publishing industry do not normally extend this sign of respect, but most Indigenous scholarly authors do. The ethical principle here is: Do what most Indigenous scholars do, and help equalize a historical, systemic, colonial power imbalance. Specifically, we need to insist that our publications will appear with appropriate capitalization. A note to a journal or book editor is all that it usually takes. In other words, we must challenge a systemic power structure masked by innocence, convention, ignorance, or racism. This is but one small, yet highly poignant, incident of decolonization. Others appear throughout this monograph.

As mentioned above but worth repeating: to decolonize school mathematics, successful educators adhere to a fundamental two-part policy. Indigenous students should be able to: (1) master and critique mathematical ways of knowing without, in the process, devaluing or setting aside their culture's ways of knowing, doing, living and being; and at the same time (2) strengthen their own cultural self-identities. The two goals are not a binary, as some educators seem to believe. They occur simultaneously.

Lipka and colleagues' (2013) Math in a Cultural Context project is an example of this policy in action. They add another important political-social dimension to keep in mind: "Indigenous groups' worldview and everyday practice differ fundamentally from Western worldview and everyday practice on the difference between subsistence and capitalism economic systems" (p. 136).

2.4. Mathematics Curricula

R&D project researchers and classroom teachers strive to meet provincial, state and federal standards, but they also address an urgent need to contextualize this content in their community's local culture (Furuto, 2014, 2017; Lunney Borden et al., 2017). "[W]e suggest that the alienation that many children in school, and adults out of school, feel towards mathematics is partly the result of the lack of connexions between their experience in mathematics classrooms and their experiences out of school" (Greer & Mukhopadhyay, 2012, p. 244). "Detached and taught in isolation, [pure] mathematics loses many of its attributes as an enormously important part of our society, culture, and science and the students lose their ability to handle complex situations where mathematics is in action" (Andersson & Ravn, 2012, p. 322).

Education standards are powerfully expressed in the form of mathematics curricula, which are certainly political documents and most often out of synchrony with culture-based school mathematics (Greer & Skovsmose, 2012). The curricula's voluminous content alone prevents most teachers from exercising the flexibility needed to provide an appropriate culturally responsive or place-based mathematics program (Lipka et al., 2005; Nicol et al., 2013). This explains the lingering tension between the legal authority of the mathematics curriculum and the evidential authority of what works well in innovative mathematics education (section 8). For the vast majority of competent teachers, curricular constraints will stifle innovation towards incorporating Indigenous perspectives in school mathematics (Matthews, 2015).

Fyhn, Jannok Nutti, Nystad, Eira and Hætta (2016) in Norway investigated what support for teachers is needed to overcome constraints that stifle culturally responsive mathematics instruction and student assessment in the context of Indigenous Sámi schools. Their research revealed "that when teachers' behaviour was [strictly] controlled by national [standards] rules, guidelines and textbooks, almost no culturally responsive mathematics took place" (p. 422).

Choosing curriculum content can "entail negative ethical implications given the currency of mathematics qualifications that are rewarded, in part, for speed and curriculum coverage" (Boylan, 2016, p. 400). Boylan conducted research into the complex, conflicting, ethical choices mathematics educators face. In a sense, a mathematics curriculum is an ethically derived document that omits communicating its ethical decisions in its content. R&D projects reported in the literature are usually successful at both expressing their ethics and achieving most of their objectives (section 8). However, the process of scaling-up (Elmore, 1996) is a major roadblock erected by the conventional curriculum and its major stakeholders (section 10.3).

The tension is exacerbated for many R&D projects whose funding agreements stipulate meeting curriculum expectations that interfere with the project's agenda of decolonizing school mathematics (FNESC, 2011; Furuto, 2017; Lunney Borden, 2013). The benefit of receiving grants to produce excellent teaching materials, however, often compensates for curricular restrictions imposed. Some projects purposefully intend to alter the mathematics curriculum someday (e.g., Donald et al., 2011; Fyhn, 2009; Lipka, 1994; Jannok Nutti, 2013).

But innovative curricula, crafted to advocate teaching culture-based mathematics, can still be inadequate unless they give: reasonably informed directives to teachers, appropriate local teaching materials, and directions for adequate professional development opportunities required to implement a decolonizing curriculum. Teachers cannot be guided by a curriculum *hypothetically envisioned* by its writers who lack the experience of introducing Indigenous perspectives into their own mathematics classrooms.

One example will illustrate how "critical analyses of our words, plans, and actions" applies to such a curriculum. The Saskatchewan Grade 6 mathematics curriculum (typical of other grade levels) is open to including Indigenous perspectives in its listing of four goals: logical thinking, number sense, spatial sense, and *mathematics as a human endeavour* (Saskatchewan Curriculum, 2016, emphasis added; all quotes are from the website). This last goal, a welcome departure from conventional curriculum frameworks, creates promising opportunities to deal with Indigenous perspectives. For example, teachers are directed to create experiences for students to develop "an understanding of mathematics as a way of knowing the world." This is expected to be achieved, first by empowering "teachers to understand that mathematics is not acultural. As a result, teachers then realize that the traditional ways of teaching mathematics are also culturally biased." This type of transformative empowerment usually requires a substantial profession development program (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, 2017; Fyhn et al., 2011; Lunney Borden et al., 2017; Michell et al., 2008).

Appropriate approaches are articulated in the Saskatchewan curriculum as follows:

> Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups

> cannot be assumed to be a direct link. … Various ways of knowing need to be celebrated to support the learning of all students. (Quoted from the curriculum's introduction)

Here is a sound idea that requires Indigenous-related teaching materials for specific areas of Saskatchewan. However, of the several resources listed in the curriculum website (e.g., a module *Salmon Fishing* from Alaska), none is applicable locally unless a teacher transposes some of its content to relate to the Canadian prairies. But where is the time, energy, and professional development for most teachers to be an Indigenous materials developer for mathematics?

The curriculum lists outcomes and indicators to define and suggest (respectively) what students are to learn or do. The Grade 6 mathematics curriculum has only one Indigenous-related outcome out of a total of 19 outcomes (more attention than other grade levels, except Grade 9). That one outcome addresses Indigenous perspectives in the context of learning about numbers, and is accompanied by three indicators (Saskatchewan Curriculum, 2016):

> *Outcome*: Research and present how First Nations and Métis peoples, past and present, envision, represent, and use quantity in their lifestyles and worldviews.
>
> *Indicators*:
>
> a. Gather and document information regarding the significance and use of quantity for at least one First Nation or Métis peoples from a variety of sources such as Elders and traditional knowledge holders.
> b. Compare the significance, representation, and use of quantity for different First Nations, Métis peoples, and other cultures.
> c. Communicate to others concretely, pictorially, orally, visually, physically, and/or in writing, what has been learned about the envisioning, representing, and use of quantity by First Nations and Métis peoples and how these understandings parallel, differ from, and enhance one's own mathematical understandings about numbers.

Out of a total of 145 indicators, five relate to Indigenous perspectives, which include the three just above. Their expectations of students seem quite unrealistic for all but a few exceptional students, unless substantial scaffolding is present to guide and support them; such as the Show Me Your Math project (Lunney Borden, 2013) (section 8.7). Otherwise, the indicator could likely encourage stereotyping found on the internet. In recommending the *Salmon Fishing* module from Alaska, the curriculum writers seem to have missed the fact that the researchers who collaboratively developed this module (Lipka et al., 2013) learned experientially from being with Elders (sections 6.1 & 8.3). Through research and development, the developers gained an understanding of what is reasonable to ask students to accomplish, guided by the wisdom of Elders. This is the standard to which curriculum writers must be held, I would think. What is being masked with innocence in the Saskatchewan curriculum?

Unlike the Saskatchewan school *science* program (Aikenhead & Elliott, 2010), the mathematics program has made no progress in producing supportive mathematics teaching materials such as textbooks, monographs, modules or websites. As a result, the mathematics curriculum exemplifies tokenism by its superficial and inadequate inclusion of Indigenous perspectives, in spite of its promising rhetoric to the contrary.

As a consequence, Saskatchewan's Indigenous students' achievement in school mathematics will continue to rely on the conventional pressure on students to persevere and work harder; a sure way to continue the status quo statistics that reflect a power imbalance between the school's curriculum and Indigenous people's expressed interest in mathematics achievement (Bang & Medin, 2010; Beatty & Blair, 2015; Neel, 2008; Lunney Borden, 2013; Pelletier, Cottrell & Hardie, 2013).

The intransigent power of the curriculum works against updating it with contemporary ideas about the cultural nature of school mathematics content; a topic discussed throughout the monograph (e.g., section 4).

2.5. Who Is the Author?

As far as readers are concerned, a major political-social context is the author's personal background and relationships with the Indigenous people with whom a R&D project is undertaken or about whom the author is writing (Bang & Medin, 2010; Lunney Borden, 2013). It is an issue of credibility revealed through self-disclosure.

Tân'si. Glen Aikenhead *nit'sîyikâsan.* I am of British ancestry and a Treaty 6 Canadian. My ancestors immigrated to the Ottawa Valley, Ontario, in 1820, to establish a farmstead on a piece of Algonkian hunting land. I acknowledge and honour the Algonkian Nation as inhabitants of that sacred land, which shadows my life today.

I grew up in rural Alberta and urban Calgary. For an accumulated five years, I was a science and mathematics teacher in Calgary and in two international schools. I have always embraced a humanistic perspective on science, which was honed during my graduate studies at Harvard University. In the early 1970s, this humanistic approach launched my career at the University of Saskatchewan with a research program making school science realistic and relevant for all students. My university teaching included science methods courses for First Nations and Métis teacher education programs. Those students inspired me to learn more about Indigenous ways of knowing.

My Caucasoid, male, middle-class, science-mathematics-related identity has always given me a highly privileged status. Since the early 1990s, I have invested this cultural capital in being an ally to my Indigenous colleagues and friends. Their visions of science and mathematics education have always defined my R&D agendas.

By volunteering for local Tribal Council projects, and being taught by patient and wise Elders, I matured to the point of offering a policy statement for Indigenous cross-cultural science education (Aikenhead, 1997). This brought me into contact with many national and international Indigenous science educators, to whom I am indebted for their mentoring and tutoring.

My 1997 policy statement led me to develop a community-based, collaborative project, *Rekindling Traditions* (Aikenhead, 2002a). Its purpose was to demonstrate how place-based science teaching materials could be developed from an Indigenous perspective.

I became Professor Emeritus in 2006. In 2008, the Saskatchewan Ministry of Education added Indigenous knowledge to its science curriculum in collaboration with Elders, and then began to publish a Grades 3-9 science textbook series with Indigenous perspectives developed in full collaboration of Elders (Aikenhead & Elliott, 2010), with whom I had the honour to share editing responsibilities. Next came an academic resource reader *Bridging Cultures* (Aikenhead & Michell, 2011), and then a collaborative project that produced a professional development monograph for implementing culturally responsive science teaching (Aikenhead et al., 2014).

2.6. Conclusion

Every country has unique and complex political-social contexts that determine educational policies. Canada’s policy now rests mainly on the social-justice issue of reconciliation. Norway’s policy is rationally formulated on government legislation protecting Sámi culture as an important heritage to maintain in Norway (Fyhn et al., 2011). Other reasons that motivate educational jurisdictions to enhance their school mathematics culturally include: to meet student assessment standards defined by governmental authority; to counter the negative consequences of globalization experienced by developing nations; and to change a government’s economic base from an Indigenous population reliant on social welfare, to a government reliance on employed Indigenous taxpayers.

3. Education Contexts

If we are to understand how Aboriginal and Eurocentric world views clash, we need to understand how the philosophy, values and customs of Aboriginal culture differs from those of Eurocentric cultures.

Kainai Elder Leroy Little Bear (2000, p. 77)

The standard-of-living gap identified in section 2.1 has a direct bearing on Indigenous high school graduation rates and most students' interest and achievement in schooling, particularly in mathematics. This influences post-secondary attendance specifically, and employment in general. Statistics are both overwhelming and well-known worldwide (Abrams, Taylor & Guo, 2013; Perso, 2012; Anderson & Richards, 2016). The vicious cycle of poverty will continue unless a host of interventions are implemented simultaneously. Education has one small but crucial part to play. Its role in developing Indigenous-related curricula and teaching materials for mathematics education can certainly help. In this section, I address three pervasive education topics that form an important context for enhancing school mathematics with Indigenous-related and mainstream culture-related teaching materials and pedagogy: (1) improvements in developing teaching materials, (2) the high status of school mathematics, and (3) culture clashes that alienate many Indigenous students.

3.1. Improvements in Developing Teaching Materials

Over the span of the past few decades, Jorgensen and Wagner (2013) observed that the emphasis for mathematics education research had shifted from "researching *on* [I]ndigenous people to researching *with* [I]ndigenous people" (p. 1, original emphasis, spelling corrected). They also drew attention to:

1. the move away from deficit assumptions about Indigenous students, and towards focussing on the assets students bring to the classroom;
2. the move away from stereotyping and romanticizing Indigenous cultures, and towards knowing the realistic diversity within any Indigenous group; and
3. the move away from grasping onto outdated ontologies (people's ideas about reality), outdated epistemologies (how people come to know what they know, and the kind of knowledge it is), axiologies (people's values and belief systems that guide their thinking and behaviour), and outdated ideologies (doctrines that determine how people or institutions treat others); all four of which are responsible for the low achievement of many

Indigenous students (e.g., Lunney Borden, 2013; Nicol et al., 2013; Russell & Chernoff, 2013).

Jorgensen and Wagner's (2013) observations highlight some fundamental values and observations that identify critical analysis criteria for this monograph.

Doolittle (2006), Meaney (2002) and Sterenberg (2013a) wrote about attempts to contextualize school mathematics in Indigenous cultures, which have ended up as tokenism, stereotyping, or other neo-colonial devices, with the negative consequence of marginalizing Indigenous students. For example, Sterenberg repeated an insulting word-problem: "Imagine a band of 250 Aboriginal People. Each tipi can hold approximately eight people. Calculate how many tipis would be needed to house the entire band" (p. 21). This is insulting to those Indigenous people whose ethics frown on using numerals to count humans. Researchers need to know this ahead of time. It is also demeaning because Indigenous people would never divide themselves in the hypothetical way stated in the word problem. Relational and spiritual factors would dominate. And the required hypothetical state of mind itself, captured in the word "imagine," is a value embraced strongly in the culture of school mathematics, but would generally be foreign to an Indigenous culture in the context of people choosing a tipi to enter. It conflicts with how to live in a good way. The word problem is an inauthentic representation of Indigenous cultures.

Fyhn (2013, p. 365) also referred to a bad example of contextualizing a word problem: A Sámi hunter stopped to determine "the exact distance across a river by means of trigonometric functions." The scene defies Indigenous reality to such an extent that it is disrespectful. Using written recipes for Indigenous cooking is quite a popular activity for teaching measurement and proportionality. But as Jannok Nutti (2013) pointed out, *authentic* Swedish Sámi cooking follows oral non-quantitative directions. Non-authentic contextualizing discourages Indigenous students' participation. Warren and Miller's (2013) research concluded that an oral language approach generally increases engagement and achievement for most young, Australian Indigenous students.

A project dedicated to teaching culture-based school mathematics can generally avoid tokenism, stereotyping, and other neo-colonial devices by *collaborating* with Indigenous people who have the relevant gifts or expertise (sections 6.1 & 6.2). Examples are found in section 8.

Unfortunately, a number of promising curriculum renewals and R&D projects never reach fruition because they became mired in the process of *consultation*, which does not sufficiently share power with, or transfer power to, Indigenous Elders or communities to make final decisions concerning education outcomes and teaching materials. There are practical advantages to reasonably reversing the status quo power imbalance so it favours an Indigenous community's interests (Aikenhead, 2002a). This did not happen in the development of Saskatchewan's 2007 curriculum. The process was only consultation (Russell, 2016).

Although exemplary R&D projects are contributing substantially to decolonizing mathematics education, a critical examination can reveal some subtle, vestigial, hegemonic presuppositions that cause privilege blindness and thereby undermine the projects' full potential for decolonization (sections 8 & 9).

3.2. High Status of School Mathematics

Educators who teach mathematics, the school's highest status subject (Venville, Wallace, Rennie, & Malone, 2002), enjoy a paucity of pressure to defend, rationalize, or modify their school mathematics curriculum content in junior and senior secondary grades (D'Ambrosio, 2007). This high status arises, in part, from school mathematics' ideology of exclusion (Pais, 2012) and its absolutist philosophy of education (Ernest, 1991). For example, students' assessment in a high status subject becomes an unquestioned screening, weeding out, or gatekeeping device for high school graduation (Boylan, 2016; Pais, 2012). "The number of mathematics courses taken by a student is commonly regarded as an indicator of his or her potential and ability, not in the least because mathematics wears the mask of impartiality so effectively" (Davis, 1996, p. 145).

Students who are most likely to fail this screening process tend to belong to marginalized and low social-economic-status groups (Fettes, 2006; Nasir, Hand, & Taylor, 2008). This gatekeeping process provides social power to the more privileged students who make it through. Certainly within Euro-American cultures, "prestige, control, authority, and power are gained by the knower" (Russell, 2016, p. 75). This reality faces all students participating in most mathematics learning environments, but especially Indigenous students; a situation that Russell and Chernoff (2013) argued, with considerable force, is ethically indefensible; as well as being systemically racist (sections 10.1–10.3).

In his critical analysis of equity in school mathematics, Pais (2012) came to two initial conclusions:

> On the one hand, the discourse that vindicates the importance of mathematics to becoming a full citizen [i.e., "the ideological injunction that you really need mathematics to attain citizenship," p. 65], and therefore requires mathematics for all, can carry the germ of exclusion. On the other hand, the way mathematics education research addresses the

> problem of equity is mostly a technical one, leaving aside a social and political comprehension of it. (p. 51)

His analysis then explored the political-social dimensions of school mathematics' screening function.

> [M]athematics empowers people not so much because it provides some kind of knowledge or competence to them, but because it gives people a value. It allows students to accumulate credit in the school system that will allow them to continue studying and later to achieve a place in the sun. Mathematics empowers people because it is posited as a socially valuable resource. (p. 61)

At the same time, school mathematics disguises its "traumatic role of … social gatekeeper" (p. 61). Therefore, from a broad political-social perspective, "mathematics is constructed as a prerequisite to citizenship because school needs mathematics to perform its role as a credit system" (p. 65). In other words, its gatekeeping function is for the benefit of school mathematics continuing its high status in schools; not so much because of what content and skills students might learn for their adult life. This all works smoothly as long as: (1) the content and skills are sufficiently challenging to create inequities among students, and (2) no one disturbs this conflict of interest owned by school mathematics. Pais concludes that equity works against the school's conventional inequitable distribution of social capital, an ideology he associates with Euro-American capitalism.

> [W]hat makes schooling such an efficient modern practice is precisely its capability of excluding people by means of promotion. Thus, schooling is possible (as an institution *overdetermined* by capitalism) only to the extent that universal schooling, where everybody will be successful, remains impossible. The motto "mathematics for all" functions as the necessary ideological double concealing the crude reality that, as any mathematics teacher knows, mathematics is not for all. (p. 58, original emphasis)

From society's and students' uncritical perspective on mathematics, perhaps its high status also stems from mathematics being "the antithesis of human activity – mechanical, detached, emotionless, value-free, and morally neutral" (Fyhn et al., 2011, p. 186), as well as appearing to be decontextualized, abstract, philosophical, and rigid, and therefore more difficult to learn (Nasir et al., 2008). This perception: (1) creates a culture clash for such students (Aikenhead, 1997; Meaney, 2002; Russell & Chernoff, 2013) (section 3.3); (2) convinces them that mathematics is not relevant to who they are and where they are going – their self-identities (Chronaki, 2011;

Ishimaru, Barajas-López & Bang, 2015; Middleton, Dupuis & Tang, 2013; Nasir, 2002); and (3) spurs them to resist this assimilationist instruction, usually by dropping out of class or “playing Fatima’s rules” (Aikenhead, 2006, p. 28) to make it appear as if meaningful learning has occurred, when it has not. These three consequences of school mathematics raise ethical issues.

If we add these three ethical concerns to the school mathematics’ hidden agenda of exclusion (Pais, 2012), and to its self-representation as the antithesis of human activity (section 4), then we are faced with a validity issue surrounding school mathematics’ maintaining its high status and gate keeping functions. Validity is defined in terms of school mathematics being what it claims to be; invalidity is about misrepresenting itself.

First, consider its screening function for entry into occupations and higher education. As mentioned above, screening is actually about acquiring social capital, defined by the credentials a student possesses; not so much the actual content taught in the courses that lead to those credentials. But what is the relationship between school mathematics content of certain high school courses and the mathematics that are actually used in occupations or in higher education institutions? In a small minority of situations, such as professional science and engineering, the relationship is understandably strong; for nursing it is surprisingly weak, as it was for industry and government labs (Aikenhead, 2005). In Pais’ (2012) review of the literature dealing with the mathematics actually used by workers in business, industry, commerce, and the public sector, the relationship was almost none existent. The workers’ “mathematics is learned…in practice, outside school” (p. 67). Moreover, academic mathematics does not underpin the information revolution. School mathematics turns out to be simply “a hoop to jump through just to prove you could…a false boundary in one’s life” (Russell, 2016, p. 44). How valid is school mathematics’ screening when it only pertains to a small minority of students, on average “24 percent of OECD high school students” (OECD, 2016, p. 362)? What about the 76 percent?

Secondly, consider what school mathematics claims to do for students personally and for a country’s economy in general. For instance, school mathematics once claimed that learning mathematics trains the mind to think logically. This was empirically tested during the 1970s and rejected as an old wives tale. The promise was then changed to one of developing a labour market to benefit a nation’s economic competitive edge (sections 8.4 & 9.1). Accumulated research in STEM employment and in economics fails to support this promise (Charette, 2013; Lunney Borden & Wiseman, 2016). A nation’s economic edge is far more influenced by “emerging technologies, industrial restructuring, poor management decisions, and government policies that affect military development, monetary exchange rates, wages, and licensing agreements" (Aikenhead, 2006, p. 35).

Although the validity of school mathematics' high status is seriously specious, it has subliminal political-social authority to maintain inequities among students (Pais, 2012), including systemic racist inequities for Indigenous students. Pais' critical analysis of equity in school mathematics reveals a clear challenge we face: How does reconciliation move forward through the resistance of school mathematics' systemic racism? Mathematics educators can take a step forward by understanding the importance of culture clashes.

3.3. Culture Clashes that Alienate Many Indigenous Students

A specific fundamental culture clash between mathematical and Indigenous ideas of reality (i.e., their ontologies) illustrates two very different cultural ideologies. The confidence that scientists hold in their mathematical representations of the physical universe can cause many people to believe that Indigenous ideas of reality are based on superstition. But at the same time, confidence in Indigenous rationality causes some Indigenous people to think that mathematical representations of reality are based on superstition. This was illustrated by Lakota Elder Deloria (1992), who stated:

> The present posture of most Western scientists is to deny any sense of purpose and direction to the world around us, believing that to do so would be to introduce mysticism and superstition. Yet *what could be more superstitious* than to believe that the world in which we live and where we have our most intimate personal experiences is not really trustworthy, and that another mathematical world exists that represents a true reality? (p. 40, emphasis added)

Thinking that someone is rational or superstitious would seem to depend upon which culture-based worldview we adhere to (Bang & Medin, 2010; Medin & Bang, 2014).

The culture of school mathematics often clashes with a student's worldview, cultural identity, or life in general (Chronaki, 2011; D'Ambrosio, 1991; Davison, 2002; Meaney, 2002). Quantification itself seems to control students' lives in arbitrary ways, such as representing their achievement with numerals assumed to validly define what they have learned; or having to perform their best on a test written between the numerals of 9:30 and 10:30 a.m. (Doolittle & Glanfield, 2007). "[S]o much of the power of [Western] Mathematics comes from the security and *control* it offers" (Bishop, 1988b, p. 151, original emphasis). Quantification encourages people to believe that they can objectify a thing, event, or person by stripping them of their qualitative, subjective, spiritual attributes. "The mathematical valuing of 'right' answers informs society which also looks (in vain of course) for right answers to its problems" (p. 152).

Boylan (2016) illustrates a much different way in which mathematics can affect people's lives:

> A significant capitalist response to the environmental crisis has been to enlist mathematics in the search for market solutions. Under the banner of green capitalism, mathematics is being used as a means to extend the commodification of natural resources in new ways… The value and worth of the natural world and our relationship to it are transmuted into valorisation; everything—water, trees, clean air, biodiversity and ecosystems—can be given a price. (pp. 402-403)

Quantification also controls the lives of R&D project co-workers who write funding proposals, final reports, and journal articles. The assessment of their project requires their creativity to generate *quantitative evidence* that will represent a successful impact of their project. This happens because *qualitative evidence* from systematic investigations is rarely valued by funding agencies and government departments, whose power to control project assessment is enacted through the concept of quantification. The source of this control is more fundamental or subliminal than saying, "It's conventional practice." The source is an ideology – the ideology of quantification, with its value cluster that includes objectivity and "cynical equations"[ii] (Sriraman, 2016, p. 12). Ideologies in general are doctrines that determine how people or institutions treat others.

State, national and international testing results exert power over citizen opinion, over education policy, and over what happens in school mathematics classrooms; all leveraged by the quantification ideology. (See section 9.4, for a detailed critical analysis.) This ideology, experienced by all students, is located in the content of school mathematics, albeit tacit. The ideology is seldom made explicit to students. Instead, students are expected to accept the ideology – an assimilation or indoctrination into the culture of school mathematics and into Eurocentric-based cultures. Some students figure out what their school mathematics is up to, and they resist in varying degrees, especially Indigenous students.

Another source of alienation arises from the fact that, for instance, axioms, sets, proofs, algebra, and geometry are all abstract inventions of the intellect (Bishop, 1988b; Boylan, 2016; Ernest, 1991; Ernest, Sriraman, & Ernest, 2016; François & Van Kerkhove, 2010). These abstract representations or metaphors can be *superimposed* on the physical world in order to make sense out of it through a mathematical lens. Skovsmose (2012, p. 352) refers to this process as "mathematical archaeology." Ernest (2013) noted a "distinction between *concept definition* (formal, explicit, publicly justifiable description) and *concept image* (visual and other representations and associations). Concept images represent a deep level of meaning" (p. 5, emphasis added).

Interestingly, these metaphoric images are originally created by the human mind, according to Einstein (1930) who wrote (quoted in Director, 2006):

> It seems that the human mind has first to construct forms independently, before we can find them in things. Kepler's marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone, but only from the comparison of the inventions of the intellect with observed fact. (p. 113)

Strong evidence that supports Einstein's assertion comes from the lives of a few remarkably unusual people. Padgett's (Padgett & Seaberg, 2014) story illustrates a condition in which some people with a mental disorder, such as an autism spectrum disorder, demonstrate extraordinary and unexpected abilities far in excess of what would be considered normal, such as suddenly becoming a mathematics genius. Some are born with savant syndrome while a few people acquire it through a brain injury as Jason did. His story is typical.

Padgett was an unmotivated college dropout, who pursued a rather aimless existence, focussed mainly on being a happy-go-lucky party animal. At the age of 31, he was robbed and severely beaten; lucky to be alive but burdened by a serious brain injury. During a painful recovery, his vision became bizarre. Everything he saw was geometric. Shapes invaded his consciousness but at the same time he was inspired by these awesome visuals. He began to draft intricate diagrams that his imagination would create. Several of these turned out to be extremely complex geometric arrangements that theoretical physicists interpreted as representing physics formulas, and what they understood to be solutions to advanced differential equations.

Of course, Einstein, Padgett, Seaberg, and most of us do not share the cult-like assumption that our thinking is somehow hard-wired into the physical world.

Just as Padgett's story seems alien to those who read it, school mathematics seems alien to students who have not yet decided to play within its ideology (i.e., they will neither try to understand it nor believe it), or alien to students who have not mastered its type of mathematizing in their everyday world (François & Van Kerkhhove, 2010).

Alienation also arises for many Indigenous students when mathematics is incorporated into school science, because the ideology of quantification tags along implicitly. In the words of chemistry professor Hogue (2011) of Métis ancestry:

> This fixation with quantifying science, in my opinion, is why Aboriginal people do not do as well in the sciences, particularly the physical sciences based upon measurement. It removes the *livingness* and as a result the experience, much like Residential School removed them from family and culture. (p. 307, emphasis in the original)

In the context of explaining the ways of thinking in high status scientific paradigms, Aikenhead and Michell (2011) pointed out the role of quantification in scientific thinking:

> The presupposition of quantification assumes that the material world is governed by objective mathematical relationships. Theoretical physicists are prone to say "the language of nature is differential equations."… Concepts such as the complexity of life, for instance, are not considered scientific unless they are measurable. (pp. 52-53)

René Descartes, a 17^{th} century founding contributor to the Age or Reason (a.k.a. the Enlightenment), wrote, "But in my opinion, all things in nature occur mathematically" (translated from Latin; https://en.wikiquote.org/wiki/Ren%C3%A9_Descartes). Interestingly, between the 17^{th} and 21^{st} centuries, I notice a change of tone from "in my opinion" to absolute or dogmatic assertions of the truth in quantifying nature (section 4.1.1).

I would emphasize that some students (Indigenous or non-Indigenous) will feel very comfortable with the ideology of quantification found in most school mathematics and physical science classes if their worldview harmonizes with a quantitative perspective that makes sense to their lived experiences (Aikenhead, 1997, 2006). For instance, Ed, who was a participant in Hogue's (2011) research with Canadian Indigenous science, mathematics, and engineering scholars, described how his gift in mathematics was noticed and encouraged at a very young age by his family. He eventually earned a Ph.D. in mathematics and is now a university professor.

In contrast to the ideology of quantification, Indigenous Elders generally live according to *qualitative* responsibilities to act in a good way, which trumps any decision to be controlled by quantitative symbols (numerals). Elders have not been colonized into allowing the ideology of quantification to dictate their behaviour. They are wise in knowing that the rectilinear concept of time is a European cultural invention (Bolter, 1984). Elders understand time as cyclical, sometimes in a spiral type of way (Aikenhead & Michell, 2011). Some wise Euro-American thinkers would concur to an extent: "Not everything that counts can be counted, and not everything that can be counted counts" (a sign purported to have hung on Einstein's office wall).

Even in Euro-American cultures, it does not make sense to ask, "What is an *average* high school student?" because the mathematics concept of average is irrelevant in this context (Hough, 2015, p. 24). In other words, calculating a mathematical average takes an *intellectual* understanding of school mathematics content, whereas knowing when and where to use that concept of average requires a *wisdom* understanding of the cultural context of school mathematics content. In other words, the wisdom component deals with, for example, mathematics' ideological-social-political context in which it is used. (Sections 4.1 & 4.2 explain how the wisdom component was suppressed when school mathematics first entered public education.) *Both the intellectual and wisdom components should be involved in understanding school mathematics*, for a number of reasons discussed in this monograph.

A distinction between intellectual and wisdom understanding represents another fundamental source of cultural clash for many Indigenous students. Indigenous cultures give much higher priority to the *wisdom tradition* of thinking, reflecting, doing, and being. When that view is normal for Indigenous students, imagine how disappointed they must feel when studying mathematics and discover the subject is *only* about an *intellectual tradition* of thinking; a much narrower view of education. Mathematics may seem inferior and largely irrelevant compared to an Indigenous student's taken-for-granted wisdom tradition of understanding. My colleague and I expressed the distinction this way (Aikenhead & Michell, 2011):

> Broadly speaking, an intellectual tradition emphasizes individual, critical, and independent cognition as exemplified by Descartes' famous dictum "I think; therefore I am." A wisdom tradition emphasizes group-oriented ways of being, exemplified by "We are, therefore I am" and by living in harmony with Mother Earth. The poet T. S. Eliot appreciated the distinction when he asked, "Where is the wisdom we have lost in knowledge?" (1963, p. 161). (p. 109)

From an Indigenous perspective, enriched by subjective relationships and responsibilities with everything in Mother Earth, objectification through quantification can show a lack of respect at the very least, and can border on oppression at worst, depending on the circumstances. That explains why, before contact with Europeans, most Indigenous groups had no need for an elaborate mathematics system (Elder Leroy Little Bear, personal communication, February 27, 1999); Mayan and Inka civilizations, for example, being exceptions.

On the one hand, analytical objectification, a component of the quantification ideology, can cause serious concerns for both Indigenous and non-Indigenous students whose wholistic[iii] qualitative worldviews do not resonate with the mathematical value of objectification. On the other hand, mathematical representations of events and mathematical problem solving are valuable cultural capital in many everyday circumstances and particularly in certain high-paying occupations. Thus, culture clashes must be resolved or at least mitigated for Indigenous students in order to help them escape the socio-economic cycle of poverty. Culture-based school mathematics has a proven potential for resolving or diminishing students' culture clashes (section 8). Fettes (2006) wisely concluded:

> There are many reasons why middle-class students from the culture that designs and runs the school system tend to do better in that system. Many of these reasons can't be addressed at the level of curriculum and pedagogy alone. The hiring of staff, communication with parents, expectations towards students, and many other aspects of school culture play a huge role. But there is no doubt that the time spent with teachers is central to children's experience of school. (p. 7)

When mathematics teachers respect Indigenous values explicitly and nurture students' cultural self-identities, learning school mathematics with academic rigor becomes easier and students do not need to devalue their cultural heritage. In addition, when examples of Indigenous mathematizing are included in a mathematics lesson, it has been shown to have a motivating effect on both Indigenous and non-Indigenous students. As a result, their mathematics achievement increases accordingly (Davison, 2002; Donald et al., 2011; Furuto, 2013a; Fyhn et al., 2011; Lipka et al., 2005; McKinley & Stewart, 2012; Perso, 2003).

Cross-cultural teaching materials that reveal the ideology of quantification to students should distinguish between what students *believe* (in the sense of making it part of their own worldview) and what they merely *understand* (in the sense of describing or using it accurately) (Aikenhead & Michell, 2011). For example, students should not be expected to believe the ideology of quantification, because that could be indoctrination. But they should be expected to understand it as pragmatic content that prepares them for living well in the dominant culture of society. If we deal solely with understanding, then we tend to reduce students' resistance to school mathematics by diminishing their culture-clash feelings. Thus, student assessment must be crafted to express this perspective explicitly; for example, when presenting a word problem to solve, use expressions such as, “How would a math person find an answer to this problem?” (See section 7.1 for other examples.)

This strategy of reducing culture clashes appeals to many high school students (both Indigenous and non-Indigenous) who see mathematics as simply foreign or inadequate to their way of thinking or their worldview. Perhaps if school mathematics were taught as if it were an economically potent foreign culture, then Indigenous students might see themselves as “math warriors,” able to appropriate rich knowledge from the dominant culture.

Intellectual clashes also stem from “ways of thinking” and “styles of communication” (Lunney Borden, 2013, p. 14). Divergent epistemologies have been detailed by Aikenhead and Michell (2011, pp. 99-120) and studied in depth by Bang and Medin (2010). Researchers and teachers who create teaching materials for Indigenous students need to be conversant with sources of cultural conflict described in this monograph, in order to develop scaffolding strategies to help students bridge what sometimes feels like an epistemic hiatus (Skovsmose, 2012; Stoet, Bailey, Moore & Geary, 2016). And researchers and teachers need to be mindful that “it is incorrect to believe that [Indigenous] students who speak English also think English” (Lunney Borden, 2013, p. 14) (section 6.4).

Most articles describing Indigenous students’ experiences with school mathematics focus on the achievement gap between Indigenous and non-Indigenous students’ test scores, which are invariably reviewed to confirm the gap’s existence. These data, however, only represent “the symptoms” of an underlying “disease” that causes those symptoms. Efforts that only address the symptoms (e.g., how do we help Indigenous students prepare for tests?) and ignore the disease (e.g., cultural power imbalances), produce insufficient outcomes.

The “disease” in this case is multifarious. As shown above, one facet lurks within the political-social fabric of society as systemic racism, and metastasizes throughout the conventional practices of schooling. Culture clashes are not a facet of the disease complex, but they will eventually lead us to an accurate diagnosis and treatment.

Within the disease complex that causes the achievement gap, our Euro-American culture’s belief about mathematics is a major issue; a topic we turn to in the next section. Other key issues are examined in due course.

4. Mathematics Clarified

Mathematics is as much an aspect of culture as it is a collection of algorithms.

Carl Benjamin Boyer (1906-1976) American historian of mathematics
Quoted from an unknown 1949 calculus textbook

Which taken-for-granted assumptions about mathematics continue to impede Indigenous students' achievement by creating culture clashes? Which deceptive assumptions have become arbitrary dogma? What are the alternatives? What role do values and ideologies play in mathematics? How are mathematics curricula related to reconciliation? These questions set the agenda for this section.

4.1. Some Fundamentals of Mathematics

As discussed in section 3.3, various culture clashes account for many students experiencing school mathematics as difficult to learn (Fowler, 2012; Nasir et al., 2008; Nichol & Robinson, 2000; Stoet et al., 2016), about "76 percent" of high school students in OECD countries (OECD, 2016, p. 362). The clashes are directly related to school mathematics' worldview-based belief system; that is, its ontological, epistemological and axiological presuppositions. These beliefs are embraced unconsciously by most mathematics educators, school administrators, Ministries of Education; and *therefore,* the beliefs are subliminally inculcated into the general public who have lived in mathematics classes during their youth.

Some time ago, Ernest (1988, website quotations) distinguished among three different sets of beliefs about the nature of mathematics:

1. an *instrumentalist belief* about mathematics as a utilitarian "accumulation of facts, rules and skills to be used in the pursuance of some external end."
2. a *Platonist belief* about mathematics "as a static but unified body of certain knowledge. Mathematics is discovered, not invented." In other words, a "doctrine that mathematical entities have real existence and the mathematical truth is independent of human thought" (Collins English Dictionary, 1994, p. 1193).
3. a *cultural belief* about mathematics "as a dynamically organized structure, located in a social and cultural context" for the benefit of problem solving; and a "continually expanding field of human creation and invention."

The instrumentalist belief is judged as being naïvely simplistic (Ernest, 1988, 1991; Ernest et al., 2016). Moreover, in Anyon's (1980) ground-breaking research into how mathematics teaching varies according to the social economic status of a school's neighbourhood, an instrumentalist approach was shown to be the pedagogy of choice in high-poverty working-class neighbourhoods. Being simplistic, classist, and racist (Jorgensen, 2016; Martin, 2006), an instrumental belief is ignored in this monograph.

4.1.1. The Platonist Belief

The ever popular Platonist belief is usually referred to in the educational literature as academic or conventional mathematics (Lipka et al., 2013; Lunney Borden et al., 2017). It characterizes mathematics as being value-free, decontextualized, acultural, non-ideological, purely objective in its use, always consistent, generalizable, universalist in the sense of being universally true, and thus *the only acceptable way of mathematizing*. This cluster of presuppositions has been analyzed critically by scholars other than Ernest (1988, 2016); for instance, Donald et al. (2011, p. 75) wrote: "Mathematics appears to be universal because of the prevalence of absolutist philosophies that view mathematics as timeless because it builds on logics of deduction." "Divine rationality" states Sriraman (2016, p. 2). Mukhopadhyay and Greer (2012) go a step further:

> The development of a multicultural and humanist view of mathematics challenges the supremacist position maintained by many mathematician educators who regard abstract mathematics as the crowning achievement of the human intellect, and school mathematics as the transmission of its products. (p. 860)

This supremacist position is championed by the National Council of Teachers of Mathematics (NCTM, 2000, p. 4) in justifying the need for mathematics education in a changing world: "Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects." (See section 10.1 for further discussions on NCTM.)

As indicated by Ernest's (1988) characterization of a Platonist belief (section 4.1 just above), a centuries-old philosophical debate underlies its foundation: Is mathematics *discovered* or *invented* by humans? Platonists champion the "discovered" position due to their belief that the universe itself is innately comprised of mathematical abstract objects; there to be discovered. Hence, mathematics must be universal, absolute, certain knowledge, beyond the influence of humans. For example, physicist Wigner (1960) justifies his Platonist ontological stance by pointing out the effectiveness of mathematics when physical scientists describe the universe. His belief has become controversial and more complex recently.

The “invented” side of the philosophical debate has countered with the following arguments (Hamming, 1980, pp. 87-89): (1) “humans see what they look for” (p. 87) (i.e., observations in science are theory-laden and technology-dependent); (2) “humans select the mathematics that fits a situation” (p. 89) and tend to ignore situations for which math is ineffective. So when scientists do find an effective mathematical idea, it would be fallacious circular reasoning to generalize as Wigner and colleagues do (Wolfram, 2013); (3) “Science in fact answers comparatively few problems. We have the illusion that science has answers to most of our questions, but this is not so” (p. 89); and (4) “Darwinian evolution would naturally select for survival those competing forms of life which had the best models of reality in their minds” (p. 89). Based on this last criterion, Indigenous knowledge systems triumph, due to their innate sustainability ontologies. Stewardship is for caretakers; sustainability is for care givers. By the way, Hamming concluded by sitting on the fence.[iv]

Wigner’s (1960) focus on *scientific* applications of mathematics draws attention away from a principal issue in this monograph, the *cultural* applications of mathematics; in other words, treating mathematizing as cultural practices developed (invented) by humans. This view is ignored by Platonists when they adhere to Plato’s words, “The highest form of pure thought is in mathematics.” Accordingly, culture is “tainted” by humanity; thus it can be ignored in comparison to the universe’s abstract objects that comprise pure thought. *Wigner has made a personal judgement that scientific applications of mathematics are superior to cultural applications.*

One significant cultural application of Platonist mathematics is the fact that political-social power groups in many countries maintain the high status Platonist mathematics enjoys in education (section 3.2). This fact might sway some mathematics educators to side with Wigner. The power exerted by a Platonist belief is critically analyzed in greater detail in this section, in sections 4.2 and 9.1, and throughout this monograph.

As a result, the 19th century conceptualization of mathematics (section 4.2.1) taught in schools is now open for a 21st century renegotiation. What should mathematics be? Sections 4.1.3, 4.2, 8, and 9 have evidence-based answers to consider.

Bishop (1990) revealed inconsistent reasoning expressed by many Platonist mathematics teachers: “[T]o decontextualize in order to be able to generalize, [that] is at the heart of Western mathematics” (p. 57). Platonist mathematics teachers will deny that school mathematics content is value-laden, and at the same time, they obviously cherish two of its *epistemic values*: decontextualization and generalizability. More examples of values are found in section 4.5.

The philosophies of mathematics aside[v] (i.e., making room for a *cultural* application of mathematics), a Platonist belief creates an image detrimental to many students' participation and achievement in school mathematics (Fyhn, 2013). Indigenous cultures, for instance, generally share presuppositions characterized as value-laden, contextualized, cultural, ideological, mostly subjective, and embracing multiple truths. Thus, a curriculum defined by a Platonist belief alone exacerbates the culture clash faced by most Indigenous students. (A detailed explanation is found in section 4.4.)

In addition, a Platonist belief promotes, very subtly yet powerfully, its universalist value that logically dismisses Indigenous mathematizing; thereby maintaining the colonial-based power imbalance between the mathematics curriculum and Indigenous communities.

Moreover, a Platonist belief ossifies privilege-blindness, thereby masking power with Platonic innocence. Its popularity in schools tends to rest on its claim to objectivity – the opiate of the academic (Aikenhead, 2008). In various contexts of this monograph, I draw attention to pervasive Platonist presuppositions that impose a neo-colonial stance on Indigenous students and a doctrinaire influence on all students (Doolittle, 2006; Doolittle & Glanfield, 2007; Meaney, 2001).

Anthropologist Hall (1976) asserted that Plato's philosophical concept "purity of mind" (p. 192) and his philosophical assumption that the universe make up of abstract objects, amounted to an intellectual mirage: "*What has been thought of as the mind is actually internalized culture*" (p. 192, original emphasis). In other words, physicist Wigner's (1960) promotion of Platonist ideas can be understood as internalized culture from the ancient past.

4.1.2. Towards a Cultural Belief

Some mathematics educators and researchers have chosen to replace the Platonist belief with a *cultural belief* or *culture-based* understanding of mathematics, with which to enhance school mathematics for the specific benefit of Indigenous students (section 8). Even though these plans and actions have been implemented, many innovative educators and researchers have yet to modify all their concepts and expressions to accurately match their intention to implement a cultural belief. Therefore, their language has not completely shifted from conveying a Platonist belief to a cultural belief, likely for at least two reasons. First, they use the old Platonist language simply out of habit, without critically reflecting on such expressions as "mathematics is all around us" (section 9.3) Secondly, they wrongly assume that students either share a worldview that harmonizes with their worldview, or students' minds are a tabula rasa.

When mathematics educators “see” mathematics all around them, they assume all students “see” what they “see.” (The quotation marks signify that in this context the word means to conceptualize; sections 4.4 & 9.3.) If a high school student challenges, “Take me to the mall and show me a quadratic equation;” a teacher has discovered a miscommunication on the teacher’s part. Arithmetic is all around us, but for about “76 percent” of high school students in OECD countries (OECD, 2016, p. 362), not the abstractions like quadratic equations.

When mathematics teachers’ and researchers’ language does not match their plans and action, they project mixed and confusing messages that interfere with the quality and success of their teaching or R&D project.

Unlike the Platonist belief, a cultural understanding of school mathematics aims to position students so they experience mathematics as a human endeavour (Ernest et al., 2016; François & Van Kerkhove, 2010; Russell, 2016; Skovsmose, 1985, 2005). “[W]e take it as self-evident that mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context” (Skovsmose & Greer, 2012, p. 379).

As such, mathematics is rooted in the culture of those who created the knowledge system (Ernest 1991), and it can be related meaningfully to students’ cultural self-identities (Chronaki, 2011; Ishimaru et al., 2015; Nasir, 2002). Bishop (1988b, p. 155) recognized this double function when he stated that mathematics is a product of its developer’s culture; and “as a cultural product, [it] is now strongly shaping Western culture as a whole.”

Bang and Medin (2010) offer a pragmatic notion of the term “culture” (Gutiérrez & Rogoff, 2003; Rogoff, 2003) that fits well with the goal of culture-based school mathematics.

> Although the construct of culture is problematic, people nevertheless “live culturally”…[by way of] a wide repertoire of sense-making *practices* that people participate in, particularly in everyday contexts. …This understanding of culture implies that there is no cultureless or “neutral” perspective any more than a photograph…could be without perspective. In this sense, everything is cultured. (pp. 1014-1015, emphasis added)

To experience mathematics as a human endeavour is to engage in a repertoire of its sense-making cultural practices (Boylan, 2016). For example, Russell (2010) asks:

> Why is it that the knitting of socks for families, with the ever changing lengths, widths, and foot shapes, is relegated to non-mathematics (it is women’s work), while aspects of pipe fitting and pipe design that use the same mathematical ideas and are recognized as “real” mathematics? (p. 36)

> Is a sign of "real" mathematics that its vocabulary does not have a connection to the concrete? What makes this a more intellectual stance? Perhaps within the ancient Greek belief that mathematics is meant for the elite. (p. 39)

Bang and Medin's (2010) view of culture as everyday practice, and Russell's analysis of mathematics as cultural practice, lend strong credibility to teaching and producing projects in which students must negotiate, supported by their teachers, among multiple cultural ways of understanding, such as an Indigenous culture and the culture of school mathematics.

4.1.3. Mathematical Pluralism

In a scholarly milieu of Cambridge University, informed by contemporary ideas such as mathematics as a cultural system (Smorynski, 1983; White, 1959; Wilder, 1981), Bishop (1988b) helped identify the cultural nature of mathematics as a pan-human activity:

> Recently, research evidence from anthropological and cross-cultural studies has emerged which demonstrates convincingly that the mathematics which we know is a culture-bound phenomenon, and that other cultures have created ideas which are clearly 'other mathematics'. (p. 145)

Worldwide, the cultural product of mathematics develops in tandem with people's everyday cultural activities. When the origin of various mathematics is linked to activities – mathematizing processes – then mathematics linguistically connects with Indigenous people's verb-based ways of expressing themselves (Ascher, 1991; Fyhn, 2013) (section 6.2). Indigenous students will benefit from a mathematics curriculum and pedagogy that convey this cultural activity perspective.

Bishop synthesized Bruner's (1964) and Vygotsky's (1978) work on how language can act like conceptual *tools*, including mathematical *symbolism*. "Mathematics, as an example of a cultural phenomenon, has an important 'technological' component," Bishop argued (1988b, p. 146). He then concluded that all mathematical cultural activities "relate to the physical and social environment in some way and…the functions of this symbolic technology called mathematics are concerned with relating [humankind] to [the] environment in a particular way" (p. 147). Bishop's general characterization of mathematics can be distilled into the following definition:

> *In any culture* (including Euro-American cultures), *their mathematics is a symbolic technology for building a relationship between humans and their social and physical environments*.

Mathematics taught in schools “cannot be separated from ‘Western’ cultural and social history” (Bishop, 1988b, p. 151) without misrepresenting a fundamental feature of the subject (Matthews, 2015); a feature that significantly reduces culture clashes for many students, especially Indigenous students. It must be emphasized: Bishop’s implication for Euro-American nations is that teaching school mathematics will be *immersed in the cultural context of their Euro-American society*. This implication, however, seems so foreign that it is beyond the conceptual grasp of most mathematics educators cited in this monograph. Their blind spot prevents them from authentically representing the connections between school mathematics and the school’s social environment; an authenticity that would make school mathematics meaningful to most students (Ernest et al., 2016). Instead, myths and dogma about mathematics being acultural – separated from the school’s cultural environment – consequently prevail (sections 4.2.1 & 7.2). This denies Indigenous students their full potential to succeed at school mathematics.

Bishop’s (1988b) research conceptualized six foundational or universal mathematizing processes (cultural practices) in mathematical systems created by various major cultures throughout history: counting, locating, measuring, designing, playing, and explaining. Very briefly, each type of cultural process is described here for the benefit of mathematics teachers and researchers who plan to collaborate with Indigenous communities to produce relevant teaching materials. Each of the six processes suggests a wide range of activities to be found in a local Indigenous culture, as well as in the local mainstream culture (section 4.5). By becoming familiar with these, teachers and researchers will be cued into an early step in producing successful teaching materials and lessons. Subsequent steps are introduced later in section 4.4.

Counting systems can be numerous within a geographical area that hosts multiple cultures. The diversity of counting systems is so rich and nuanced that their popular dichotomy of civilized versus primitive is meaningless. The diversity reflects environmental interactions in which humans are, or were, engaged.

Locating encompasses “mapping, navigation and the spatial organisation of objects” (p. 148), which leads to key mathematical ideas, such as geometrical concepts and what we call compass points.

“*Measuring* is concerned with comparing, with ordering and with valuing” (p. 148, emphasis added). Body-based measuring activities are found in all cultures. Bishop also mentioned the distance unit known as a day’s travel. Such units of measurement can depend on the space-time component of a culture’s collective worldview, a concept that varies greatly among cultures (Duran, 2013). For example, an authentic translation of the English question, "How far is it from here to Regina?" in the Lakota language is "When is Regina?" (Lakota Elder Darlene Speidel, June 23, 2016, personal communication). Obviously Eurocentric cultures do not share the Lakota Nation's space-time measurement concept.

Designing considers "aspects of the designed form without having to actually make the object" (p. 149). This involves the "perceived spatial relationship between object and purpose" (p. 149). Because designing requires abstracting, it is recognizable in Western mathematics as dealing with "shape, size, scale, ratio, proportions and many other geometric concepts" (p. 149).

Playing relates to mathematizing in the design and rules of a game, often documented by a number of anthropologists. Bishop describes it as an abundant Indigenous source for developing teaching materials.

Explaining, a "meta-conceptual characteristic" (p. 150) of mathematics, is most often manifested as story, in which logical connections are made, such as formulating or responding to a proposition. Our notion of mathematical proof stems from a need to explain in a Eurocentric mathematizing way.

Throughout history, the intellectual invention called "mathematics" has produced radically different, culture-based, mathematical systems that have built symbolic relationships between humans and their environment. Consider, for example, Indigenous Australian mathematics in use tens of thousands of years ago (Watson & Chambers, 1989), Yup′ik mathematics (unknown date; Lipka et al., 2013), Mayan mathematics (about 2,000 years ago; NOVA, 2016), Polynesian mathematics (about 1,000 years ago; Ball, 2013), and advanced Japanese mathematics (about 400 years ago; Wikipedia, 2016).

Every major culture has had a mathematics knowledge system; therefore, many different mathematics knowledge systems exist; ergo, mathematics is culturally *pluralist*, but not relativist. Platonists value binaries very highly, for example, universalist or relativist, and nothing in-between. But pluralism rests comfortably in-between. Pluralism is simply a logical implication of Bishop's (1988) characterization of mathematics.

From a grammar perspective, this means that the term "mathematics" functions as a *superordinate* concept, representing a set of multiple mathematics knowledge systems that grammatically function as *subordinate* concepts. By analogy, the term "furniture" is a superordinate concept with respect to the subordinate concepts: chair, table, bed, and book case.

Ogawa (1995) established the same distinction for a superordinate "science," elaborated upon by Aikenhead and Ogawa (2007; p. 566-567) who identified as subordinate sciences: Eurocentric science, Indigenous ways of living in nature, Islamic science, and Japanese ways of knowing seigyo-shizen. Similarly, Sternberg and McDonnell (2010) focused on three categories: Western, Indigenous, and personal mathematics in their study of students' experiences learning mathematics from place. However, their narrow definition of Western mathematics – "ideas of space, of time, of numbers, of relationships" (p. 12) – exposes their blind spot, described just above (section 4.1.3), which makes invisible the connections between the content of school mathematics and Western culture, to the partial detriment of Indigenous students' success (Russell, 2010) (section 4.2.1).

Mathematical pluralism directly relates to epistemological pluralism in general, and it could serve as an anti-hegemonic mathematics specifically. Fasheh (2012) explains:

> Searching for universals today is not treated as a search that adds to our understanding but it has become a list of facts that are imposed on peoples and countries as the only absolute truth. [We are compelled] to regain "pluralism" as a most distinctive characteristic in life, nature, human beings, and knowledge and to realize that universals are contrary to humanity and Nature. (p. 100)

For clarity and accuracy, the context of pluralism requires that the mathematical system conventionally taught in schools and universities to be *identified by its cultural association*. And what is that cultural association?

4.2. Yousan

An answer is found in the following historical evidence. Japanese mathematics that Japanese people call "Wasan" (和算) effectively served Japan's highly sophisticated Edo society period, 1603-1868. When Japan was suddenly forced by American navy gunboats to join the economic globalization movement, in 1868, a mathematical system Japanese people called "Yousan" (洋算) was introduced officially into Japan. Yousan is taught in schools worldwide today.

Historically, Yousan had been developed within Euro-American cultures through invention (Ernest, 2016b) and much appropriation from other cultures over time (e.g., Babylonian, Egyptian, Greek, Hindu-Arabic and Chinese cultures) (section 4.4).

A Japanese point of view is revealed by an approximate literal translation of "Yousan" into English: foreign over the ocean calculation (Uegaki, 1990). Japanese scholars realized that Yousan was a cultural creation that harboured ideologies, values, and perspectives foreign to those of Japan's (Kawasaki, 2002). For example, the Yousan counting system uses the same names for numbers no matter what is being counted; "Mathematics is the art of giving the same name to different things" – Henri Poincaré, French mathematician, early 20th century, cited in Verhulst, 2012, p. 154). On the other hand, the Wasan counting system, which continues today to some degree, has different number names depending on the group of objects being counted. For instance, number names differ somewhat when counting: people, ages, kitchen cupfuls of ingredients, long skinny things, thin flat things, clothes, etc.; all of which subtly relate to an endemic Japanese worldview.

According to Japanese common sense, therefore, Yousan was consistent with Ernest's (1988) *cultural belief* about mathematics, and its cultural association was indeed Western, or more accurately, Euro-American. But Platonist content was also present in Yousan, of course. As a result, Yousan combines Ernest's two categories, a *Platonist belief* and a *cultural belief* about mathematics, in a yin-yang paradoxical way. This means that both beliefs function simultaneously. They are an amalgam. One belief is not subsumed within the other belief.

Accordingly, I propose to replace the literal translation of Yousan ("foreign over the ocean calculation") with a more precise translation: "*Euro-American mathematics*." The phrase expresses: a cultural association, a pluralist stance, and an amalgam of the *cultural* content of mathematics along with its conventional *Platonist* content. Andersson and Ravn (2012, p. 313) describe this relationship as "intertwined and intimately connected and can only secondarily be divided into different domains."

Grammatically speaking, Euro-American mathematics (EAM) is a subordinate concept to the superordinate mathematics. Moreover, *EAM articulates a meaningful concept of what can be taught in school mathematics; one that leads naturally to updating school mathematics applicable to the 21st century, for the benefit of most students*.

If what is taught in schools is just called "mathematics" or "Mathematics," then its cultural association is supressed in accordance with the Platonist belief (section 4.2.1), and consequently the name continues a hegemonic agenda that claims school mathematics is acultural, value-free, non-ideological, universalist, and no other mathematical system is worth considering (Greer & Mukhopadhyay, 2012). In short, from an Indigenous perspective, the school subject's name dismisses Indigenous mathematizing.

According to Bishop (1990, p. 1), Western (i.e., Euro-American) mathematics is “one of the most powerful weapons in the imposition of Western culture." This agenda of power, enacted through the ideology of quantification among other ideologies (section 4.5), is one aspect of the culture of conventional school mathematics that tends to alienate Indigenous students, thereby ultimately contributing to: high dropout rates in schools, the cycle of socio-economic poverty, and its concomitant devastating consequences for the wellbeing of students and their communities.

A Haida First Nation carpenter identified this cycle of oppression (Neal, 2011):

> In Aboriginal communities across the country, there is a system of institutionalized racism at work. What I see is that Aboriginal people that have gone through the school system have not had a very good experience. So they return to their communities with their bad experiences; their kids continue to struggle through it and they probably have bad experiences. (p. 121).

Thus, many Indigenous students enter mathematics classrooms saddled with an “educational debt” (Ladson-Billings, 2006, p. 3) caused by mainstream schooling’s institutionalized racism implicitly conveyed by conventional mathematics curriculum and instruction. Breaking the cycle is a high priority agenda item for 21st century school mathematics in an era of reconciliation.

4.2.1. A Hidden Platonist Agenda

When and how did the Platonist belief become ensconced in the school mathematics curriculum? How has the Platonist belief prevailed to this day? Who is served by it? Can conventional school mathematics evolve to become in tune with today’s political-social and educational realities (sections 2 & 3)? A thumbnail historical sketch initiates this discussion.

Emerging from Renaissance Europe, radically new institutions of learning called “universities,” the generators and keepers of humanity’s knowledge, grew in size, number, and power at the expense of monasteries of the time.

Mathematics (the superordinate meaning of the term) (section 4.1.3) evolved over the ages in various civilizations (section 4.4), and then slowly found a home in elite universities during 17th century England and 18th century America, among other countries. During this Age of Reason (a.k.a. The Enlightenment), mathematics had to compete with the classics, history, and ancient languages for a place in the curriculum. At that time the philosophy of education was simple: subject matter difficulty prepared the male mind for any future event, occupation, or profession (Willoughby, 1967). Females were not in the picture. Mathematics, with its deductively proven theorems based on fundamental axioms, became the poster boy for the Age of Reason. Its high degree of difficulty caused mathematics to flourish in the competition for a place in the school curriculum. Elite Latin Grammar Schools began to teach arithmetic as a prerequisite for university entrance, then came algebra, geometry, etc. (Willoughby, 1967).

During the 19th century in the era of Queen Victoria (1819-1901), the Industrial Revolution caused the development of the British public education system, quickly adopted in Canada. Mathematics would be a core subject, but which version: the elite academic mathematics that had no context, or a practical relevant mathematics that would connect students with their real world around them (Layton, 1981; Nikolakaki, 2016)?

The elitist Platonists, channelling ancient Greek beliefs in the superiority of pure abstract thought (Plato's World of Ideas), insisted on *decontextualized* content. This view is consistent with the idea that mathematics content is *discovered* as abstract objects that constitute the universe, not invented by humans (section 4.1.1). Platonists eschewed worldly practical knowledge (Plato's Phenomenal World) connected with human activity, which would produce a curriculum of *contextualized* content. These two views mirror the ancient Greek class-based society: the elite aristocracy versus the artisans and slaves.

The Platonists' proposal for the public school subject of mathematics mimicked the elite 19th century Latin Grammar Schools. They won the battle over their utilitarian colleagues by using a deceptive strategy for defining their school subject; a strategy that rendered itself invisible to future generations – a hidden agenda at work, perhaps?

Pais (2012) pointed out the hidden agenda's effect on today's mathematics teaching and research.

> This concealment is essential to maintain the role of school as an ideological state apparatus. Seeing school as a place free of ideology disables bringing ideological struggles to school. All enterprises undertaken by teachers to unmask the "invisible" ideology are immediately accused of being ideological acts. In this way, the dominant ideology ensures that no ideology is present in school except, of course, the dominant one. The dominant one is precisely the one that presents itself as ideologically free, by positing the importance of mathematics as knowledge and competence [for citizens]. (p. 70)

The 19th century Platonists' reach into the 21st century to stifle much needed innovation speaks to the urgency to explore this hidden agenda further in order to challenge its non-ideological ideology.

At the close of the previous subsection, Bishop (1990, p. 1) connected "Western mathematics" directly to the act of colonization. D'Ambrosio (1991, p. 12) concurred: Euro-American "[m]athematics is also the imprint of Western culture." Its cultural identity expresses itself, for instance, when it is used to control people. A Platonist objection to this assertion may be that mathematics does not control people, *authorities* in power control people by making decisions based on mathematics. This response parallels: "Guns don't kill people, people do." This objection, a rhetorical device for dismissing the cultural identity of Platonist mathematics content, is known as the "fallacy of mistaking the relevance of proximate causation" (Johnson, 2013, website quotation).

Ernest (1991, p. 259) critically countered with a more sophisticated response to the Platonists' belief "in the absolute objectivity and neutrality of mathematics." Platonists vehemently claim that mathematics is value-free, in spite of a public repository of mathematics values and ideologies (already mentioned, but expanded upon in section 4.5). Ernest answered, "[T]he values of the absolutists are smuggled into mathematics, either consciously or unconsciously, through *the definition of the field*" (p. 259, emphasis added). A deceptive hidden agenda?

Ernest is referring to the following rhetorical sleight of hand[vi]. First, the Platonists drew on a binary, "logical versus irrational," invented by "Western culture dating back to Socrates, Plato, and Aristotle" (Hall, 1976, p. 213), in order to construct their own theoretical binary: formal mathematical discourse versus informal mathematical discourse.

Then they arbitrarily assigned the highly abstract decontextualized aspects of 19th century European mathematics to the *formal* discourse category, which they declared the discipline of school mathematics. It contained, for instance, deductive proofs and theorems, geometry, and the scientific application of EAM Platonist content.

The *informal* mathematical discourse category comprised everything that made their subject a human endeavour; for example: its societal contexts, its ideologies and values by which it operates, and its preferences that guide mathematicians; that is, EAM cultural content.[vii]

Because Platonists bifurcated (formal versus informal) what had once been just European mathematics, it allowed the developers of the 19th century public school curriculum to lay claim to mathematics' formal discourse, and to jettison the informal discourse into obscurity. This, I believe, is the deceptive hidden agenda. What does Einstein think?

Einstein (1921, website quotation) provided a parallel explanation by noting a "new departure in mathematics which is known by the name of mathematical logic or axiomatics." Axiomatics concerns axiomatic knowledge systems. The truth and certainty of a mathematical theorem is based on a set of axioms, from which those theorems were derived by the logic of deduction. Lakatos (1976) describes deduction this way:

> In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever increasing set of eternal, immutable truths. Counter examples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguised monster-barring and proof-generated definitions and with the fully-fledged theorem, and by suppressing the primitive conjecture, the refutations, and the criticism of the proof. (p. 142)

In other words, an axiomatic system is a self-consistent system by design. Truth of its theorems, therefore, is *relative to* the set of axioms from which the theorem was deduced. Hence, each theorem's truth-generating power is specifically relativistic. Although an axiomatic system conveys an aura of truth and certainty, the sum of relativistic axiomatic theorems comprising mathematical formal discourse seems to make that formal discourse somewhat *relativistic*.

Einstein (1921, website quotations) also wrote that "logical-formal" content was defined as an axiomatic system, and therefore, this definition separated "logical-formal" from "intuitive" content. "[A]ccording to axiomatics the logical-formal alone forms the subject-matter of mathematics, which is not concerned with …intuitive or other content *associated with* the logical-formal" (emphasis added).

The explanations from Ernest and Einstein converge. In essence, Platonists' strategy rests on an arbitrary, Age-of-Reason type of social licence to *define* their Platonist mathematics content as axiomatic systems only (i.e., decontextualized, value- and ideology-free within closed, self-consistent, deductive systems). Ernest (1991) hints at a magic trick: "[A]t the heart of the absolutist neutral view of mathematics is a set of values and a cultural perspective, as well as an ideology which renders them invisible" (p. 260). This ideology is the Greek-based European impulse to reify and vigorously promote arbitrary binaries that only exist in the mind.

Is this a case of masking power with a rhetorical sleight of hand? *In the context of educational policy and curriculum development* for schools, has the philosophic absolutists' social licence not expired well before the 21st century? In today's political-social context of reconciliation dedicated to rid society of the same type of Age-of-Reason social licence that enacted residential schools (section 2.1), is it not time to revoke the Platonists' social licence that needlessly depresses high school graduation rates of Indigenous and non-Indigenous students? Because the composition of school mathematics was mainly established by an arbitrary 19th century definition; it seems reasonable that it could be redefined by a 21st century definition.

The issue can stir emotional reaction because the culture of school mathematics is tied up with graduation requirements and people's professional self-identities and worldview-based beliefs (section 3.2) – people who have had the power to control the school mathematics curriculum (section 10.3).

The above critical analyses of Platonists' hidden agenda not only expose fallacious reasoning, rhetorical tricks, and political deception, the analyses echo the lived experiences of Indigenous and non-Indigenous students in school mathematics classrooms. By doing so, the analyses help set mathematics education on a path toward a negotiated sharing of power between mathematics curricula and Indigenous and non-Indigenous communities.

This coincides with Recommendation 10 from Canada's Truth and Reconciliation Commission (2016a): "improving education attainment levels and success rates" and "developing culturally appropriate curricula" (p. 165). Examples of restoring a more equitable balance of power are highlighted in section 8.

I place a 21st century challenge at the feet of mathematics educators, education administrators, and curriculum developers, especially in Canada and Australia where their Prime Ministers officially apologized to the country's original peoples for the devastating treatment and cripplingly consequences of residential schools. I understand it is now time to revise, regain, and reconceptualize school mathematics' curriculum content in terms of cultural practices, thereby replacing its 19th century Platonist belief with a 20th century Euro-American cultural belief contextualized by reconciliation.

4.3. Definitions Clarified

Now that this monograph has explicitly taken on a contemporary cultural belief about mathematics, I pause to acknowledge that some words and expressions have taken on a more precise meaning. Definitions are required.

"*Mathematics*," without a qualifier, *will have a superordinate meaning only*, the content of which is all the mathematical knowledge systems that have developed historically in specific cultures. In grammatical terms, these knowledge systems, such as Japanese mathematics (Wasan), carry a *subordinate* meaning of "mathematics," which always requires a qualifier (e.g., Japanese) or a totally unambiguous context.

Another existing subordinate knowledge systems could accurately be named "*Euro-American mathematics*" (EAM). Its content draws from content taught in schools and universities (Platonist content) wholistically integrated with the content associated with EAM's cultural identity (e.g., its political-social-economic-military functions, its history, and its ontological, epistemological and axiological presuppositions; Corrigan, Gunstone, Bishop & Clarke, 2004; Bishop, 1988b, 1990; Donald et al., 2011; Ernest, 1988, 1991, 2016; Skovsmose, 1985, 2005, 2012, 2016). This cultural content is suppressed by the Platonist belief about the nature of mathematics (sections 4.2 & 4.5). Thus, EAM content has amalgamated (intimately intertwined) what conventionally was explicit (Platonist) and implicit (cultural), but will all now be explicit according to the intellectual maturity of students.

For the sake of clear precise communication, EAM content will continue to have separate names (Platonist content and cultural content) although in practice they are wholistically interwoven just like Indigenous epistemologies would conceptualize them. Features of EAM cultural content appear in sections 4.5, 7.1, and throughout this monograph.

A second point of clear communication needs to be mentioned: What is the consequence of *not* distinguishing precisely between the superordinate and subordinate meanings of the term "mathematics"? The answer is fuzzy or confused reasoning. If someone unconsciously switches between the two, then any conclusion they reach is likely illogical; a situation Aristotle labelled the Fallacy of Equivocation (Philosophy Department, 2017). For example (Neel, 2011), in Haida Gwaii, the

> director of the Swan Bay Rediscovery Program, mentions that the staff "teach concepts that can be tied to [students'] culture – anything from just simple food gathering to safe hunting protocol to reading the tides. All the activities require a lot of problem solving and focus on *mathematical* skills" (p. 121, emphasis added).

Is the director talking about Haida or Euro-American mathematizing skills (both represent the subordinate meaning of mathematics) or skills to be found in most major cultures (the superordinate meaning)? Or has the director confused himself and others by insinuating that the expression "mathematical skills" refers to Platonist content and is therefore misunderstood as *superordinate* because Platonist content has been defined by the Platonist agenda as being *universal*? As evidenced in the mathematics research literature, the Platonist agenda frequently creates such fuzzy or illogical reasoning for those not vigilantly critical of the term “mathematics” (sections 9.1, 9.2, & 9.3). In short, Platonists claim the term “mathematics” as theirs alone; thus perpetuating the myth that Platonist mathematics is the superordinate meaning of mathematics. In precise terms, perhaps what the director meant to say was not “focus on mathematics,” but “focus on EAM Platonist content.” The former insinuates oppressive treatment of Indigenous students; the latter, culturally responsive treatment.

To summarize, from this point forward I shall use the following expressions as defined here:

- EAM or EAM content – both Platonist content and EAM’s cultural identity content combined as an amalgam
- EAM Platonist content – the conventional content currently taught in most schools and universities (representing an intellectual understanding)
- EAM cultural content – the cultural identity content of EAM (representing a wisdom understanding): EAM’s ideologies, values, and presuppositions; as well as its influence on the political-social-economic-military contexts – mathematics-in-action[viii] (Skovsmose, 2005) (sections 4.5 & 7.1).
- EAM educators – educators who deal with EAM content explicitly.

Although EAM cultural content has been suppressed, it can easily become explicit when, for example, Indigenous mathematizing is involved in a lesson, or when a teacher is diagnosing a student’s reluctance to learn EAM Platonist content due to culture clashes.

The expression “s*chool mathematics*” will refer to EAM content principally, in keeping with Ernest’s (1988) “culture belief” category. To identify school mathematics that does not include cultural content, the phrase “*Platonist school mathematics*” will appear, meaning the same as conventional or traditional school mathematics.

When school mathematics does combine EAM content with content from another subordinate mathematical knowledge system, such as Mi′kmaw mathematizing, then “school mathematics” will have a qualifier such as *cross-cultural* or *culture-based* school mathematics; as in “cross-cultural Euro-American mathematics.” The phrase “enhancing school mathematics culturally” will have the same meaning, that is, focussed on both EAM cultural content and Indigenous mathematizing.

For Indigenous groups, such as the Mi′kmaw First Nation, I use the action form – mathematizing – to convey their verb-based language-laden cognition (section 6.2).

Knowing that mathematics knowledge systems in some ancient cultures have contributed to today's EAM Platonist content, we could ask: How have these cultures all but disappeared in school mathematics, making it appear as if it is acultural? An answer lies in the Eurocentric habit of appropriating from other cultures, a topic addressed in section 4.4.

Of course, contemporary appropriation and marginalization are processes to be avoided when producing teaching materials for culture-based school mathematics. These two issues are addressed in sections 6.1 and 6.2, which draw heavily from the following discussion explaining a mechanistic process of historical appropriation.

4.4. Historical Appropriation

About 5,000 years ago, "mathematics had ritual functions" (Ernest, 2016b, p. 382) as well as record keeping, accounting, and land surveying. "[T]he reliability of calculation, measures and numerical records was also understood as part of the idea of justice, taking on ethical as well as utilitarian and ultimately epistemological value" (p. 382). Ernest added:

> By the time of the Pythagoreans, elements of number mysticism had developed. …Three centuries later, Plato conceptualized numbers as self-subsistent entities existing in an ontological category apart from things bodily and mundane. Plato's ontology included numbers as ideal abstract forms, unchanging entities that could be known with certainty. …A thousand years later, the emergence of modern science in the work of Galileo, Descartes, Newton and others prioritized numbers as the real existents behind our sensory experiences. Numbers were understood as invariant objects that characterized the most reliable and permanent knowledge of the world, part of the structure of the universe. (pp. 384-385)

The Eurocentric impulse to appropriate from other cultures can account for how European mathematicians throughout the centuries seem to have imported ways of mathematizing from earlier cultures, but then reconstructed those ideas to fit the European mathematical philosophy or ideology of the time. How does this appropriation process occur exactly?

A mechanism is constructed out of Kawasaki's (2002) linguistic domain of *language-laden cognition*. It reveals that a word's meaning in one culture's language (e.g., renaissance European) is attached to a cluster of associated concepts related to that culture's collective worldview. For instance, to understand an English or French concept beyond superficiality is to be aware of some of its associated concepts. A non-English or non-French language (e.g., Arabic) will reflect a very different collective worldview and likely a different cluster of peripheral concepts that create a connotation of a word or expression. This perspective resonates with how Ojalehto and Medin (2015) characterize the relation between culture and concepts:

> Emerging trends in semantics, agency, and causal concepts converge on the idea that individual minds are grounded in systems of social relations, from the languages we speak to the (cultural) practices we engage in, and concepts must be understood as elements of those systems. (p. 9)

As a contemporary illustration of peripheral concepts, consider the following Grade 9 indicator in the Saskatchewan Mathematics Curriculum (2007, emphasis added): "Describe examples of where First Nations and Métis, past and present, lifestyles and worldviews demonstrate one or more of the *circle properties* (e.g., tipi and medicine wheel)." Note that the phrase "circle properties" refers to a *decontextualized* meaning of the term "circle;" with its cluster of peripheral Platonist concepts such as point and plane, as well as associated values such as decontextualization, intellectual purity, consistency, and objectivity – all examples of Platonists' public façade. The phrase "circle properties" does not refer to the Indigenous concept of something circular (in Plains Cree, wâweyiyâw). Instead, wâweyiyâw has peripheral concepts of a Plains Cree community's subjective, wholistic and spiritual meanings of something circular. The connotation of "circle" differs between the two cultures.

The curriculum's indicator merely encourages students to *superimpose a EAM concept* (e.g., circle) *onto Indigenous artifacts or processes*. This neo-colonial act of projectionism is revisited with other examples in section 9.3. Doolittle (2006, p. 20) criticized projectionism because it ignores substantial peripheral concepts:

> "The tipi is a cone," I have heard countless times. But that is surely wrong; the tipi is not a cone. …It bulges here, sinks in there, has holes for people and smoke and bugs to pass, a floor made of dirt and grass, various smells and sounds and textures. There is a body of tradition and ceremony attached to the tipi which is completely different from and rivals that of the cone.

Without identifying "a body of tradition and ceremony" as peripheral concepts, mathematics professor Doolittle of Mohawk ancestry illustrated their importance for when we move between the culture of EAM and the culture of an Indigenous community.

Peripheral concepts may subtly address three different types of cultural presuppositions, stated here with Indigenous examples: (1) *ontological*; dependent relational existence, and sacredness; (2) *epistemological*; wholistic and paradoxical reasoning (Maryboy, Begay & Nichol, 2006); and/or (3) *axiological*; "community solidarity, respect for the earth, and respect for elders" (Lunney Borden, 2013, p. 9); and Native Hawaiian values of: mākana (to care for), lokomaika′i (to share with each other) and olakino maika′i (to live healthily) (Furuto, 2013a, pp. 42-43). Meaney (2002, p. 17) explained, "The mathematics register carries with it values which influence how mathematics is perceived." More examples are found at the end of section 6.4.

A worldview encompasses ontological ideas of "what ought to exist" (Kawasaki, 2002, p. 26). For European mathematical thinkers who speak Standard Average European (SAE) languages (Whorf, 1959), their ideas fit Plato's "World of Ideas," characterized by the purity of universalist objectivity that celebrates abstract nouns, as Kawasaki (p. 25) explains. For non-SAE mathematicians whose first language is Egyptian, Hindu, Arabic, or Chinese, for example, their "what ought to exist" tends to be more like Plato's "Phenomenal [material, tangible] World" that celebrates subjective placed-based processes or action. For each group, their peripheral concepts will reinforce their cultural ontological stance.

The historical European appropriation from ancient cultures is a translation issue. An SAE mathematician: (1) may not even understand the peripheral concepts of a non-SAE mathematician; or (2) may find those peripheral concepts irrelevant to the SAE mathematician's World of Ideas. In either case, the peripheral concepts are lost in translation. In short, the historical appropriation process is one of deconstructing an idea from one culture (i.e., removing its peripheral concepts) and then reconstructing it to fit another culture (i.e., adding peripheral concepts that make sense in that culture). This mimics what engineers or the general public do when they "apply scientific ideas" to an everyday situation or issue of interest to them (Aikenhead, 2006, p. 30).

In a rather parallel way, Ernest (2016b), explained historical appropriation by European mathematicians:

> Eurocentric ideology…has dominated historical and philosophical thought for the past 200 years. This ideology elevates rationality based on deductive reason as the highest intellectual good. …The "afro-asiatic roots of classical civilization" have been neglected, discarded and denied. Thus, [their] vital developments in number and calculation…are unrecognized as the essential foundation for all of mathematics including proof. (p. 385)

What might seem as a challenge to the grammatically subordinate identity of *Euro-American mathematics*, Donald and colleagues (2011, p. 75) mentioned "the myth of mathematics as a European discipline." They understood school mathematics as a pluralist collection of mathematical knowledge systems from several ancient civilizations (see the end of section 4.1). This idea of a collection is indeed correct, in keeping with Ernest's explanation. But EAM has a subordinate meaning of "mathematics," with a cultural content component that includes a more complex explanation involving language-laden cognition and peripheral concepts.

Thus, I respond to their challenge by noting we are both correct, I believe. This paradox can be easily resolved. We have two different stories to tell about ancient cultures' contributions to EAM. Each story is relevant for different audiences, depending upon their need for simplicity or complexity, respectively: either (1) school mathematics is mainly a collection of multicultural knowledge systems with historical roots that Platonist mathematicians neglected, discarded or denied; or (2) school mathematics is mainly appropriated knowledge with a Eurocentric imprint (i.e., the original culture's peripheral concepts were deconstructed and then reconstructed with Eurocentric peripheral concepts); plus its own mathematical inventions described in Ernest (2016b). This more complex explanation happens to describe a cultural feature of EAM (i.e., EAM cultural content related to the history of EAM). EAM certainly owes a debt of gratitude to the cultures that originally inspired earlier European mathematicians to appropriate ideas from them.

Burton (1995, p. 279) combined both stories when she wrote: "Colonization of mathematics has been so successful that the history of their own mathematical culture and its contribution to knowledge is often unknown to students in Africa, Asia, and Latin America."

The imprint of appropriation tends to alienate many Indigenous students because it manifests Eurocentric cultural features associated with superiority and aggressive authority that have the capacity to conspire in campaigns of colonization. At the same time, this imprint of superiority and aggressive authority helps explain the high status enjoyed by Platonist school mathematics in a Euro-American culture (section 3.2).

Showing agreement with Bishop's (1990) comment that Western mathematics is "one of the most powerful weapons in the imposition of Western culture" (p. 1) (section 4.2), D'Ambrosio (1991) identified Platonist school mathematics' identity and ideology this way:

> The history of Mathematics is identified with its progress through colonialism, industrialization and the emergence of the great European Empires of the XIXth century. Mathematics is also the imprint of Western culture. This is easily identified with the decline and subordination, conquest and colonization and destruction of old empires of what is now called the Third World and with populations labelled as "minorities." (pp. 12-13)

The Eurocentric impulse to appropriate from other cultures drew upon its own what-ought-to-exist, objectivity-conscious, and language-laden cognition, thereby ignoring or not being able to “see” (i.e., conceptualize) (section 9.3) the subjectivity-conscious content of other cultures’ language-laden cognition.

This describes, for instance, the British colonizers who could not understand the complex genealogy-based mathematics system in use by Indigenous families in what is known today as the State of Western Australia (Watson & Chambers, 1989). The colonizers’ inability to understand Indigenous mathematizing illustrates how the universalist stance of Platonist school mathematics can ignore and therefore demean Indigenous mathematizing. Is this history repeating itself today?

To prevent school mathematics from re-enacting what happed in Australia, mathematics educators can participate in today’s evolution of school mathematics (section 8). Boylan (2016) wrote:

> Mathematics is a cultural product of our ancestors and positions humans as “participants in the great, age-old human conversation that sustains and extends our common knowledge and cultural heritage”; such a recognition entails “acknowledging that the conversation is greater than yourself” (Ernest, 2013, p. 11). This suggests a responsibility to mathematics itself. (p. 402)

Accordingly, this monograph emphasizes mathematics educators’ responsibility to critically scrutinize school mathematics and then determine appropriate 21st century Euro-American and Indigenous cultural practices that could serve as contexts for teaching school mathematics in an era of reconciliation; contexts such as “what happens in the real world” (Wolfram, 2012, p. 2). For example, Neel and Fettes (2010) described how geometric transformations such as line and rotational symmetry could be taught at a Haida Gwaii school on the West Coast of Canada. A Grade 9 teacher looked “for symbols that evoke some of the same associations of identity, pride, power, and display as crest designs do” (p. 47). Students were encouraged to select either a familiar local Haida Gwaii piece of art work or a cherished sports logo that held personal significance to them. The teacher wanted the selection to stir a “deep source of emotional engagement” (p. 47) in the student. In other words, for one geometric topic two different cultural practices (traditional and contemporary) established a meaningful context in which students could explore geometric transformations. If appropriate cultural practices cannot be found, how can the EAM Platonist content be adequately learned? The availability of an appropriate cultural practice could be one criterion for deciding what is essential to include in a mathematics curriculum, and what is not (sections 2.4 & 10).

Looking ahead, what heritage of school mathematics do we want to pass on to future generations (section 10)?

4.5. Cultural Content Made Explicit

To summarize, Eurocentric features of peripheral concepts attached to Platonist mathematical concepts assert an objectivity-conscious epistemology, a particular what-ought-to-exist ontology, and an axiology that includes the ideology of quantification among others. Masking power with innocence thrives if this framework is kept invisible. “Detached and taught in isolation, mathematics loses many of its attributes as an enormously important part of our society, culture, and science and the students lose their ability to handle complex situations where mathematics is in action” (Andersson & Ravn, 2012, p. 322). As Bishop (1988a, p. 82) pointed out, “Mathematics, as a cultural phenomenon, only makes sense if [its] values are also made explicit.” His idea transfers well to the experience of most Indigenous students (Furuto, 2013a).

Andersson and Ravn (2012), Bishop (1988b), and Furuto (2013) offered excellent advice for curriculum developers and teachers. A value-free knowledge system does not make common sense to students whose worldview is replete with, and relies on, values. Making EAM cultural content explicit, therefore, repositions school mathematics as a value-laden knowledge system that can make common sense to more students, Indigenous or non-Indigenous.

Whether or not students agree with EAM’s values can be sorted out by requiring students to least *understand* those values, even if they do not *believe* them (section 3.3). As mentioned before, assessment in cross-cultural school mathematics is restricted to what students understand and can do. Expecting students to believe what they are taught (making it part of their worldviews) reflects a type of assimilation that visited the classrooms of residential schools. Certainly, students should be afforded the intellectual independence to believe what they want to believe.

Ernest’s (1988) cultural belief about mathematics led him to identify cultural values inherent in Platonist content. The following are expressed in the form of non-binary dyads with respect to the value’s importance: “Abstract is valued over concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, general over particular, theory over practice, the work of the brain over the work of the hand, and so on” (Ernest, 1991, p. 259).

These pure/applied dyads suggest that EAM cultural content has two non-dichotomous dimensions: (1) a human dimension *internal* to the enterprise of EAM: its history and its ontological, epistemological and axiological presuppositions; and (2) an *external* human dimension; in short, mathematics-in-action (Skovsmose, 2005): EAM's roles or functions in society, such as, on the one hand, mainstream cultural practices connected to specific Platonist content; and on the other, Platonist mathematics' influences and how they go largely unnoticed in political, social, economic, military and personal contexts of use.

Andersson and Ravn (2012) developed a philosophical framework to encompass what I am calling EAM Platonist content (the authors' "the core;" p. 313) and EAM cultural content (the authors' "the context;" p. 313). "Importantly, this human-oriented conception of mathematics…states that these two poles or perspectives [core and context] from which mathematics can be conceived are, in fact, intertwined and intimately connected and can only secondarily be divided into different domains" (p. 313). Interestingly, their "context" perspective ignores the *internal* human dimension of EAM cultural content, as do many mathematics educators and researchers who have not quite escaped the Platonist's hidden agenda (section 4.2.1).

Defined as "How mathematical conceptions are projected into reality," Greer and Mukhopadhyay (2012, p. 233), explored the phenomena of mathematics-in-action and how our physical and political-social-educational worlds are being mathematized today:

> A prominent development within the uses of mathematics is the extent to which [mathematical] modelling is being applied, not just to physical phenomena, but also social phenomena. …[T]echnical solutions are often substituted for the more complex human solutions that are needed. Improving mathematics education, for example, is a human problem, not a technical problem. Yet recent educational policy in the USA, and well beyond, treats the system, and the students, teachers, and schools within it, as a black box that can be controlled by external levers of test data, financial incentives and disincentives, punitive measures, and market forces. (p. 232)

Mathematics modelling causes citizens, teachers, and students to become powerless when these external human dimensions of EAM are ignored by school mathematics[ix]. The standard official rhetoric, "All students must have a solid grounding in mathematics to function effectively in today's world" (p. 239), never includes critical thinking about mathematics-in-action in political-social-economic-military contexts. The rhetoric refers only to Platonist content that presents itself as acultural and value-free. Who do schools serve, therefore? Future citizens who could learn to be analytically rational, or powerful citizens who try to control others? As described in section 3.2, Pais (2012) would answer that schools ultimately serve Western capitalism.

Eight short specific examples illustrate mathematics-in-action, the *external* human dimension of EAM cultural content. My first example concerns the influence of Platonist content in education, in the form of high-stakes testing. Most often, such testing determines what content schools teach. Thus, students are invariably subjected to the full force of the hegemony of Platonist content, as teachers feel pressured to teach to the test. This issue is critically analyzed in sections 9.1 and 9.4.

My second example is a story about creating mathematical models to be put into action. NASA in the late 1950s and early 1960s was being challenged to get astronaut John Glenn into orbit for the first time. The mathematicians who created the breakthrough were unknown heroes until the release of the 2016 Hollywood film *Hidden Figures*. Katherine Johnson, Dorothy Vaughan, and Mary Jackson were responsible for "doing the math," which meant designing the equations that modelled the complex trajectories required by John Glenn. Not only was the mathematics model created by three women; they were Black women who are now heroes.

Third, mathematics modelling used by airline companies calculates how many seats should be overbooked on a particular flight in order to maximize profits for the company and capital gains for its shareholders. The modelling attempts to duplicate reality through balancing such factors as: the usual number of passengers who do not show up, circumstances that affect passengers arriving at the airport on time, and the cost of placating passengers "bumped" when the flight has more passengers than seats. Greer and Mukhopadhyay (2012) observed:

> Since the passengers so affected modify their behaviour in response to their experience, and these reactions in turn lead to adjustments to the models, an important aspect that this example illustrates is that "when part of reality becomes modelled and remodelled, then this process also influences reality itself." (p. 233)

A fourth example concerns mathematics modelling that poignantly requires input concerning the dollar value placed on: a child's life, an American soldier's life, and a missile fired by a drone or aircraft. Other input concerns the context of the military target (e.g., school, Mosque, apartment building, hospital, bridge, etc.) and the priority for "removing the target." The parameters are entered into a mathematics model, up until someone pushes the "fire" button. The computer's mathematics model decides if the drone or aircraft will fire a missile or not. The human operator does not decide.

A fifth example shows how mathematical thinking can structure the way we conceptualize reality, such as climate change (Boylan, 2016):

> [T]he descriptive, predictive and communicative aspects of climate science involve the use of mathematics and mathematical literacy. The idea of climate change is a "realised abstraction" (Barwell, 2013, p. 10) that, through mathematics, formats the world, but excludes the human narratives of changing weather or the anguish of the disruption of people's lives. (p. 402)

Indigenous students who figure out this "formats the world" function will tend to resist learning mathematics because of its hegemony (Greer and Mukhopadhyay, 2012). This monograph ends with a story of what happens when a mathematician attempts to develop a mathematics model to format a physical phenomenon that includes a human value (section 11.2).

These five examples reveal hidden mathematics-in-action and lend themselves to student discussions and perhaps decisions.

My next example introduces a different type of mathematics-in-action, one which contextualizes Platonic content in the everyday world of activities. These are not events sanitized and simplified for typical textbook use, but real events based on mainstream culture in which students actually see themselves participating, or participating vicariously. For the purpose of this monograph, the expression "relevant for students" will mean any situation in which they can realistically see themselves.

The sixth example is a CT scanner, a medical instrument that takes multiple x-ray photos from 360 degrees around a patient's body. These data are fed into an elaborate mathematics model of simultaneous equations that create a series of two-dimensional images for analysis by a medical expert. A workable set of three simultaneous equations to solve could then be introduced to students.

My seventh and eighth examples were trialed by Andersson and Ravn (2012). The examples were typically open-ended activities but structured to give enough guidance to senior secondary students. Their directions for the activity "Newspaper Posters with Mathematics Argumentation" began this way:

> The task of today is, in small groups, to create a number of newspaper posters that hit people, engage people, arouse curiosity, reflections and/or emotions – with a mathematical

> content! The goal is for you to acquire insight into how big the penetrating power of numbers can be in advertisements and newspaper articles. There are 54 articles in "Convention on the Rights of the Child". Choose the one that interests you the most and focus on that specific one. (p. 320)

My last example of mathematics-in-action is Andersson and Ravn's activity "Making Your Dreams Come True."

> Reflect on something you would like to do, experience, or buy, for yourself or others, which costs so much that you need to borrow the money to cover the expenses. You have to find out how much money you require to finance the project and what repayment (including interest) the bank expects you to pay.
>
> We suggest the following: the repayments are made to the bank once a year, and you pay back the loan within five years. If this is not possible for you we will discuss that.
>
> • How much will you be paying back each year in interest? In total over the five years?
>
> • How much do you need to pay back in total per year?
>
> • What did the total cost add up to?
>
> • Was it worth it? Why/why not? (p. 319)

To summarize, the phrase "EAM cultural content" embodies external and internal human dimensions of EAM, its cultural identity so to speak. The external domain comprises mathematics-in-action found in two types of contexts: (1) contexts in which the Platonist mathematics is highly complex, too abstract for teaching its sophisticated Platonist school mathematics, but contexts related to public issues to be identified and perhaps discussed; and (2) contexts related to everyday mathematizing in which students can participate in order to learn concepts, skills, and strategies associated with the Platonist content in the curriculum.

Although the eight examples above illustrate mathematics-in-action (the *external* human dimension to EAM cultural content), Andersson and Ravn's (2012) activities associated with Platonist content could have included follow-up questions that introduced the *internal* human dimension to EAM cultural content as well. Such questions would have required students to identify a presupposition or values inherent in the Platonist content mentioned in their write-up; assuming some of that internal content had been taught.

Both the internal and external human dimensions were described further in an axiological study of "Western mathematical knowledge." Corrigan and her colleagues' (2004) research-based catalogue of values-qualities-features is one place where curriculum developers can begin to identify the internal culture of Platonist mathematics (shown in Table 1, on the next page). Mathematics educators need to work out how to make the values-qualities-features explicit for students. Table 1 has three continuums (Ideology of Mathematics Knowledge, Mathematics Knowledge and Society, and How a Student Relates to Mathematics Knowledge). They capture a diversity of 35 values-qualities-features that characterize the internal culture of Platonist mathematics based on earlier research by Bishop (1988a). The values-qualities-features generally comprise epistemological and axiological presuppositions of Platonist mathematics.

However, neither Ernest (1988) nor Corrigan and colleagues (2004) mentioned a cluster of aesthetic values dear to the hearts of mathematicians: elegance, simplicity, and beauty; any of which would apply to my favourite Platonist equation, Euler's unity ($e^{\pi i} + 1 = 0$). While many mathematicians have eloquently and poetically expressed their emotive understanding of Platonist mathematics, British mathematician G. H. Hardy articulated the rule: "Beauty is the first test: there is no permanent place in the world for ugly mathematics;" a well-known quote from his 1941 book A Mathematician's Apology.

Whether or not students agree with these values-qualities-features is not the point. The important is for students to *understand* those values-qualities-features, even if they do not *believe* them or wish to make them their own (section 3.3). Assessment in culture-based school mathematics is restricted to what students understand and can do. Expecting students to believe these values-qualities-features is indoctrination, of course. For instance, assessing students on their understanding of analytical thinking (an epistemological presupposition), a teacher could expect students to identify such thinking among four examples of different types of thinking.

School mathematics should not necessarily "foster" these values-qualities-features as mathematics educators often suggests. As described in endnote xx, only a minority of about 28, 26, and 31 percent of Canadian, Saskatchewan, or U.S. high school mathematics enthusiasts (respectively) would tend to fully embrace these values-qualities-features (OECD, 2016, pp. 362, 447, & 362, respectively). The other 74 percent tend to find them foreign to varying degrees.

By making the values-qualities-features explicit, we reduce the culture clashes that tend to exist between the 72, 74, or 69 percent group (Canadian, Saskatchewan or U.S. figures) and the culture of Platonist mathematics. A reduced culture clash tends to raise the achievement of those students (section 3.3). To assist teachers, Corrigan and her colleagues (2004) matched the values-qualities-features with students' intellectual and social capabilities at different grade levels.

Table 1. Platonist Mathematics Values-Qualities-Features[1] (Based on Corrigan et al., 2004)

Ideology of Mathematics Knowledge	
RATIONALISM[2] Values, Qualities, and Features Explanations Abstractions Theories Reason Hypothetical reasoning Logical thinking	EMPIRICISM[3] Values, Qualities, and Features Atomism Materialism Determinism Objectivizing Concretizing Symbolizing Analytical thinking
Mathematics Knowledge and Society	
CONTROL[4] Values, Qualities, and Features Prediction Knowing Security Rules Power Mastery over the environment	PROGRESS[5] Values, Qualities, and Features Growth Alternativism Generalization Questioning Cumulative development of knowledge
How a Student Relates to Mathematics Knowledge	
OPENNESS[6] Values, Qualities, and Features Facts Articulation Demonstration Verification Universality Individual liberty Sharing	MYSTERY[7] Values, Qualities, and Features Abstractness Wonder Mystique Dehumanized knowledge

1 "Values in mathematics education are deep affective qualities which education aims to foster though the subject of mathematics" (Bishop, 1996, p. 96). Bishop (1988, p. 62) concluded that mathematical values cluster around three continuums: an ideological dimension, a sociological dimension, and a personal self-identity dimension.
2 Rationalism involves the separation of an idea from any associated object, utilizing "deductive reasoning as the only true way of achieving explanations and conclusions" (p. 62).
3 Empiricism involves abstracting ideas and treating these ideas as if they were objects.
4 Mathematics can be used as a tool to control the environment and people.
5 Progress represents the idea that mathematics is dynamic and develops with time. It often promotes new technological and economic developments in society, but ethical issues can arise as a consequence.
6 Openness "concerns the fact that mathematical truths, propositions and ideas generally, are open to examination by all" (Bishop, 1988, p. 75).
7 Mystery refers to the idea that many people, including mathematicians, feel mystified about what mathematics is, including the recurrence of entities such π.

When discussing mathematics and mathematics teachers' preferences of values, Bishop (2008) concluded:

> Could the dehumanised, highly abstract and mystique-laden value of Mystery of mathematics which appears to be such an obstacle to mathematics learners be made more explicit so that it could be challenged by the more humanised and personal intuitive nature of that value which science appears to enjoy? …Teachers' values in the classroom are shaped to some extent by the values embedded in each subject, as perceived by them. This implies that changing teachers' perceptions and understandings of the subject being taught may well change the values they can emphasise in class." (p. 56)

The values-qualities-features comprise an important portion of the interior culture of Platonist mathematics (i.e., Euro-American mathematics' cultural content) by addressing its history, its ontology (e.g., the dualistic presupposition about the nature of reality), its epistemology, and its axiology. However, Platonist mathematics has an external portion to its culture. This includes its math-in-use in the everyday world of home, community, and work; its math-in-action behind the scenes of business, industry, military and government that have political, social and ethical consequences; and its ideologies (i.e., doctrines that determine how people or institutions treat others). To prepare for adult life in Canada or anywhere, most students should be acquainted with the ideologies of: quantification, reification, certainty, exclusion, schooling, Eurocentrism, colonization, elitism, authoritarianism, and formal mathematics discourse. Some would refer to this outcome as being culturally savvy.

Teaching values-qualities-features of Platonist mathematics could naturally occur spontaneously as a teachable moment. But cleverly developed teaching materials can set up a discussion related to a specific value-quality-feature.

We can think of Euro-American mathematics as Yousan (section 4.2). The Japanese people recognized that EAM cultural content is inseparable from and wholistically interpenetrates EAM Platonist content. As mentioned in section 4.3, EAM cultural content is treated in this monograph as a separate category only for the convenience of discussing its content.

5. Early Innovations

The real voyage of discovery consists not in seeking new landscapes but in having new eyes.

A famous paraphrase from French novelist Marcel Proust (1923, website quotation)

D'Ambrosio is often credited as being the pioneer educator for teaching school mathematics enmeshed in local culture. He drew upon the cultures of Brazilian communities when teaching school and university mathematics and science in the 1950s and 1960s (Ascher, 1991). But in his humility, he credits the pioneer moniker to John Dewey who was one source of inspiration for D'Ambrosio's (1991) ethnomathematics. Another source of inspiration was the Brazilian politics of social justice in the context of globalization. "Ethnomathematics was forged in the experiences, reflections, and hopes for a better quality of life" (Furuto, 2013a, p. 39). D'Ambrosio's (2006, p. 308) basic mathematizing list for all cultures includes: "arithmetic, classifying, ordering, inferring, and modeling."

D'Ambrosio has many followers, some of whom embrace agendas modified from his original agenda. Today, meanings of ethnomathematics include: making school mathematics more relevant to different cultural or ethnic groups, describing how cultural values influence school mathematics, studying the mathematizing in different cultures, using that information to teach EAM Platonist content, and teaching the ethical principle human survival with dignity (Adam, Alangui & Barton, 2003; Ascher, 1991; Barton, 1995; D'Ambrosio, 2010; Doolittle, 2006; Jablonka & Gellert, 2012; Neel, 2008; Nova, 2016; Jannok Nutti, 2013; Rosa & Orey, 2011). This diversity has turned D'Ambrosio's term "ethnomathematics" into a slogan or vague metaphor, which serves as an influential rallying cry to improve students' experience in school mathematics. Thus, if researchers wish to describe their work as ethnomathematics, they must clearly define exactly what they mean, as many do. But there is still a problems with ethnomathematics (section 7.2) other than its ambiguity.

Another early innovation occurred in Aotearoa New Zealand in the 1970s and 1980s. A large minority of Māori people, fearing the extinction of their language (te reo Māori), began establishing independent total-immersion schools that offered a "bicultural" approach to school mathematics (McMurchy-Pilkington & Trinick, 2002). Losing one's Indigenous language severely diminishes a person's Indigenous cultural identity. In 1989, along with te reo Māori becoming an official language of the country, the Ministry of Education took responsibility for the Māori education system. Next came the 1992 decision to *translate* national curriculum documents into te reo Māori, which led to a Māori version of the 1996 mathematics curriculum (Pāngarau) that was used in bilingual and total immersion Māori schools.

As discussed in section 4.4, much can get lost or distorted in translation. Pāngarau included Māori vocabulary (including newly invented terms) and syntax, but only to the extent that the vocabulary and syntax conformed to an English version of EAM Platonist content (McMurchy-Pilkington & Trinick, 2002). This ensured that students who studied Pāngarau would be guided to think in a Euro-American way. “Some of the contexts and the exemplars are from a Māori perspective, but the concepts and ideas are underpinned by Western thinking” (p. 468).

Because the Māori education innovation gave equal attention to promoting te reo Māori and to nurturing academic achievement in Platonist school mathematics, some Māori educators challenged the equivalence and cultural validity of the Māori version of the country’s high-stake examinations (Meaney et al., 2012). They were a Māori translation into English thinking.

McMurchy-Pilkington and Trinick (2002) warned that Pāngarau (the te reo Māori version of the Platonist mathematics curriculum) acted like a Trojan horse by surreptitiously assimilating Māori students into thinking in a Euro-American way (i.e., “cognitive imperialism,” Battiste, 1986, p. 23), thereby continuing the country’s ongoing agenda of neo-colonization. Their concern has materialized, according to Russell and Chernoff (2013, p. 114): “Unintentionally, these endeavours to bring school mathematics into the Indigenous languages have actually resulted in further losses to the languages and culture.” In 2008, Pāngarau was updated, mostly by modifications that harmonized it wholistically with Pūtaiao (science), Tikanga-a-Iwi (social sciences), Ngā Toi (the arts), etc., altogether referred to as The Curriculum of Aotearoa. However, Pāngarau more or less continues its Trojan horse role.

Another pioneering group of educators, this time in Alaska, recognized the potential for positive outcomes from reducing culture conflicts between Indigenous students and EAM Platonist content (Lipka, 1994). They set about to reverse the alienation felt by many (but certainly not all) Indigenous students in conventional school mathematics classes (Lipka et al., 2013):

> In the early 1980s, Jerry Lipka and Dora Andrew-Ihrke met when Lipka was hired by the UAF [University of Alaska Fairbanks] as a faculty member in the Cross-Cultural Education Development Program in Bristol Bay, in a southwest Yup′ik…region. He and many others went on to develop Math in a Cultural Context (MCC) project as a way to incorporate local Indigenous knowledge…into schooling through mathematics curriculum materials and professional development. This resulted in a long-term exploration and a

> vision for mathematics education where insiders and outsiders work together to develop locally based mathematics curriculum to improve learning outcomes for Alaska Native students. (p. 130)

The MCC (2016) R&D project continues to collaborate extensively with Yupʹik Elders and knowledge holders to include local Indigenous perspectives in school mathematics (section 8.3). Fundamentally, the MCC program is about interrogating the culture of school mathematics conventionally taught in local Indigenous communities, in order to renegotiate with those communities an improved culture of schooling (Lipka, 1994).

Other R&D mathematics teams have been inspired by D'Ambrosio's ethnomathematics, the Māori bilingual Pāngarau program, or the Yupʹik Alaskan MCC project. These early innovations have helped the field mature to the point of having a noticeable presence in the research literature. We are indebted to these innovators and their colleagues for having fresh eyes so we too could find a new landscape of school mathematics.

6. Avoiding Present Day Appropriation and Marginalization

Without mutual respect and mutual responsibility, the truth is we can achieve very little.
Prime Minister Kevin Rudd, February 13, 2008
Apologized to Australia's Indigenous peoples for the stolen generations (Rudd, 2009)

6.1. Appropriation

Keene (2016, podcast 6.33-6.44 minutes) of Cherokee ancestry, defined cultural appropriation as: “Taking content from another culture, erasing the original meaning, and using it as you see fit;” that is, using it for a purpose not intended by the source. At one extreme of appropriation, the appropriator’s purpose may be well intended; and at the other extreme, solely for profit. Nevertheless, either case represents appropriation. Appropriation is even more subtle for researchers who ask permission to use the knowledge for the purpose of transforming it into a Euro-American mathematics (EAM) instruction activity We know the transformation process necessarily erases the Indigenous peripheral concepts (section 4.4). Thus, Keene’s view may seem too stringent, but indirectly it raises the question: What happens to the peripheral concepts that contribute original meaning to that knowledge? An answer is forthcoming in section 6.4.

In a contrasting view of appropriation, Beatty and Blair (2015, p. 5) describe it as “taking of Indigenous knowledge to use within a different cultural context, without truly understanding the cultural significance of the knowledge.” It raises an important point that we should understand what meaning (what peripheral concepts) is being erased from the original piece of knowledge.

Both views of appropriation express important essences, but they both ignore a tenet of Indigenous epistemology: knowledge belongs to (is related to) the person who holds that knowledge and shared it. This implies that if I want to use that knowledge, permission must be given by the knowledge holder according to local protocol, and permission depends on how I intend to use the knowledge. My request for permission is not a yes or no proposition; there are conditions and responsibilities I must follow. For instance, do I intend to incorporate it into my personal life only? Or can I share it: orally with my immediate family only, orally with students, in writing in teaching materials for other teachers to use, or digitally on a website available to the world?

And when I do use it, my obligation is to show respect by explicitly acknowledging the permission granted by the Elder or knowledge holder, so that others know to whom the knowledge is related. In short, to avoid appropriation we must ensure that the knowledge holder has the power to make a fully informed decision: “Yes, go ahead;” “No, that wouldn’t be appropriate;” or “OK, provided that… .”

It is informative that Keene’s condition “erasing the original meaning” points to the “deconstruction” process of stripping away Indigenous peripheral concepts (section 4.4). An Elder or knowledge holder can give me permission to do that, but only if I am aware of the key peripheral concepts. To become aware usually entails experiential learning, plus engaging in a type of in-depth exchange of taken-for-granted ideas about our worldviews, our language-laden cognition, and our values; a conversation many call “third-space dialogue” (Lipka, Sharp, Adams & Sharp, 2007). Vickers (2007, p. 592) of Tsm’syen ancestry called such meetings “camping spots where we can dialogue” between cultures. The self-explanatory metaphor *camping spots of dialogue* captures a coming to know each other in a way expressed in Nehinuwehin (Swampy Cree language), as kiskinaumatowin (learning/teaching each other as equals; described further in section 6.4).

In summary, mathematics education researchers and teachers can avoid appropriating from an Indigenous community by simply: (1) collaborating with Elders or knowledge holders and *receiving permission* to use their artifacts, processes, or ideas in a specific agreed upon way; (2) identifying the artifacts, processes, or ideas in the teaching materials as coming from the specific Elders or knowledge holders; and (3) acknowledging their permission to use them. For example, see section 8.5, among other examples in section 8, for an explicit illustration.

However, if we are not cognizant of the cultural significance being stripped away, we run the high risk of marginalizing Indigenous students by not being transparent about the transformation process that turns an Indigenous artifact, process or idea into a school mathematics lesson.

6.2. Marginalization

Obviously we want to avoid, even unconsciously, marginalizing Indigenous perspectives. Therefore, a critical analysis of the process or mechanism by which marginalization can occur will be helpful. The issue has fairly complex nuances to explore.

The following linguistic explanation of marginalization parallels the language-laden cognition explanation of historical appropriation found in section 4.4. But in the present section, it is specific to producing teaching materials. When mainstream researchers and teachers visit Nehiyaw (Plains Cree) communities, for instance, each culture group brings its own language-laden cognition to their conversations. Of course Indigenous language-laden cognition will vary slightly from community to community.

Kawasaki (2002, p. 24) clarifies who the actors are typically: one the one hand, the well-intended “objectivity-conscious” researchers and teachers who occupy mainly a World of Ideas expressed in noun-based, abstract, decontextualized, symbolic language; and on the other hand, the “subjectivity-conscious” Nehiyaw community mainly in their Phenomenal World expressed in verb-based Nehiyawewin (Plains Cree language). Of course, Indigenous languages contain highly abstract concepts as well.

A SAE (Standard Average European) noun-based language is often deficient in capturing the complex peripheral concepts attached to an Indigenous verb-based construct. (Examples were given in section 4.4.) As a result, mainstream educational researchers and teachers may tend to leave out peripheral concepts associated with Indigenous Phenomenal Worlds. Moreover, subjectivity-conscious ideas are not usually relevant to the objectivity-conscious mind, and will tend to be ignored. This subtle disappearance or dilution of an Indigenous perspective in the teaching materials – an instance of marginalization, to be sure – can make an Indigenous student or community feel violated (Meaney, 2002). According to Doolittle (2006), a mathematics professor of Mohawk ancestry, students will likely

> feel that their culture has been [marginalized] by a powerful force for the purpose of leading them away from their culture. …Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures. (p. 20)

What can teachers and researchers do to prevent it from happening?

Many high quality R&D projects discussed in this monograph begin with dialogues of respect, humility, and patience with Elders and knowledge holders in order to discuss language-laden cognition. They inform each other about, for example, what-ought-to-exist presuppositions might cause misunderstandings. Most researchers and teachers quickly lose their fear of accidentally insulting an Indigenous student or knowledge holder after talking about such fears with an Elder. Elders invariably put us at ease (Aikenhead et al., 2014). As discussed in section 6.1, a heart-felt discussion takes place in a “third-space” dialogue (Lipka et al., 2007, p. 97): “[T]he space where two cultures or linguistic styles meet but co-evolve into a practice that is not strictly either and becomes a new creation. … These third spaces have the potential to become productive uncharted zones between school and local cultural knowledge and norms.” Hogue

(2011, p. 71) of Métis ancestry prefers the metaphor "liminal space." As mentioned in section 6.1, Vickers (2007, p. 592) called such meetings "camping spots where we can dialogue" between cultures. This type of dialogue requires us to maintain an open mind in order to avoid a Eurocentric impulse of authority, but instead develop humility in our detailed appreciation of the local Indigenous subjectivity-conscious activities from which we may develop culture-based teaching materials. This exchange is one example of a cultural immersion.

Participating in a camping spot of dialogue is a *foundational prerequisite*: (1) for anyone initiating a project that develops teaching materials (section 8); (2) for teachers expected to implement them (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, 2017; Fyhn et al., 2011; Lunney Borden et al., 2017; Michell et al., 2008); and (3) for curriculum writers who compose outcomes and indicators (sections 2.4 & 10.3). Cultural immersions are filled with superb learning moments, including those that guide us around linguistic pitfalls such as confusion or misconceptions.

6.3. English/French Language-Laden Confusion/Misconceptions

The English and French languages seem to have a built-in colonizing vocabulary and syntax that hamper mathematics educators in *expressing* respect for the integrity of Indigenous cultures (Garroutte, 1999), in spite of their heart-felt respect for those cultures. A critical analysis of three expressions reveals neo-colonial messages detrimental to a researcher's R&D project or a teacher's instruction.

1. "Tlingits manipulated their natural environment with great skill, requiring understandings of mathematical concepts and physics" (Bradley & Taylor, 2002, p. 53).

On the contrary, the Tlingit people of eastern Alaska survived for millenniums without having contact with EAM Platonist concepts or Newtonian physics. Although meant as a compliment, the statement privileges Euro-American thinking by using it as the universal standard against which to compare Indigenous thinking. Bradley and Taylor unintentionally projected their colonizing epistemology onto an Indigenous group – "cognitive imperialism" (Battiste, 1986, p. 23).

When is a compliment not a compliment? Two everyday examples may help clarify this question. Suppose a Euro-Canadian customer is pleased with the service provided by a third generation Asian-Canadian store clerk. The customer compliments the clerk on speaking English so well. The clerk, however, hears the customer's subtext as: "I do not perceive you as a true

Canadian." The customer's compliment turns out to be an unconscious putdown; but felt as a racist putdown nonetheless.

My second example is a school principal at a Grade 12 graduation ceremony. An Indigenous student receives a diploma with a top-of-the-class distinction. Instead of complimenting the student by saying, "We're proud of you; we knew you could do it;" the principal exclaims with enthusiasm, "You've done exceptionally well for an Aboriginal student." The student does not feel the praise, but an unconscious putdown of the student's ethnicity. Both examples highlight a neo-colonial masking power with a compliment.

Bradley and Taylor's (2002) example also exemplifies the epistemological privilege that envelops academia in general (Kovach, 2009) and school mathematics specifically. Such universal-like privilege causes many educators to be blind to an Indigenous epistemology, and so they project their own epistemology onto an Indigenous artifact, activity, or idea. In response to this and other types of privilege blindness, my monograph seeks to: expose such privileges, examine their consequences, explore their political racialized roots (Russell, 2010), and propose an alternative for the benefit of all students.

2. "These [Yup'ik] practices form a coherent and generative set of concepts which *incorporate* geometry, fractions, ratios, and proportional reasoning" (Lipka et al., 2013, p. 132, emphasis added).

An English vocabulary (e.g., geometry, fractions, etc.) is used here to describe Indigenous concepts, rather than either using an Indigenous vocabulary or restating the English to read, "These Yup'ik practices form a coherent and generative set of practices, which Euro-American mathematicians may recognize as analogous to geometry… ." Without being edited, Lipka and colleagues' statement, at best, implicitly depreciates the integrity of Yup′ik concepts. At worst, it conveys epistemological privilege and cognitive imperialism.

3. "To create the square, which is the base shape for these [embroidered] products, Yup'ik elders *used symmetry/splitting…*" (Lipka et al., 2013, p. 133, emphasis added).

The EAM Platonist content mentioned in both quotes from Lipka and colleagues was neither *incorporated* nor *used* by the Yup'ik embroiderers. These English terms in this context serve to marginalize Indigenous perspectives. The act is accomplished by a process identical to the marginalizing process described in section 6.2 on how to avoid such action. A respect for Yup′ik culture integrity could be restored as suggested for the second example just above.

Just because the processes used by Yup'ik *embroiderers can be described in English or French and with Platonist concepts does not mean the embroiderers were using those concepts*. And if embroiderers did not use those concepts, the concepts do not exist in Yup′ik culture. We need to respect the integrity of Indigenous cultures in this way (Garroutte, 1999).

The erroneous act of attributing Euro-American content to Indigenous activities is found much too often in the literature (section 9). The error suggests either: (1) the authors do not sufficiently understand their Indigenous collaborators, and as a consequence, the authors are inadvertently recolonizing the people they intend to serve; or (2) the authors have not critically examined their English or French words or phrases closely enough – one aim of this monograph. When teachers come across resources that commit this kind of error, it becomes a teachable moment to point out the error and to discuss an accurate wording.

To avoid this type of error in the first place, we need to remember to describe Indigenous content using a vocabulary that *clearly conveys an Indigenous perspective*, not a Euro-American one. For instance, rather than theoretical physicists announcing that "the language of nature is differential equations;" a reasonable revision might read: "Some scientists understand the physical world in terms of differential equations;" or perhaps, "Paradigmatic preferences among some scientists are metaphors borrowed from the field of differential equations." Or as anthropologist Nespor (1994, p. 20) expressed it, "mathematics equations in physics are representational technologies."

When we describe an Indigenous idea in English or French but feel a bit uncertain or inadequate about understanding its peripheral concepts, we can request a translation. But a more powerful technique is a back-translation. For instance, ask a Nehiyaw speaking person who is well acquainted with English or French, to translate into Nehiyawewin (the Cree language) the word "knowledge" in the context of learning. In Nehiyawewin, as in many Indigenous languages, there is no such word. Thus, the person will utter a Nehiyaw expression normally used by Nehiyawak (Cree people) in the context of learning. Then get another independent Nehiyaw speaker to translate whatever the first speaker said back into English or French in a literal way. This is a back-translation. A Nehiyaw back-translation of "knowledge" usually comes out as "ways of knowing, living, and being" (Aikenhead & Ogawa, 2007, p. 553). This back-translation helps a person appreciate that the English expression "Indigenous knowledge," when spoken by Nehiyawak, very subtly imposes a Eurocentric epistemology on what they say, but do not really mean. The word "knowledge" acts as an assimilating Trojan horse.

We can certainly ask Indigenous students to inquire into a translation and a back-translation by talking with an Indigenous speaker. The role of the teacher is now the learner while the role of the student is the teacher. When this happens, be prepared for an amazing improvement in the quality of teacher-student relations even from students who did not talk with an Elder or knowledge holder.

Epistemically assimilated Indigenous scholars have been known to claim, for example, that their kokum (grandmother in Plains Cree) was doing chemistry when she tanned hides, because tanning is all about chemistry. Garroutte (1999, p. 107), Princeton doctoral graduate of Cherokee ancestry, commented on this misconstrued type of belief: "It occurs when we suggest that Indian models of inquiry constituted proto-sciences." The confusion and/or misconception happens as a result of Euro-American epistemology being unconsciously held as superior to Indigenous epistemology; it subtly but effectively degrades Indigenous epistemologies. A wisdom tradition of understanding used by a kokum (e.g., her balancing the *intellectual*, emotional, physical and spiritual dimensions of tanning hides wholistically related to the universe) is far richer than the more narrow *intellectual* tradition of chemists' understanding hide tanning processes.

An implication for educators is to be open to the fact that Euro-American mathematics (EAM) Platonist concepts can be so narrowly specific that it would be confusing to some Indigenous students to use that EAM Platonist concept in an Indigenous context; that is, they would appear to be out of context. But if the concept's name does come up in a community context, a teacher could point out that the EAM Platonist word has a technically narrow meaning, and then learn either an Indigenous translation of the word, or a back-translation into English or French, if possible. If a translation is not possible, then a severe clash exits between a noun-based SAE (Standard Average European) language and a verb-based Indigenous language in this case.

In terms of validity, Indigenous understandings of tanning do not need chemistry to establish or augment their validity. Both cultures establish and regulate their own knowledge system's validity by employing very different processes and values. The two coexist independently. They do not validate each other (Aikenhead & Ogawa, 2007; Garroutte, 1999).

How do we describe the complex process that appears to begin with an Indigenous group's everyday object, activity, or understanding, and ends with a culturally contextualized EAM lesson or unit? The literature is replete with verbs such as: to interpret, to transfer, to connect, and to relate; all of which give no hint as to how it is done exactly. (More context-specific examples are discussed in section 9.3.)

As described in section 6.2, an Indigenous everyday artifact, activity, or understanding must be deconstructed, and then reconstructed to fit into the culture of school mathematics. But how does a researcher select an Indigenous artifact, activity, or understanding to be deconstructed? A four-step process can be a general guide:

1. We forge a relationship with an Indigenous knowledge holder or Elder, adhering to local protocols (Lipka et al., 2013; Sterenberg, 2013a).
2. We become familiar with some of the group's ways of counting, locating, measuring, designing, playing, and/or explaining (Bishop, 1988b, pp. 147-151) (section 4.1).
3. Guided by Einstein's insight that "It seems that the human mind has first to construct forms independently, before we can find them in things" (Einstein, 1930; quoted by Director, 2006, p. 113), mathematics educators will draw upon their professional constructed forms or images (mathematical abstractions, concepts, and processes) to superimpose their mental forms or images on Indigenous groups' mathematizing, *within the educator's understanding of* that mathematizing. This is accomplished, in part, by ignoring irrelevant, peripheral, ontological, epistemological and axiological concepts – the same "*stripping*" process described in section 4.4; but with help from an Elder, we become cognizant of some of the peripheral concepts being stripped away.
4. We keep track of the content stripped away to use later as needed in order to make the stripping process transparent to students. For example, Fyhn (2013) showed how Indigenous mathematizing is integrated with cultural values. When making a square embroidering pattern on a fur parka, "The construction process is explained by the need for balance and harmony between the wearer and the spirit world, and between the user and the task" (p. 355). This will be some of the content stripped away, which should be made explicit to students at some appropriate time. Furuto's (2013a, p. 53) Native Hawaiian R&D project emphasizes "[H]ow universal values bind us [Indigenous and non-Indigenous] together," as practised by the circumnavigating Hōkūleʻa voyage (section 8.4).

This mechanism can be summarized as: s*uperimposing* a Platonist content image on an Indigenous artifact, process, or idea by using a best-fit trial-and-error method; *deconstructing* it in an Indigenous culture; and then *reconstructing* it in the culture of EAM with EAM peripheral concepts. This three-part process can be represented by the metaphor of transformation (Aikenhead & Ogawa, 2007; Jannok Nutti, 2013). The word "transformation" itself has various context-dependent peripheral concepts that create various connotations, some of which may not be appropriate for a specific cross-cultural superimposing-deconstructing-reconstructing process described here.

6.4. Centrality of Indigenous Languages

An English or French teacher's or researcher's sensitivity towards the issue of a mainstream language monopoly (i.e., language privilege) is sharpened by critically asking oneself: "Whose language is being spoken? Did I acknowledge that I am thereby privileged by the use of my language in this project?" When we learn some local key Indigenous phrases, we convey sincerity with our acknowledgement.

Analyzing Indigenous language-laden cognition is central to the success of cross-cultural school mathematics. For example, when Elders observe a natural phenomenon, they might ask, "*Who* did that?" and "What might I do to show my responsibility to my relations in return?" Whereas schools only ask students, "*How* does the phenomenon happen"?

Actually, Nehiyaw Elders do not observe, in the English objectivity-conscious meaning of the word. Instead they engage in kanawapamew (Nehiyawewin for "observing;" Beaudet, 1995), a meaning with peripheral concepts associated with *contemplating* the interrelationships that contribute to a *wholistic* understanding of the phenomenon in terms of everything in creation. In other words: Elders do not *explain* the universe, they *inhabit* it; and Elders do not ask *how* the universe works, they ask what the universe *is* (Battiste & Henderson, 2000, p. 121).

These questions remind us of the differing ontologies, epistemologies, and axiologies between the culture of Euro-American mathematics (EAM) and a culture of an Indigenous community. The chance of "intercultural misunderstandings" is great (Hall, 1976, p. 165). Hall explains these misunderstandings in terms of the limitations of language itself. "The paradox of culture is that language, the system most frequently used to describe culture, is by nature poorly adapted to this difficult task" (p. 57).

Language, the encyclopedia of a culture's collective worldview or an individual's personal worldview, allows a person to organize and express their thoughts and then engage in a language-based activity that *anticipates* a potential meaningful response. According to Hall (1976),

> language is not (as is commonly thought) a system for transferring thoughts or meaning from one brain to another, but a system for organizing information and for releasing thoughts and responses in other organisms. …[I]t is impossible to plant [thoughts or

> meanings] in the minds of others. *Experience* does that for us instead. (p. 57, emphasis added)

An ancient Chinese saying captures Hall's paradox: If you want to understand water, don't ask a fish." Hall describes the paradox this way:

> All one can say when studying any aspect of a strange culture is: there is a system; the people who live by the system can tell you very little about the laws that govern the way the system works (they can only tell you if you are using the system correctly or not); there is little relationship between the manifest way in which the system is expressed (the meanings derived from it) and how it is organized. (pp. 165-166)

Here Hall suggests that language functions along with an *explicit* cultural system, and that there are culturally unconscious or "out-of-awareness cultural systems that have yet to be made explicit" (p. 166). The two categories (explicit and out-of-awareness) are not dichotomous, but instead they co-exist. Hall concluded that the out-of-awareness cultural systems "probably outnumber the explicit systems by a factor of one thousand or more. …The investigation of out-of-awareness culture can be accomplished only by actual observation of real events in normal settings and contexts" (p. 166).

Mathematics education researchers described in this monograph investigate, from the point of view of their own worldviews, an Indigenous community's out-of-awareness cultural systems, such as beading moccasins, for the purpose of transforming an Indigenous activity into an explicit language-laden cognition expressed in mathematics teaching materials. Skovsmose (2010, p. 351) called this transformation process an "articulation;" a bridging between "in-school and out-of-school practices." The superimposition stage of the transformation process requires a researcher to imagine a Euro-American image or process by observing and *experiencing* an Indigenous out-of-awareness system. Success at the next two stages (deconstruction and reconstruction), however, relies much more on a person's awareness or familiarity with Indigenous language-laden cognition and peripheral concepts. In short, the strategy is to make the "strange culture" familiar.

Success at the deconstruction and reconstruction stages also leads to Indigenous students having their cultural self-identities reaffirmed by their: (1) encountering aspects of the community's language in their mathematics classes, (2) becoming aware of some peripheral concepts being stripped away during the deconstruction stage, and (3) being introduced explicitly to some peripheral concepts associated with EAM Platonist content. As mentioned above (end of section 6.3), the transformation of Indigenous artifacts, processes, or ideas into mathematics teaching materials can become transparent in an age-appropriate way.

Consider, for example, a culturally contextualized EAM lesson on probabilities that draw upon playing an authentic Mohawk peach pit bowl game of chance called Lahal (Doolittle, 2006). Suppose the teacher asked students to graph the results in order to discuss a EAM Platonist concept of probability. The game's ceremonial and spiritual content could certainly be experienced by informed Mohawk students and perhaps explained in a wholistic way by a Mohawk knowledge holder whose explanation would likely entail how to live in a good way. *But what happens to that Indigenous, peripheral, out-of-awareness understanding of living in a good way, when the graphs are constructed?*

If ignored, as Ascher (1991) and FNESC (2011) did with games similar to Lahal, then Mohawk culture has been marginalized. To get around this consequence, the EAM lesson must have a way to draw students' attention to the process of stripping the game of its subjectivity-consciousness – its peripheral meanings. The lesson has now entered into a EAM cultural content discussion, along with Mohawk mathematizing. In other words, the superimposing-deconstructing-reconstructing process is being made transparent to students in order for them to learn some peripheral concepts of Lahal, and some EAM peripheral concepts that replaced them – EAM cultural content. Non-transparency contributes to the marginalization of Indigenous students.

Could students, Indigenous and non-Indigenous, be challenged to articulate the marginalization problem as they draw their graphs? Perhaps students might be able to create a method of combining qualitative ideas on a quantitative graph; that is, a form of hybridized knowledge (Aikenhead & Michell, 2011; Enyedy et al., 2011; Lipka et al., 2007; Jannok Nutti, 2013). Perhaps an Elder might have a solution. Perhaps students' unsuccessful attempts would be sufficient to teach them a rational limitation of EAM Platonist content – it was not created to communicate a subjective consciousness. This in itself is a very powerful lesson about a cultural aspect of EAM. Perhaps students will learn an epistemic presupposition of conventional school mathematics' problem solving: Ignore the context even though one is presented to you.

At the same time, students who learn the Lahal game in an experiential way could gain an insight into Mohawk ceremonies and spirituality. In other words, students can acquire an *intellectual* understanding of the EAM Platonist concept of probabilities, while in the same lesson, students can appreciate a *wisdom* understanding embraced by Mohawk traditions. This lesson illustrates two-way learning.

Indigenous language-laden cognition shares some similarities with Japanese language-laden cognition. As Kawasaki (2002, p. 42) pointed out, the Anglo meaning of "to observe" invokes Plato's World of Ideas (abstractions). He went on to explain that in Japan, Euro-American mathematics (Yousan) instruction creates a challenge for Japanese teachers because their Japanese language has no abstract nouns; only nouns that fit Plato's Phenomenal World; in this case "kansatsu" (to observe). Some Japanese scholars believe, as Kawasaki does, that Japanese students are being assimilated (indoctrinated) into a Euro-American global culture by being forced to set aside their traditional meaning of kansatsu, which includes a subjectivity-conscious set of peripheral concepts, and adopt the objectivity-conscious meaning of "to observe" into their self-identity; another example of cognitive imperialism (Battiste, 1986).

It is worth repeating: to gain insights into Indigenous language-laden cognition, a researcher might learn *some features of the language* held by Indigenous participants in a R&D project (Fyhn, 2013; Matthews, 2015; Lunney Borden, 2013). For instance, the Māori word "ako" refers to both the act of teaching and the act of learning (Bishop & Glynn, 1999). The English distinction between teaching and learning is not found in te reo Māori (the Māori language). Moreover, there is no Māori equivalent to the English "teacher" or "student." The closest is "tuakana" (a teacher who learns from students) and "teina" (a student who teaches their teacher); a fact that speaks volumes about a Māori perspective on education.

In a fairly similar way, the Nehinuw language (Swampy Cree, in western Canada) uses the same stem "kiskinauma" to refer to both teaching and learning, but the word would never be used on its own (Goulet & Goulet, 2014, pp. 65-69). The speaker must also specify one of three modes of teaching/learning (emphasis added for clarity of reading): (1) kiskinauma*gehin* – teaching another; learning is from someone who has more expertise or wisdom; (2) kiskinauma*towin* – teaching each other; learning is interactive and based on equality; and (3) kiskinauma*sowin* – teaching oneself; learning is autonomous and the learner takes responsibility. Thus, if a teacher spoke of kiskinaumatowin, a student would anticipate a much different interaction than if a teacher spoke of kiskinaumagehin. This information illustrates "some features of the" Nehinuw language.

In the eastern coast lands of Canada, a Mi′kmaw speaker may say "mawikinutimatimk" (translated as "coming together"). But as Lunney Borden (2013, p. 9) pointed out, the translation loses the peripheral concepts of "equity and mutual respect." Perhaps the English translation of mawikinutimatimk should be "camping spots of dialogue" (Vickers, 2007, p. 592).

Lunney Borden, in Sterenberg and colleagues (2010), discussed the depth to which students can get lost in translation between the teacher's English or French and students' Mi′kmaw first language. The Mi′kmaw school principal where Lunney Borden taught, pointed out

> the difference between *lnuitasi* (Our people's ways of thinking) and *aklasiweitasi* (Anglophone ways of thinking). She argued that many conflicts arise for children when their ways of thinking (*lnuitasi*) come into conflict with teachers who have different ways of thinking (*aklasiweitasi*). There is something being lost in the translation of worldviews, of ways of thinking and styles of communication. She claimed that she sees the conflicts arising on a daily basis as the students in her school struggle to find their way through a colonizing curriculum. (p. 11, original emphasis)

As mentioned earlier, just because students can speak English does not mean they necessarily have aklasiweitasi. Lunney Borden (p. 12) offered specific illustrations of: (1) "the importance of reclaiming mathematical words and supporting Mi'kmaw speaking teachers to develop a lexicon of words that could be used in their classes;" (2) how to gain new insights into lnuitasi (our people's way of thinking) by asking students "What's the word for…?" or "Is there a word for…?" These questions often lead to insightful learnable moments for teachers; and (3) "there is a sense of motion embedded in the Mi'kmaw language that is not apparent in school-based mathematics. Shape and space words act as verbs in Mi'kmaw and are dynamic" (sections 4.1.3 & 6.2). For a very detailed discussion on these points, see Lunney Borden (2010).

As discussed in section 5, the Māori translation of the English mathematics curriculum could not alter its Anglophone epistemic structure. As a result, Māori students were surreptitiously being taught to think in a Euro-American way – cognitive imperialism. Barton (2008) has experienced similar struggles. Lunney Borden (in Sterenberg et al., 2010) pointed out that Barton (2008) believes that mathematics evolves with language. Consequently: "A proper understanding of the link between language and mathematics may be the key to finally throwing off the shadow of imperialism and colonialisation that continues to haunt education for [I]ndigenous groups in a modern world of international languages and global curricula" (Barton, 2008, p. 9, quoted by Lunney Borden, in Sterenberg et al., 2010, p. 11).

In an attempt to avoid cognitive imperialism, Denny (1981) collaborated with a group of Inuit Elders in Artic Canada to explore mathematical words in the Inuktitut language. Lunney

Borden (in Sterenberg et al., 2010) described Denny's work: "Rather than developing curriculum and translating it into Inuktitut, they used the mathematical words to develop the curriculum and associated school mathematics activities" (p. 11). But it was not until 1996 that a wholistic comprehensive school curriculum, *Inuuqatigiit: The Curriculum from the Inuit Perspective*, was developed from an Inuit perspective (Inuit Subject Advisory Committee, 1996). This inspired the production of *School Mathematics Glossary – English-Inuktitut Glossary*, produced by the Nunavut Arctic College, Nunatta Campus (http://www.arcticcollege.ca/nunatta-campus). Inuktitut syllabics, shown in Figure 1, require a separate font software for computers.

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Figure 1. "Nunavut Artic College"

The document *Inuuqatigiit* conveys Inuit Elders' knowledge and wisdom with which to contextualize all school subjects in Inuit culture. To encourage teachers to integrate *Inuuqatigiit* into their mathematics instruction, the Government of Nunavut's Department of Human Resources (2005) developed conjointly with its citizens the document *Inuti Qaujimajatuqangit* (*IQ*), which captures "all aspects of traditional Inuit culture including values, world-view, language, social organization, knowledge, life skills, perceptions and expectations" (Higgins, 2011, p. 18); in other words, the customs, ground rules, and correct ways to behave for Inuit people. The goal is to balance school mathematics with Inuit culture, usually by focusing on eight key values to guide curriculum developers, teachers, and student assessment. Each value is correlated with their mathematics curriculum or program (Kivalliq Math Education Panel, 2014); for example, "Piliriqatigiingniq (collaboration)" with "Collaboration requires communication and communication is a mathematical process that is strengthened through group work" (p. 6). Similar emphasis is given to values in projects found in sections 8.4, 8.6, and 8.7.

A mathematics curriculum could be developed from *Inuuqatigiit* and *IQ* as Denny (1981) envisioned, but the authority of the conventional Alberta mathematics curriculum mandated in Artic Canada has damped interest in such a major project.

Generally in the northern hemisphere, there are very few EAM programs fully immersed in students' Indigenous languages. Indigenous immersion classrooms in preschool and primary grades occur more frequently, but locally developed methods and materials are usually the only resources for mathematics. For example, Russell (2016) explained that Inuit primary teachers in northern Québec were required to speak Inuktitut fluently. But those who did, seldom had an academic background. Their students were always very well prepared for Grade 4 where English or French instruction began, *except for mathematics*. It was discovered that the Inuit teachers had taught Inuit mathematizing in keeping with Inuktitut, which uses base-20 (and subbase-5). It is also done mentally, according to the oral tradition, not on paper. Similar to some other cultures, Inuit numbers have different names depending on the context, of which there are six in Inuktitut. No wonder Grade 4 mathematics was a disaster! These revelations give modern impetus to Denny's (1981) agenda to find or invent Inuktitut expressions for a base-10 number system in which names of numbers are universal and everything is written in symbols. While this incident is an extreme example, it makes us vigilant to more subtle instances involving Indigenous students steeped in their language and culture.

A Lil'wat (L̓il̓wat7úl) First Nation group of Elders translated a Grade 1 mathematics textbook at the Xit'olacw Community School in Mount Currie, British Columbia (Sterenberg et al., 2010, p. 13). The Sámi Nation in Norway and Sweden has immersion programs (Fyhn et al., 2011; Nutti, 2013, respectively). Similar to the Māori bilingual situation, the Sámi translations were criticized for attempting to assimilate Sámi students into a Eurocentric epistemology and for disadvantaging Sámi students on national examinations (Fyhn, 2013).

The translation of a country's Platonist mathematics curriculum into an Indigenous language is highly problematic because the former is considered value-free, while the latter envelops cherished values. All learners, but especially Indigenous learners, come to anticipate the centrality of values in every language because that is their linguistic experience. However, the language of Platonist mathematics seems to students to be an exception. Most Euro-American educators do not recognize this problem when students become confused by the purportedly value-free feature of Platonist content. Teachers tend to respond with the intuitive pedagogy of repeated practice and memorized algorithms.

This partial explanation for students' confusion with school mathematics fits many non-Indigenous students as well. But it certainly does not fit a small number of mathematics-*oriented* students, Indigenous or non-Indigenous. How small a number? The proportion is 24 percent of the 15-year-olds in OECD countries, according to the 2015 PISA results (OECD, 2016).

A student's orientation to school mathematics depends on the degree to which their worldview harmonizes with school mathematics' purported value-free, abstract, decontextualized objective characteristics (Aikenhead, 1997, 2006) (sections 3.3 & 8.7). As a result, culture clashes can range from modest to severe for “76 percent” of 15-year-olds in OECD countries (OECD, 2016, p. 362).

A solution? Ministries of Education need to establish a cross-cultural Euro-American mathematics (EAM) curriculum with: (1) an amalgam of Platonist content and EAM *cultural content* that includes values (section 4.5); and (2) the inclusion of local Indigenous mathematizing, beyond tokenism (sections 7 & 8). Then the transition for students will be from one cultural practice (their home culture) to another cultural practice (EAM), which tends to mitigate mathematics-shy students’ modest to severe culture clashes.

Other challenges remain for translators and for writers of teaching materials, because there is clearly a close connection between the amount of Indigenous language used in class or in resources and the degree of decolonization achieved. For instance, similar to the language of Japan, Nehiyawewin (Plains Cree language) does not use a definite article – the word “the.” This word has the power to essentialize an idea; for instance, *the* law of gravity or *the* scientific method. So when one hears some Nehiyawak (Plains Cree people) say, “They offer prayers to Creator,” that is their authentic traditional view expressed in English. If they said, “They offer prayers to the Creator,” one could surmise that the person has unconsciously used the expression “the Creator” due to either having been assimilated into thinking according to SAE-laden cognition, or wanting to convey an allegiance to a Euro-American religion. High quality teaching materials avoid a perception of cognitive imperialism by not using definite articles, if appropriate, when writing about Indigenous understandings; thereby pervasively nurturing an Indigenous student’s cultural self-identity, while diminishing students’ feelings of marginalization.

Life-long learning describes the experience of becoming acquainted with an Indigenous community’s language-laden cognition. Thus, the point is not to be proficient before embarking on developing materials or teaching them. The point is to be vigilant to opportunities (learnable moments) to add to the language-laden cognition ideas we have already accumulated. Our willingness to learn demonstrates respect, which in turn is rewarded by Indigenous collaborators or students not being concerned when “mistakes” happen. Learnable moments frequently arise from cross-cultural misunderstandings (Aikenhead et al., 2014). In addition, Lunney Borden (2013, p. 5) again wisely counsels teachers and researchers to ask, “What’s the Mi’kmaw word for…?” and be open to learning more about an Indigenous person’s vocabulary, syntax, and worldview.

Another excellent source of learnable moments is the process of back-translation, in which the peripheral concepts associated with an Indigenous central concept are, to some degree, exposed (section 6.3). It is a translation into English that conveys Indigenous thinking.

A Nehiyaw back-translation of the word “language” is “taking something apart from the female body that has life of the wind” (“pikiskwewin” in Nehiyawewin; Halfe, 2015). Back-translations can reveal just how much information, such as peripheral concepts, is lost in translation. This lost information can usually be made explicit during camping spots of dialogue (Vickers, 2007) or cultural immersions. Accordingly, it is incumbent on teachers and researchers to discover the back-translation of certain key EAM words when developing teaching materials relevant to Indigenous students. Back-translations can play a crucial role in EAM classrooms.

Lipka and colleagues (2013) reported, “…when elders are asked what Yup′ik word or concept best describes mathematics, ‘cuqete’ (to measure) is given” (p.133). It is interesting to see the “cuqete” translation, but imagine how much more could have been gained if Lipka and colleagues had reported a back-translation of “mathematics.” A small window into Yup′ik worldviews was inadvertently closed.

As mentioned before, a group’s language is an encyclopedic repository of the group’s culture. Accordingly, the more we become acquainted with at least some aspects of our Indigenous students’ language, the better we can appreciate their thinking and culture; and consequently, the more authentic our teaching materials will be. In other words, the risk of cognitive imperialism, culture clashes, and marginalization is minimized when the descriptions of Indigenous culture are least influenced by a Euro-American perspective. This is where back-translations are particularly useful.

At the same time, we must realize that a SAE (Standard Average European) language is also an encyclopedic repository of a Euro-American culture. If the teaching materials for culture-based school mathematics are going to be highly successful, then *it is absolutely essential to edit the SAE language* so it conveys the fact that EAM is a human, social and cultural enterprise with its own ideologies and its own set of cultural practices. In other words, *a writer must critically analyze their SAE language to detect and rewrite expressions that suggest EAM content is acultural and objective when applied, and that no other mathematical system is worth considering*.

7. Diversity Related to Teaching Materials Produced

Mathematics is commonly seen as consisting essentially of computation and formulas, yielding exact and infallible answers, without relevance to everyday life, accessible only by experts, and not open to criticism.

Swapna Mukhopadhyay and Brian Greer (2001, p. 297)

The simplistic view of mathematics described by Mukhopadhyay and Greer, as commonly held by the general public, can be attributed to the experiences the public accumulated in Platonist school mathematics classrooms. On the one hand, this state of affairs sadly reflects the serious shortcomings in conventional school mathematics. On the other, it echoes Platonist school mathematics' success at intimidating the general public from questioning the validity of the school subject's gate-keeping function (section 3.2) and from criticizing the subject matter itself, outside of grumbling about its difficulty and irrelevance. But as documented in this monograph, some mathematics educators have produced teaching materials that offer students alternative classroom experiences that lead to outcomes much different from those described by Mukhopadhyay and Greer (2001) (section 4.5 & 8); that is, culture-based Euro-American mathematics (EAM).

There is great diversity among the innovative R&D projects that include Indigenous mathematizing[x]. Projects involve a wide variety of students: from Indigenous students steeped in their culture, to those who seem to be assimilated into the mainstream culture. The contexts in which the projects take place also differ. There are Indigenous schools located on reserves, under the jurisdiction of a federal government, and with varying degrees of autonomy by the reserve's local government; as well as non-reserve (rural and urban) schools under the jurisdiction of provincial or state governments with a mixture of non-Indigenous and Indigenous students. Many of the projects have been undertaken by non-Indigenous researchers with varying degrees of collaboration with Indigenous colleagues, Elders, and communities. There are large R&D projects blessed with ample funding, and small projects blessed with one or a few passionately innovative mathematics educators whose accomplishments rarely become known outside their local domain.

All of these factors influence the processes and products of R&D projects that draw on Indigenous perspectives. Some will be add-ons to a conventional mathematics curriculum, while others will give weight to the culture of an Indigenous community. These two positions represent two ends of a continuum, from *contextualized* to *cross-cultural* school mathematics.

The contextualized end strongly emphasizes Platonist content outcomes, motivated from time to time by using an Indigenous artifact or activity to introduce a Platonist idea. The cross-cultural end gives priority to EAM Platonist content but highlights detailed features of Indigenous perspectives and EAM cultural content. Students move back and forth between the two cultures, learning and discussing each culture's mathematizing. Student assessment addresses both cultures but emphasizes EAM. The continuum reflects the degree to which Indigenous perspectives contribute a significant presence to the materials produced.

Of course, the true test of significance only comes with evidence of how classroom teachers use those materials (Lipka, et al., 2005; Jannok Nutti, 2013; Russell and Chernoff, 2013); a topic rarely included in the literature. Two exceptions are worth noting, however. First, evidence from case studies helps make that decision. Four case studies from the Alaskan Math in Cultural Contexts project have been published in the *Journal of American Indian Education*, 2005, Issue 3). Secondly, Russell and Chernoff's (2015) research in classrooms identified critical incidents in which teacher's worldviews and Platonist beliefs disrupted their engagement in Indigenous-inspired teaching strategies valuable to Indigenous students.

7.1. Making Connections

Both contextualized and cross-cultural approaches aim to resolve an oft repeated complaint about school mathematics:

> A former Blackfoot student, now a medical doctor, who struggled greatly with the sciences and mathematics, said, "It helps if you show it first with something that makes everyday sense to me so that I can see the relationship for myself then let me do it so I can see how it works first". Students need context to make connections and the more familiar that context is, the easier the connections are. (Hogue, 2013, p. 1)

In other words, the Blackfoot MD is advising a mathematics-in-action approach to teaching. However, this sound advice comes with two crucial qualifications. First, if a SAE word for a EAM concept does not have a direct Indigenous translation, then in accordance with language-laden cognition, the concept will most likely be foreign to an Indigenous speaker. For instance, the English word "middle," as in "point to the object in the middle," has no equivalent in the Mi′kmaw language in eastern Canada (Lunney Borden, 2013, p. 19). A back-translation is "go half way" (aqatayik). This is a reminder that not all noun-based concepts can be directly translated into verb-based concepts, and vice versa (Neel & Fettes, 2010).

Secondly, Enyedy and colleagues (2011) discovered that students differ according to how strongly they hold on to the peripheral concepts attached to mathematizing in an everyday situation. At one end of this spectrum, some students with a strong allegiance to the everyday Indigenous context cannot let go of a construct's peripheral concepts, and consequently on their part there is little movement from their local everyday cultural mathematizing towards the school classroom Platonist mathematizing (see also Gellert & Jablonka, 2009). At the other end of the spectrum, some students have already figured out ideological features of school mathematics: "[T]he real game [is] to ignore the context" (Enyedy et al., 2011, p. 276); or the teacher's expression, "I am not talking about reality here" (Gellert & Jablonka, 2009, p. 39); or the phrase "just suppose." This protocol in the culture of Platonist school mathematics, suspend reality, is foreign to many students, especially Indigenous students.

Greer and Mukhopadhyay (2012, p. 237) explained that this protocol is found worldwide when students are presented with word problems such as, "John's best time to run 100 metre is 17 seconds. How long would it take him to run 1 kilometre?" Greer and Mukhopadhyay noted,

> With remarkable uniformity, [students] predominantly answer in a way that appears to negate their real-world knowledge [i.e., 170 seconds or about 2 minutes and 50 seconds]. …The implication is that, after some years at school, children learn to play a game in which one of the rules is that when asked a mathematical word problem, what they know about the world should be ignored. (p. 237)

The researchers concluded, "It does not seem too far-fetched to conjecture that this training in school mathematics may contribute to an attitude in later life whereby a veneer of mathematics inhibits critical application of real-world knowledge" (p. 238).

It can be difficult to move back and forth between a culture that celebrates useful subjectivity-conscious features of mathematizing, and a culture that celebrates objectivity-conscious decontextualized EAM Platonist content. In other words, many Indigenous students negotiate between, one the one hand, a knowledge-in-action value-laden Indigenous culture, and on the other, an apparent "let's pretend," value-free, EAM Platonist content. There would be less culture clash if they moved into a cross-cultural EAM that dealt with both the "let's pretend" (EAM Platonist content) and the "mind the hidden values, ideologies, and applications" (EAM cultural content). This situation will be familiar to Indigenous students raised on paradoxical reasoning (Maryboy et al., 2006) or indeterminism (Sriraman, 2005), or to any student experienced in "two-eyed seeing" (Hatcher, Bartlett, Marshall, & Marshall, 2009). The two categories, EAM Platonist and cultural content, wholistically interprenatrate within most

Indigenous students' minds naturally; but not so easily in the minds of mathematical and scientific educators (sections 4.3 & 4.5).

This deficiency is illustrated by Russell (2010). First, a cross-cultural mathematics classroom "provides all students the opportunity to understand the connection between mathematics, place, and need. It engages students in rich and deep meaning making, providing students with alternative representations and ways of knowing and understanding ideas" (p. 43). These alternative representations that come from different cultures are often identified with a particular race. Hence, a school mathematics curriculum that promotes Euro-American numeracy but denies teaching students a non-Euro-American culture's numeracy, is "unintentionally telling students with other backgrounds [and races], other beliefs, and other thoughts that they are wrong, inferior, and incapable of attaining superior intellect" (p. 43). Russell points out that, unintended or not, this promotes a racist-based stance – "racism by numbers" (p. 36) in her words. Being associated with a racist ideology is not easy for mathematics educators to grasp or accept.

Moreover, when the conception of Western mathematics is synonymous with EAM Platonist content, mathematics educators seem blind to Western (Euro-American) mathematics' cultural content conventionally suppressed by Platonists (e.g., Euro-American cultural features associated with superiority, aggressive authority, and control). If this suppressed cultural content is made explicit to Indigenous students, then those students have a more transparent view of a connection between their culture and the culture of mathematics. The process is myth busting. Then students are free to treat EAM Platonist content as being just another culture's way of mathematizing; different but not superior to their own culture's ways of mathematizing (section 4.5). In other words, EAM cultural content taught in mathematics lessons can make the connection between students' place-based culture's mathematizing and EAM Platonist content; thereby easing students' transition from their culture to the culture of EAM. The concept "different but equitably valid" helps erase vestiges of neo-colonial racism from our school mathematics; but only when we replace this blind spot with an equitable connection.

Therefore, an important issue for cross-cultural teaching and R&D projects is how to make the transition between the two cultures clearer and easier; and then include that strategy in the teaching materials produced. The issue is indeed complex (Skovsmose, 2012). Lunney Borden (2013, p. 10) offers an evidence-inspired concept map to introduce the complexity of "meaningful personal connections to [school] mathematics." Her concept map is well worth studying. Students and their teacher became involved in what Lunney Borden (2013, p. 9) called "mawikinutimatimk," Mi′kmaw for "coming together…[with] equity and mutual respect."

In the graphing lesson discussed in section 6.4, there were three approaches suggested to resolve the problem of what gets lost in graphing data that arise from an Indigenous activity (the Lahal game). One approach was to ignore what gets lost; but this only helps to marginalize Indigenous perspectives. Secondly, if sufficient attention were paid to the meaning of the Indigenous activity, sufficient enough to have Indigenous students feel they had developed a stronger understanding of their culture, then the mathematics lesson would have been contextualized very appropriately. The third strategy was to embrace both cultures explicitly by discussing similarities, differences, strengths and limitations of each culture, EAM and Indigenous.

If teachers and researchers treat Indigenous artifacts, practices, and ideas as "*merely* objects of mathematical thinking and knowledge" (Russell & Chernoff, 2013, p. 114), they risk enacting a neo-colonial agenda by ignoring the foundational policy that Indigenous students need to strengthen their cultural self-identities while engaging in school mathematics (Nasir, 2002).

One example, is a work sheet for British Columbia teachers attending a conference (Neel & Pusic, 2009). Its purpose was: "to explore mathematics and culture *through the lens of* the Squamish Lil'Wat Nations" (p. 1, emphasis added). Each of 11 Indigenous artifacts was analyzed to promote mathematical conversations around Platonist school mathematics content. However, nothing about Squamish Lil'Wat culture could have been learned by participating in this work sheet; thus, objects were explored not through an Indigenous lens as advertised, but only through a Platonist lens, which promotes the myth that the Squamish Lil'Wat people have no mathematizing (section 4.1.1). These Squamish Lil'Wat cultural possessions are treated *merely* as objects (Russell & Chernoff, 2013). No permission to analyze them is stated in the document. The work sheet is simply an exercise in Platonist projectionism (section 9.3). Teacher conferences should be prime venues for illustrating the ethos of cross-cultural mathematics education.

A minimally appropriate project may nurture stronger Indigenous self-identities for a few students to some degree, but negative reactions can result for the others. Partially quoted earlier but well worth being reminded by university mathematics professor Doolittle (2006):

> My feeling is that Indigenous students who are presented with such oversimplification feel that their culture has been appropriated by a powerful force for the purpose of leading them away from the culture. The [contextualized teaching materials] may be reasonable but the

> direction is away from the culture and toward some strange and uncomfortable place. Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures. (p. 20)

More appropriate teaching or projects will meet Davison's (2002, p. 24) challenge to produce teaching materials "so that they make sense both in the context of the [Indigenous] culture and in the context of the school mathematics curriculum." The standard of appropriateness increases even higher with Enyedy and colleagues' (2011) position on the relationship between researchers and Indigenous community members:

> Both parties must legitimately be engaged in the negotiation process in order to avoid creating artificial activities under the guise of authenticity; engaging both parties in *co-construction* of a local meaning of relevance helps to ensure that the end product is, in fact, relevant and meaningful to all of the participants. (pp. 288-289, emphasis added)

The potential quality of contextualized teaching and R&D projects generally increases when they are characterized by descriptors such as co-constructive, collaborative, cooperative, co-leadership, and co-governance.

Doolittle (2006, p. 19) shared word problems that meet the minimum criterion of appropriate contextualized school mathematics. Sometimes a teacher is rewarded by a student's honest answer when it reveals aspects of a student's Indigenous culture of which the teacher was unaware – a learnable moment for the teacher.

1. *Question*: Imagine you and three friends are sitting on the ground with 72 pennies piled in front of you. What would you do so that each of you got the same number of pennies?

   A student's answer: Pass them out until they are all gone.

The student gave a correct answer according to his/her Indigenous mathematizing. Indigenous verb-based languages reflect action. It is no surprise, therefore, that Indigenous mathematizing tends to favour action. For the Kankanaey people in northern Philippines, their concept of a circle (a noun) is encircling (a verb) (Salleh, 2006). Another example of action is a back-translation of "circumference" – running around a small stone.

2. *Question*: If your big brother took his truck to Calgary, how much would he have to spend on gas?

   A student's answer: My brother doesn't have a truck.

Again, a correct answer. The student took the situation seriously and personally, rather than hypothetically, thereby revealing a difference of values between the culture of EAM Platonist content (objectivity-consciousness associated with Plato's World of Ideas) and the student's Indigenous culture (subjectivity-consciousness associated with Plato's Phenomenal World).

Sterenberg (2013a) noted that because of a question's wording, some Indigenous students *could not see themselves reflected* in the situation described, and could not relate to it. Their honest response to questions will hint at what they do relate to. On a much larger scale when students do not see themselves reflected in the mathematics curriculum, their reactions to it are much the same (Lunney Borden, 2013).

The second question (above) reveals an attribute of school mathematics: A student is expected to answer problems as if the students belonged to the culture of EAM. The question, therefore, should have included the direction: "Answer these problems like your mathematics teacher would answer them." This shifts the focus from *believing* the EAM content (the go-to position of the student's response) to merely *understanding* the Platonist content hypothetically as the teacher actually expects (section 3.3). Teachers and their instructional materials must make the connections explicit. Straight talk works well; inference-laden talk by typically middleclass teachers, not so much.

At the beginning of this subsection, Greer and Mukhopadhyay (2012) noted how conventional school mathematics encouraged students to ignore their real-world knowledge. Their example was about using proportional reasoning to extrapolate how fast John would run a kilometer, given his 100 meter best time. This contextualized problem would create a challenge for Indigenous or non-Indigenous students who understood proportional reasoning but had strong allegiances to their local knowledge. How would they bridge "in-school and out-of-school practices that might include mathematics?" (Skovsmose, 2012, p. 351).

In the example above, the out-of-school context generally pertained to any culture, and involved a type of mathematics-in-action introduced in section 4.5 with examples by Andersson & Ravn (2012). Similar examples include: estimating health risks when choosing a sport, deciding on a cellphone plan that will give you the most for your money, figuring out the cheapest trip for a beach holiday, and "knitting a pullover" (Skovsmose, 2012, p. 349). Both non-Indigenous and Indigenous students benefit from learning mathematics when engaged in this type of out-of-school situations – mathematics-in-action (Andersson & Ravn, 2012).

Depending on the degree of harmony or discord between a student's worldview and the worldview endemic to Platonist mathematics, culture clashes will exist between out-of-school mathematizing and in-school Platonist formal discourse (Aikenhead, 1997, 2006) (sections 3.3 &

8.7). These clashes are exacerbated by differing values and peripheral concepts in the two contexts (sections 4.4 & 6).

A second type of mathematics-in-action described in section 4.5 addresses the influence of Platonist content on political-social-economic-military features of mainstream culture (Skovsmose, 2005, 2016). Skovsmose (2012, p. 350) also introduced the idea of mathematisation. “Mathematisation refers to the phenomenon that many social practices have becomes structured through mathematics-based technological systems.” Benevolent or malevolent results can occur. This societal influence of Platonist mathematics extends to Indigenous cultures as well.

Skovsmose (2012) promotes updating school mathematics into a subject that embraces all types of mathematics-in-action that he expects will occupy a significant portion of the school curriculum. “When relationships are made visible between what is happening in the classroom and some practices outside school in which the students might become involved, a resource for students’ construction of meaning has been established” (p. 352). He suggested that bridging out-of-school mathematizing and in-school Platonist content “could be a sense-making activity for many students” (p. 353). Students could be assessed in terms of how well they did this, as Andersson and Ravn (2012) demonstrated.

This subsection 7.1, “Making Connections,” has described many components that can comprise contextualized and cross-cultural Euro-American school mathematics. One task remains, however: to summarize the components and identify their interrelationships. The following outline serves as a guide for exploring Figure 2, which depicts the connections among all components of cross-cultural school mathematics. There is some overlap among the components; they are not discrete categories. The letters A to D in the outline below refer to regions A to D in Figure 2.

A. The outer circle represents the content of a country’s Euro-American mainstream culture.
B. The inner three circles represent content in Euro-American mathematics (EAM), comprised of:
    1. the culture of Platonist mathematics: its history, ideologies, values, and presuppositions
    2. mathematics-in-action in a country
        i. mainstream cultural artifacts, activities or ideas that explicitly or implicitly involve either Platonist mathematics or analogues of it.
        ii. explicit or implicit societal influences that the use of Platonist mathematics has on mainstream and Indigenous cultures, in which “one faces social and ethical responsibilities” (Skovsmose & Greer, 2012, p. 381)
    3. Platonist content.

C. The ellipse represents the mathematizing found in an Indigenous culture; that is, a mathematical knowledge system, among many worldwide, which differs from the knowledge system used in mainstream Euro-American countries.

D. The shaded rectangle represents school mathematics that draws from four content areas shown in Figure 2: B.1, B.2, B.3, and C. School mathematics also includes the instruction, assessment, and learning environment experienced by students (section 8), not represented in Figure 2.

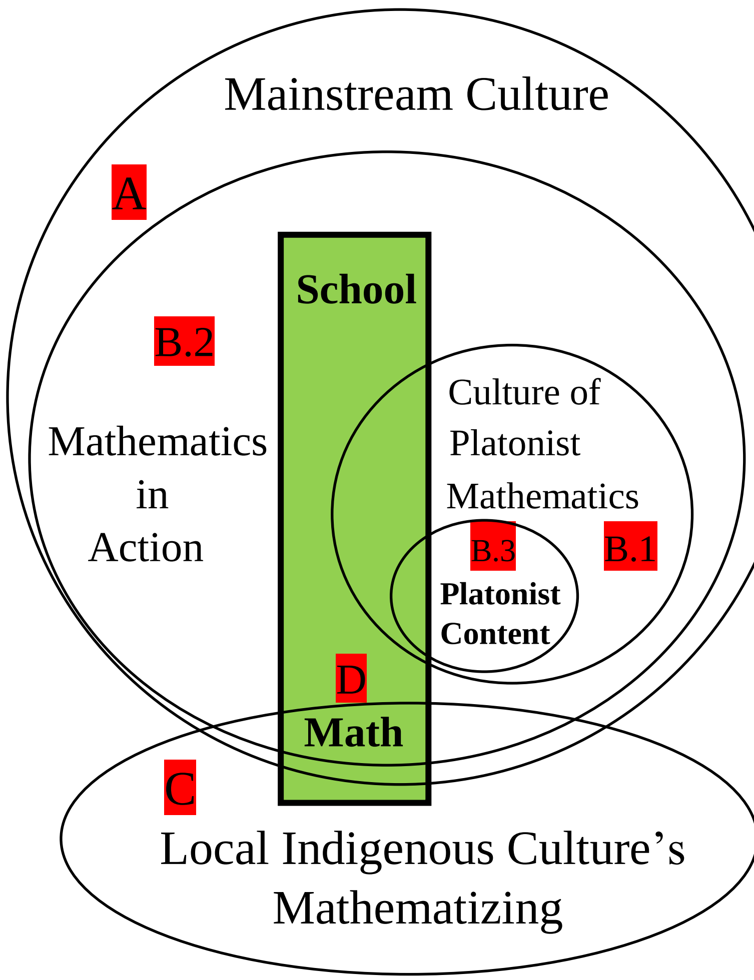

Figure 2. A schematic showing relationships among the components of cross-cultural Euro-American school mathematics.

7.2. Ethnomathematics

The social-justice politics of D'Ambrosio's (2003) ethnomathematics does counter some neo-colonizing ideologies that place high status on conventionally taught, decontextualized, school mathematics courses. He introduced his ethnomathematics to make EAM Platonist content more inviting and accessible to under-represented minority students (D'Ambrosio, 1991). Ethnomathematics is expected to be "far more reflective and respectful to Indigenous traditions of thought" (Doolittle, 2006, p. 20).

D'Ambrosio's pedagogical strategies are recognized as one approach to implementing culturally responsive, place-based, or culture-based school mathematics; among other terms (Chronaki, Moutzouri & Magos, 2015; Furuto, 2017; Sterenberg, 2013b). In his pursuit of peace education, D'Ambrosio (2007, p. 34) wrote: "The main goal of Ethnomathematics is building up a civilization free of truculence, arrogance, intolerance, discrimination, inequity, bigotry and hatred." He gives special attention to the political-social contexts of use for Platonist content (mathematics-in-action), focussed on "the technological, industrial, military, economic and political complexes [that are] responsible for the growing crises threatening humanity. Survival with dignity is the *most universal problem facing mankind*" (p. 25, original emphasis). He points, for instance, to mathematicians who work in the "weapons industry" (p. 27).

Boylan (2016, p. 403) concluded from D'Ambrosio's (2010) recognition of the intimate mutual relationships among the individual, the other, and nature, that ecological ethics requires "an environmentally informed critical mathematics education."

D'Ambrosio pedagogy can also be characterized as contextualized EAM Platonist content that highlights *human interactions and responsibilities* found in students' local community (Greer et al., 2009). Therefore, his ethnomathematics celebrates the policy of strengthening Indigenous students' self-identities.

As defined in sections 4.3 and 7.1 (Figure 2, area B.2), EAM cultural content includes the mathematics-in-action features found within Ethnomathematics, which Skovsmose (2016) explores in detail. But EAM cultural content also comprises the history and the ontological, epistemological, and axiological presuppositions that wholistically intertwine with Platonist content (Figure 2, area B1).

The history and presuppositions portion of EAM cultural content, however, does not seem to exist for D'Ambrosio (1991, 2003, 2006, 2007, 2016). By accepting the myth that Platonist mathematics is value-free, decontextualized, acultural, non-ideological, and purely objective in its use, his term "mathematics" in the expression "school mathematics content" consistently seems to convey the myth, given the fact he does not discuss any of its internal cultural values or ideologies (section 4.5), but only its association with the imprint of Euro-American cultures; that is, only mathematics-in-action. In other words, he does not extend pluralism to include Platonist mathematics. Every other culture's mathematizing is recognized by him as being cultural, but not Euro-American's mathematizing. Thus, he embraces an amputated version of mathematics' pluralism. This amputated piece – the presuppositions and history of Platonist content – happens to help diminish the culture clash between most Indigenous students and their Platonist school mathematics classroom (section 3.3). Further evidence of D'Ambrosio's blindness to the internal cultural nature of Platonist content comes from some projects described in section 8. Jablonka and Gellert (2012, p. 293-294) pose more "pitfalls" they perceive in D'Ambrosio's ethnomathematics.

As a result of ethnomathematics restricting itself to a truncated pluralism, the hegemonic power imbalance favouring mathematics curricula over Indigenous communities is not *sufficiently* challenged by ethnomathematics. Indigenous students' self-identities could certainly be strengthened much further than they are.

Unfortunately, the name "ethnomathematics" itself suggests that school mathematics is "real mathematics" because it is mainstream and differs from the subordinately prefixed *ethno*mathematics. Given the Euro-American ethnicity of Euro-American mathematics (EAM), logically it too is an ethnomathematics. An identical point was made in science education by Medin and Bang (2014). "Worse yet is the term 'ethnoscience,' which is used to refer to the study of 'non-Western systems of understanding the world.' As Hess (1995) notes, one key problem with this term is that all knowledge systems are "culturally rooted," including Western science" (pp. 163-164). In other words, the literal meaning of ethnomathematics is a tautology, thus causing confusion and multiple conceptions found among mathematics educators.

Bishop's (1988b) implication for the superordinate designation of the term "mathematics" causes all subordinate mathematics, such as EAM, to become an example of an ethnomathematics. This pluralism helps to decolonize the political-social power imbalance between Indigenous communities and the mathematics curriculum. Therefore, the name "ethnomathematics" becomes a discriminatory influence against today's decolonizing agenda for school mathematics; and as a result, it is not fully supportive of today's reconciliatory renewal of Canadian education.

Treating school mathematics content as culture-laden is foundational to teaching mathematics culturally. To reiterate, the expression "Euro-American mathematics" means the amalgam of EAM cultural content and EAM Platonist content (sections 4.3 & 4.5). As detailed in section 9, however, some researchers who subscribe to culturally responsive, place-based, or culture-based school mathematics (or other labels of choice), which includes EAM cultural content for the full benefit of students, also insist their project is based on ethnomathematics, which excludes an important portion of EAM cultural content. Hence, researchers find themselves in a logical fix; a problem they usually just ignore, except for Jannok Nutti (2013).

She identified the core of her project as “the ethnomathematics research field based on Bishops six activities” (p. 69) (section 4.1). In that statement I detect a paradox.

Jannok Nutti correctly relied on a well-known academic for a definition of ethnomathematics repeated here: “the study and presentation of mathematical ideas of traditional peoples” (Ascher, 1991, p. 188). Ascher’s “mathematical ideas of traditional peoples” seems to eliminate Euro-American mathematics from ethnomathematics.

Here is the paradox: Ascher appears to ignore pluralism applied to Platonist school mathematics, but Jannok Nutti embraces both Ascher’s definition and Bishop’s (1990, p. 1) pluralist conclusion that school mathematics is “Western mathematics” (i.e., EAM); just one of the many culture-based mathematical knowledge systems in existence. But for Jannok Nutti (May 12, 2016, personal communication), Ascher’s ethnomathematics content had *political capital* for legitimizing local Sámi perspectives in her school’s mathematics classes. This legitimacy helped to rebalance colonial-initiated power imbalances that have always favoured Platonist content. In other words, for *pragmatic* reasons, EAM’s cultural status was purposefully ignored because it may have interfered with the political capital gained from identifying Jannok Nutti’s project with ethnomathematics in the context of her project’s Sámi school.

Another credible definition of ethnomathematics came from D’Ambrosio (2006) and was mapped out by Furuto (2014, p. 362): “the intersection of culture, historical traditions, sociocultural roots and mathematics, among others.” Notice that the word “mathematics” sits as an independent item at the end of a four-item list. This definition of ethnomathematics refers to EAM Platonist content only (section 8.4). Ethnomathematics certainly appears to eschew a significant portion of EAM cultural content.

However, in keeping with what Bishop (1988b) advocated, Neel (2008, p. 23) composed a unique definition that seemed to affirm the suppressed EAM cultural content: “Ethnomathematics has been identified as the study of mathematics that takes into consideration the culture in which mathematics arises (Ascher, 1991; Bishop, 1988b; D’Ambrosio, 1985).” Yet Neel consistently abandons this definition either by treating school mathematics as an acultural commodity or by assuming that Euro-American cultures had nothing to do with the evolution of EAM Platonist content.

By maintaining an acultural stance towards Platonist content, teachers and researchers unwittingly maintain a source of culture clash for Indigenous students (Doolittle, 2006; Meaney, 2002); a clash that need not be. The intention to mitigate such culture clashes seems contradicted by the allegiance to ethnomathematics as defined by those teachers and researchers.

Although many different ethnomathematical approaches work towards decolonizing school mathematics to some degree, mathematics educators could “up their game” to full advantage for all students, especially Indigenous students. Is it not time to retire ethnomathematics in the domain of mathematics education due to its evolved slogan status (section 5), its usual truncated pluralist stance, its discriminatory moniker, and its ambiguity among teachers and researchers?

In its place, we could recognize the political capital and good will of D’Ambrosio’s enormous contributions to school mathematics by establishing the expression “*D’Ambrosio mathematics*,” which would: (1) embrace *the cultural nature of* Platonist mathematics; (2) teach essential EAM Platonist content on a need-to-know basis (section 10.2) primarily based on artifacts, processes, and ideas found in an educational jurisdiction’s mainstream culture; (3) maintain the reality of mathematical pluralism that applies to EAM Platonist content, and (4) express a cross-cultural feature of school mathematics in a non-tokenistic way by incorporating artifacts, processes, and ideas found in various local communities’ mathematizing.

This makes D’Ambrosio mathematics a synonym for cross-cultural EAM. This updated version of D’Ambrosio’s social justice agenda is timely for Canada’s and Australia’s era of reconciliation, I submit.

Recently, François (2016, p. 187) emphasized ethnomathematics as a “field of enquiry” and discussed an “enriched meaning of ethnomathematics as an alternative, implicit philosophy of school mathematical practice.” In her argument to make ethnomathematics a human right, she mentioned three implications for research: (1) “to reveal the (explicit and implicit) values within mathematics,” (2)”the use and integration of pupils’ [home culture] mathematical practice in the curriculum,” and (3) “pupils’ daily mathematical practices [in the mainstream culture]” (p. 195). No specific examples were cited, but her enriched meaning of ethnomathematics seems to converge *towards* cross-cultural EAM.

8. Examples of Culture-Based School Mathematics

They could have come up with an education plan that would have complemented Native cultures and, perhaps, even enriched White culture at the same time.

Thomas King (2012) *The Inconvenient Indian* (p. 119).

As mentioned at the outset, this monograph is neither a comprehensive review of the literature nor a clearinghouse for R&D projects. Instead, it critically analyzes school mathematics and suggests what mathematics researchers and teachers should know when initiating culture-based school mathematics that embraces reconciliation.

A critical analysis entails identifying positive features that demonstrate best practices (section 8), as well as negative features that should be improved or avoided in the future (sections 8 & 9): Which innovative taken-for-granted notions (presuppositions) of school mathematics improve student achievement? and Which notions found in conventional school mathematics continue to serve students' interests?

In the context of this monograph, achievement includes four principal dimensions: (1) students' test scores on relevant EAM Platonist content compiled in a culture-based curriculum; (2) students' coming to know – a back-translation (section 6.3) of "to learn in a deep way" (Cajete, 2000, p. 110) –Euro-American mathematics (EAM) cultural content and examples of Indigenous mathematizing; (3) show-what-you-know assessment activities (Fyhn et al., 2016; Lunney Borden, 2013) (sections 8.6.1 & 8.7, respectively); and (4) Indigenous students strengthening their cultural self-identities (Nasir, 2002), while joining non-Indigenous students in reconciliation.

Many innovative cross-cultural projects are highly localized in keeping with the place-based nature of Indigenous perspectives (Michell et al., 2008; Sterenberg, 2013b). Most projects, therefore, are known only within their local educational jurisdiction. I would like to honour those many dedicated, resourceful, creative people who are unknown to the mathematics education literature. I do so by noting one typical example, the Indigenous education consultant for Saskatchewan's North East School Division, Sharon Meyer, of Nehiyaw (Plains Cree) ancestry. In collaboration with local Elders and extended family members, she has developed and photographed many Indigenous mathematizing activities that led students to learning EAM Platonist content and learning more about their community's culture. These activities and photos are shared with teachers throughout her large rural school division during workshops and classroom visits. Her Medicine Wheel Teachings, for example, integrate school subjects around certain topics or themes. With it, she can teach the abstraction of mathematical quadrants to Grade 1 students who understand their First Nation's medicine wheel's four directions.

Several larger innovative cross-cultural projects funnel all their human, time, and financial resources into cross-cultural school mathematics, putting research into practice locally without a publication trail to tell their story. This is particularly true for start-up projects, such as the northwestern Ontario-based Ojibwe primary mathematics "Gaa-maamawi-asigagindaasoyang Collective" (Caswell, et al., 2016). The project teaches "high quality mathematics in a way that is more inclusive, accessible, playful, culturally responsive and engaging" (p. 2). Grades 2 and 3 children in several communities have been involved in either experimental classrooms (inquiry, hands-on, minds-on, and contextualized in Ojibwe culture) or in control-group classrooms (conventional pedagogies), developing spatial visualization skills and foundational aspects of geometry. Their initial results a very encouraging.

A cross-cultural approach to creating teaching materials connects to both place-based and culturally responsive mathematics teaching/learning (section 1). Both approaches to teaching/learning wholistically embody the emotional, spiritual, intellectual and physical dimensions of a location where Indigenous people reside. Both combine a wholistic array of students' languages spoken, knowledge developed, and wisdom accumulated. Both are succinctly described here as a context for sections 8.1 to 8.8.

Place, in "place-based" education, is verb-like because it envelopes the interacting processes among "all my relations," centrally situated in familial, historical, ceremonial, social, economic, political, and memorable human experiences. Place constitutes self-identities: We are the land, and the land is us. Although place is associated with what is local land, its outer boundaries reach into the night sky, as experienced by that locale. Place includes everything, but it is unique to where a certain people live in Mother Earth. Guujaaw, an experienced carver and president of the Haida Nation, counsels: "We should not just focus on the curriculum but also find a way to integrate the learning with the land. …The flexibility has got to be built into the curriculum to allow our kids to learn about those things that are relevant to the life in the place that we live" (Neel, 2011, p. 120).

Culturally responsive mathematics teaching takes on many forms, but normally it promotes specific attention to: (1) the cultural identity of students and other assets/gifts they bring to a classroom; (2) EAM and Indigenous content relevant to students, as judged by students and Elders, by and large; (3) teaching strategies and methods amenable to Indigenous students, which when itemized read like what excellent teachers have always done; (4) culturally valid student assessment in the context of assessment for learning; (5) interpersonal relationships between students and their teacher, and among students themselves; and (6) the wholistic integration of the first five items from a student's perspective, often recognized as the classroom environment.

The following R&D projects and research studies serve as concrete guides for creating or adapting teaching materials for culture-based school mathematics. But it is not "a teacher merely using a handout or a worksheet to make a concept culturally relevant. That defeats the purpose. The teaching needs to be imaginative and creative to motivate students to learn in ways that are interactive and engaging" (Neel, 2011, p. 120).

8.1. British Columbia Projects

Two very different projects are discussed in section 8.1. A Haida Gwaii project established at Simon Fraser University illustrates meticulous attention given to Indigenous mathematizing practices that teachers or researchers can transform into mainstream mathematizing practices in school mathematics. The project also prescribes a four-point guide to organize a sequence of cross-cultural mathematics lessons.

A second project by the First Nations Education Steering Committee (FNESC) produced a Grade 8-9 teaching resource that provides teaching materials that have generally been contextualized in Indigenous mathematizing practices found throughout British Columbia, plus advice on how to emphasize relationships in the lessons taught.

8.1.1. Haida Gwaii

The Haida Gwaii project[xi] (Neel & Fettes, 2010) emphasized contextualized teaching materials related to highly authentic Haida First Nation mathematizing. The project is conceptualized here as a convergence of two overlapping harmonious projects: Learning for Understanding through Culturally Inclusive Imaginative Development (LUCID) (Fettes, 2006) and Numeracy in Haida Gwaii (Neel, 2008).

LUCID stressed collaborative, teacher professional development that created teaching units, outlines, and frameworks. Fettes (2006) attributed successful contextualized learning in all core school subjects to:

- workshops led by First Nations educators
- teachers who participated in community events and debrief with a mentor

- groups that read articles and books and watched films on Aboriginal education
- teachers from public schools and band schools who planned joint units and events. (p. 9)

This coordinated integration of resource personnel could be adopted by any educational jurisdiction. LUCID expanded teachers' knowledge of, and connections with, their local First Nations community.

"The LUCID framework identifies four main phases to a typical [local] curriculum narrative: First Encounter, Going Deeper, Creating/Inventing/Re-Imagining, and Integrating/Celebrating" (Neel & Fettes, 2010, p. 47). The "curriculum narrative" flows from: (1) identifying or creating a problem, (2) encountering increasing complexity resulting in increasingly detailed understandings by students, (3) implementing tentative remedial action, and finally (4) including a denouement in which the original problem situation is resolved and most of the participants recognize that a transformation has occurred within themselves by coming to know something worthwhile. These four phases resemble what happens with many students in Lunney Borden's Show Me Your Math (section 8.7).

Neel and Fettes (2010) illustrated the ethos of the four phases:

> In this transformation unit, the central tension is that between a [student's] definite public identity, symbolized by a crest or logo, and the relative powerlessness and uncertainty of an indefinite private identity. This is a tension experienced directly, on a daily basis, by our Grade 9 students, and we will be using it as a source of emotional energy for learning mathematics. (p. 47)

Learning occurs with both the head and the heart. "With the overall narrative arc of the unit in mind, we can now think about the scope and sequence of activities as they unfold" (p. 48). An essential feature is "that activities be continually related back to the 'why' of the unit" (p. 48).

During his year-long stay in Haida Gwaii, Fettes (2007) developed a teaching resource guide for teachers. It is highly relevant to all educational jurisdictions; not for its rich Haida details, but for being a template what for topics should appear in any mathematics teacher resource guide, for instance, including a section on how to pronounce words in the local Indigenous language.

In this unusually extensive Haida Gwaii study into Indigenous mathematizing, Neel (2008) spent months interviewing a diversity of Haida and non-Haida community members, including: Elders and other community leaders, parents or grandparents, school administers, teachers, and students. The objective was to understand their ideas of Haida mathematizing. Neel explored how those "practices could be integrated into the present curriculum" (p. 62) to increase students' participation and achievement. These everyday practices involved "quantitative, relational, and spatial aspects of peoples' lives" (p. 206). During the interviews, Neel attended to the relationship between student achievement and the degree to which "students could see themselves included and represented in the curriculum" (p. iii).

As reported in many studies, the adults interviewed in Neel's (2008) Numeracy in Haida Gwaii project "were adamant that their children should learn mathematics that is 'authentic'. They wanted to see their culture acknowledged and represented in the curriculum, but not as some kind of 'watered down' curriculum for their children" (p. 246). By "authentic," I believe adults meant Platonist school mathematics. The parents would not have known that this elitist school mathematics systemically favours middleclass Euro-Canadian students (Fettes, 2006; Nazir et al., 2008). Therefore, these adults did not realize that their "authentic" mathematics would create *stronger culture clashes* for Indigenous students (i.e., greater marginalization), thereby functioning as a neo-colonial educational practice (sections 4.1 & 6.2).

Importantly but unknowingly, these parents identified a two-variable problem for mathematics educators to resolve: (1) Indigenous perspectives being absent from the "authentic" mathematics curriculum, but also (2) the "authentic" curriculum being overcrowded with elitist non-essential content (sections 2.4 & 10). Parents and many mathematics educators uncritically assume the problem is unidimensional, pertaining to the first variable only. But this can only lead to *partial* student success. *Both parts to the problem need to be resolved*, for maximum success.

Moreover, a one-dimensional solution presents mathematics teachers with a type of oxymoron to deal with: combining a *cultural* practice with a purported *acultural* practice. Maximum student engagement and achievement arises from combining a cultural practice (Haida mathematizing) with another cultural practice (Euro-American mathematics) (sections 4.2, 4.3, & 4.5). This arrangement eliminates both an oxymoron and an elitist ideology. An equitable balance in the political-social power relationship between the Haida community and a renewed mathematics curriculum would be reached.

In addition to the tension between parents wanting their Haida culture evident in the local curriculum and parents wanting their children to earn high school credentials in mathematics, Neel (2008, p. 255) noted three other tensions: (1) "a tension between maintaining [Haida] culture and letting culture evolve" into a contemporary culture; (2) "a tension over how school mathematics should connect with daily numeracy practices [Haida mathematizing],” that is, a concern over the way mathematics teachers and researchers conduct the transformation of a Haida cultural process into classroom content; and lastly, (3) a tension over which “cultural lens” – school or Haida – should measure student performance in schools; a false binary to be sure.

“Many students find themselves participating in two cultures – the culture of the home/community and the culture of the school;” but “students see little connection between the two” (Neel, 2008, p. 255). Navigating between very different epistemologies requires instruction in two-eyed seeing (Hatcher et al., 2009); that is, epistemological scaffolding offered by teaching materials and teachers (Bang & Medin, 2010).

8.1.2. FNESC Teacher Resource

Teaching Mathematics in a First Peoples Context: Grades 8 and 9 (FNESC, 2011) is a very pragmatic document to help mathematics teachers “extend their existing practice to incorporate new approaches that make the BC school system more reflective of the realities of First Peoples in this province” (p. 7). Reading between the lines, I recognize a great deal of appropriate effort went into the document’s development that represents a contextual approach to culture-based mathematics (section 7). However, individual teachers can take the initiative to augment their chosen unit into a cross-cultural approach, as the resource suggests they do.

The document contains 10 unit plans (e.g., Cooking with Fractions, Bentwood Boxes, Hunting, Statistics and Salmon, and The Water Keepers). Each unit explicitly draws upon artifacts, processes, or ideas found in British Columbia’s Indigenous cultures, and advises teachers to forge a relationship with a local Indigenous knowledge holder or Elder. Some units simply show how to use EAM Platonist content (e.g., statistics) in an Indigenous context (e.g., salmon fishing), without much explicit instruction on Indigenous values and relationships attached to the meaning of an activity such as salmon fishing. Teachers’ self-initiative could fill in this gap. Other units provide this specific information. It would have been helpful had the project cited documentation on how the resource was created or what its relationship was to the mathematics education literature.

One of many strengths of this resource is its chapter entitled “Making Connections.” “Building strong community links – engaging in *consultation* with local First Peoples and seeking their support for what is being taught – will allow you to provide active, participatory, experiential learning and to localize course content” (FNESC, p. 15, emphasis added). The resource, however, gives weight to the weaker process of consultation, rather than the stronger *collaboration* (section 3.1).

Two very thoughtful lists, “First Peoples Principles of Learning” and “First Peoples Principles of Mathematical Teaching” are included. Teachers and researchers will find these ideas helpful in discussions with a knowledge holder or Elder.

The document offers a province-wide list of administrators who can help a teacher find a knowledge holder or Elder with whom to seek advice and to help lead a cultural immersion. But on the other hand, it is silent on the *necessity* of teachers engaging, first and foremost, in a transformational cultural immersion experience out in the land, organized by a group of Indigenous people, lasting at least two consecutive days, and including an overnight stay if feasible (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, 2017; Fyhn et al., 2011; Lunney Borden et al., 2017; Michell et al., 2008), as discussed in sections 2.3, 2.4, and 6.2.

Teacher autonomy favours flexibility in what should be done in class. However, a few specific teacher-developed lesson plans, from which teachers will naturally deviate, could have been helpful. Importantly, lesson plans are a concrete place to begin. They motivate many teachers into getting started at a critical time when they feel most uncertain and vulnerable (Aikenhead, 2002a; Anderson, Comay & Chiarotto, 2017). A resource could also offer a teacher’s story about what they had done to compose a lesson plan, thereby helping novice teachers decide how they want to proceed.

At one point in the document, the deficit model of teaching (section 3.1) is emphasized: “The students who come to our school have serious gaps in their education” (p. 29). When mathematics teachers complain that Indigenous students lack the background for doing well in their class, these teachers are masking power with innocence. By framing the issue as a lack of background knowledge, they implicitly fault the students. A critical analysis exposes a much different perspective: the country’s dominant political-social agenda of colonization forced an “educational debt” on Indigenous students (Bang & Medin, 2010, p. 1023; Ladson-Billings, 2006, p. 3). It is this educational debt that teachers are actually complaining about, and perhaps about the fact that mathematics teachers are expected to help “pay off” the debt through teaching mathematics cross-culturally. What began as an “objective assessment” of Indigenous students’ background knowledge has turned out to be an ethical self-revealing judgement concerning a teacher’s personal responsibilities towards Indigenous students. In other words, these students have unknowingly invited mathematics teachers to participate in reconciliation.

Many sensible suggestions for responsible action are offered in the FNESC resource, but the context could have been the asset model of teaching; that is, tapping into the wholistic and experiential resources that students bring to a classroom, and treat these resources as assets for academic success (Aikenhead, 2002a; Aikenhead et al, 2014; Bang & Medin, 2010).

The FNESC teacher resource had sufficient financial backing to employ a project manager, four writers, a diverse advisory team (including people from the Ministry of Education), and a publishing company to coordinate, write, edit, and do the layout. In spite of this, the word "Indigenous" is misspelled "indigenous" inconsistently about half the time throughout the document.

The resource deals well with making connections: (1) between school mathematics and Indigenous perspectives, (2) between teachers and students, and (3) between teachers and Indigenous themes and topics. In other words, teachers are offered detailed support to enact one of several foundational aspects of Indigenous worldviews: reality is comprised of a web of dependent interrelationships and sacred responsibilities to those relationships (Aikenhead & Michell, 2011).

8.2. Blackfoot First Nations Confederacy Studies

Rarely can readers watch the dynamics unfold as a mathematics teacher evolves into using a cross-cultural approach to her instruction. Sterenberg (2013a) gives a clear account of a teacher's professional transformation; fundamental to anyone who creates materials for place-based or culturally responsive teaching. In this instance, the material was a lesson plan contextualized in the real world of teaching. But the teacher's journey is replete with ideas for teachers and researchers who are reticent about making "mistakes."

Sterenberg, whose ancestry is non-Indigenous, collaboratively co-investigated with Bryony, a Blackfoot teacher in an Alberta Blackfoot reserve school. Bryony was beginning to integrate Indigenous perspectives with the province's Grade 9 mathematics curriculum. Sterenberg's two-part study immersed itself in the local culture by adhering to an Indigenous paradigm of social science research (Donald et al, 2011; Tuhiwai Smith, 2012; Wilson, 2008). The first part focused on how Bryony modified a published unit that taught geometry in the context of house construction. The second part followed Bryony's journey as she moved along a continuum from emphasizing EAM Platonist content contextualized in an Indigenous community's housing, to a more equitably balanced emphasis on both cultures' mathematizing; that is, a cross-cultural approach (introduction to section 7).

For Bryony (Sterenberg, 2013a):

> Teaching from an Aboriginal perspective is simply finding what is meaningful and relevant to the students that honours the ancestors of the host territory in which teachers live and teach. It means teaching the curriculum and addressing silent identity issues simultaneously by revering the land and people from which the students came. (p. 23)

When examining other teaching resources at hand, Bryony was offended by three features: tokenism, the "pan-Indian homogenization of Indigenous culture" (p. 25), and the exclusion of contemporary Indigenous cultures.

Guided by a published unit on house construction, she conscientiously engaged her students in: meeting with the reserve's housing committee, drawing scaled maps of the local land and of blueprints of reserve houses, adding furniture icons in scaled proportions to the blueprints, creating a furniture budget, calculating how long it would take to pay for the furniture, and discussing tax structure and band policies. All of this involved ratios, proportional reasoning, and much arithmetic. There was no mention, however, of relational, spiritual or emotional aspects of the community's Indigenous perspectives. Peripheral concepts connected to these aspects give epistemological, ontological, or axiological meaning to Indigenous artifacts, processes, or ideas. Thus, one might ask: How authentically Indigenous did the housing unit and furniture activities appear to the students?

Bryony had worked diligently to make her adaptation relevant to her students, but the "house construction unit did not resonate with their experiences" (Sterenberg, 2013a, p. 26). Next "she wondered what a mathematics project that started from Indigenous mathematizing might look like" (p. 27).

Bryony then made the crucial decision to talk with an Elder about Blackfoot mathematizing and school mathematics. From those conversations and from professional reading, she learned more about an Indigenous notion of teaching from place (Sterenberg, 2013b). In other words, Bryony continued her journey into cross-cultural school mathematics.

Part two of Sterenberg's (2013a) study was based on the fact that "an emphasis on quantifying procedures rather than *a focus on relationships* often becomes the sole priority in school mathematics" (p. 24, emphasis added). A Blackfoot priority is a placed-based, wholistic, and relational coming to know – "to learn in a deep way" (Cajete, 2000, p. 110). Cross-cultural school mathematics embraces *both* Euro-American mathematics and Indigenous perspectives. And now, so does Bryony.

Following proper protocol, Bryony arranged a fieldtrip to a sacred medicine wheel site, measuring 27 meters in diameter and having 28 “spokes” emanating from its centre. Collaborating with the Blackfoot Elder, she designed four learning centres, a sacred number for many First Nations peoples. Two centres involved EAM Platonist content, where students would measure features of the medicine wheel and investigate quantitative relationships among the measurements. The other two centres dealt with information about the sacred place. Other survival-related activities were planned. Importantly, Bryony’s idea of “authentically relating Indigenous knowledges and mathematics curricula” (Sterenberg, 2013a, p. 27) had become linked to an Indigenous sense of place.

On the fieldtrip, the class was accompanied by four resource experts: the Elder, an archaeologist acquainted with the site, a reflective writing instructor, and a Blackfoot cultural teacher who knew details about Blackfoot mathematizing and the cultural significance of the site. The site visit began in a good way with the Elder saying a prayer, in one of the three Blackfoot confederacy’s languages, and leading everyone in making sacred offerings of tobacco to Mother Earth. The Elder also described the preparatory ceremony conducted the day before. Then the cultural teacher told “the story of the Iniskim, which are sacred calling stones; the Iniskim call out to people to find them” (Sterenberg, 2013a, p. 28). It is an ancient story that relates the site’s small stones to the buffalo. The students became fascinated. At the story’s end they were carried away and wanted to act out part of the story. This would alter the fieldtrip lesson plan. Bryony’s intuition advised her to go with the flow. The day turned out to be an exciting and educational success for the students, and a highly worthwhile learnable moment for Bryony.

When she reflected on the day, she realized that rather than begin a lesson with EAM Platonist content (i.e., two of the teaching centres), “she had provided students with the opportunity to respond to the teachings of the land. … She was flexible in recognizing the significance of learning from place” (Sterenberg, 2013a, p. 28). Bryony stated, “They’d understood why this place was important and were now able to incorporate the math” (p. 28). Sterenberg wrote, “Students seemed to have a positive attitude about their own perceptions that engaging in mathematics was a human and social endeavor” (p. 29).

This vignette points out the power that stories hold, the power of the land as a teacher, and the power of a particular teaching sequence – first give priority to the local Indigenous culture, then follow up with EAM directly associated with students coming to know the land; and finally with students motivated by success, follow with EAM unrelated to the land but important in the curriculum. The other way around treats Indigenous perspectives as add-ons, Bryony claimed. Sterenberg (2013a) concluded that EAM Platonist content and Blackfoot mathematizing can only be successfully *integrated* if a teacher begins "from Blackfoot knowledge of the land" (p. 29).

But what does integration mean in this context? There are at least two legitimate meanings of "to integrate" that should be clarified. The first reflects the sense of *including* both Indigenous mathematizing and EAM content in school mathematics, but in a *binary way*; that is, a teacher will address either EAM or Indigenous mathematizing at any given moment. As mentioned earlier (section 6.3), this emphasizes the integrity and independence of each culture's knowledge system (Garroutte, 1999). However, the binary can subtly obliterate the common ground the two knowledge systems importantly share ontologically, epistemologically, and axiologically (Aikenhead & Michell, 2011, pp. 99-120). Similarities do not seem to appear in the mathematics education research literature. Bishop's (1988b) six mathematizing processes found in all major cultures would be a good place to find similarities.

A second meaning of "to integrate" comes from Elder Marshall of the Mi′kmaw First Nation when he explains his "two-eyed seeing" concept (Hatcher et al., 2009); an idea that emphasizes a person learning the strengths of both Indigenous mathematizing and EAM, and then the person can idiosyncratically draw bits and pieces from either knowledge system in order to solve a specific problem or make sense out of a specific issue. This process creates "hybridized" knowledge (Aikenhead & Michell, 2011; Enyedy et al., 2011; Lipka et al., 2007; Jannok Nutti, 2013), thus *integrating* Indigenous and Euro-American knowledge systems for a particular context.

It is only the individual who can create hybridized knowledge due to the idiosyncratic nature of the coming-to-know process. Thus, teachers should integrate (the first meaning) and then students will create a hybridization if they see a need to (the second meaning of "to integrate"). But teachers should not attempt to hybridize EAM and First Nations mathematizing *for* students. However, showing how someone (the teacher included) did it themselves could make an excellent concrete illustration.

For me, five important points arise from Sterenberg's inquiry into Bryony's project to create teaching materials that will enhance her mathematics pedagogy in a cultural way. First, there are many other mathematics teachers who have parallel stories to tell but are not published. Second, a very worthwhile future research program would be to discover, write up, and publish a compendium of these powerful professional development stories (e.g., Aikenhead et al., 2014;

Anderson, Comay & Chiarotto, 2017; Lipka et al., 2005). Third, Bryony's lesson plan (the product of her project) is naturally place-based and therefore, it is not easily repeatable except for those few communities on Turtle Island (North America) that are located near a medicine wheel or some other sacred site. But it certainly has inspirational value for other teachers. Fourth, Sterenberg's research illustrates the richness of results that qualitative research produces. To transpose her research into a numbers-generating instrument, such as a theory-based questionnaire, would strip away her study's core significance, and it would lose its power to influence teachers and policy makers. However, questionnaires have been very useful in identifying people to include in a qualitative study. And fifth, it is all about relationships, relationships, relationships: Bryony's relationship with an Elder moved her planning forward; her students' relationship with the sacred site propelled their engagement forward; and her relationship with her students ensured she could go with the flow with excellent results.

A key feature, significant by its absence from Sterenberg's (2013a) article, is Blackfoot language and back-translations of important terms or expressions. Standard-setting Indigenous and non-Indigenous education scholars add a crucial dimension to their work by attending to features of an Indigenous language. For example, Lipka (section 8.3), Furuto (8.4), Fyhn (8.6.1), Jannok Nutti (8.6.2), and Lunney Borden (8.7) developed at least a rudimentary functional use of an Indigenous language over a number of years. As a result, they have a great deal more to explore, to offer students, and to report on. They understood the centrality of Indigenous languages (section 6.4).

Equally important to include is information on how contact was originally made and what protocols were followed. As illustrated in section 2.5, each publication should also include a description of a researcher's or author's relationship to Indigenous people in general, and to the R&D project's Indigenous participants in particular.

In a later study in the same school but collaborating with a different Grade 9 teacher (Theresa McDonnell of Cree ancestry), Sterenberg and McDonnell (2010) "experimented with intertwining Indigenous, Western, and personal mathematics through learning from place" (place-based school mathematics) (p. 13). The researchers adapted this triad from Ogawa's (1995) triad: "Indigenous science, Western modern science, and personal science" (Sterenberg & McDonnell, 2010, p. 11). Perhaps the Blackfoot sacred braided sweetgrass inspired this intertwining model. The three strands expressed three different types of mathematical understanding:

- Blackfoot mathematizing expressed Bishop's (1988b) fundamental mathematical process of locating.
- Western mathematics expressed Platonist school geometry content, but it ignored EAM cultural content; a situation discussed at the end of section 4.1 and in sections 7.2 and 9.1.
- Personal mathematics of each student was developed naturally and idiosyncratically, and was expressed through students' writing and talking about their experiences.

This list needs a short explanation. The first two strands relate to *pluralism* by representing two of many culture-based mathematics systems. The last strand exemplifies *relativism*; a collection of individual fairly unique understandings, from which commonalities can be abstracted.

Sterenberg and McDonnell (2010) developed four lessons around this intertwining model; the first two related to the provincial curriculum (i.e., similar and congruent triangles, the use of the Pythagorean Theorem, coordinate geometry, and measuring). The last two lessons related to both Blackfoot and Platonist mathematics perspectives, which intertwined during a field trip to a Blackfoot sacred site called Big Rock. An Elder was in charge of teaching Blackfoot mathematizing through protocol, ceremony, and stories about the site. The Blackfoot mathematizing turned to create deep relationships between the site and each student.

The experiences of a fairly typical student were reported. Dallas' facility with EAM Platonic content (i.e., mentally reflecting, sliding, and rotating objects in space) were noted as he determined the height of Big Rock using similar triangles. He also succeeded to map the site using GPS technology supplied by the school. After the field trip, he talked about how mathematics could be learned in different ways: "You can make math fun or boring" (p. 18). Importantly, Dallas described his increased awareness of his Blackfoot identity, history, and how he came to feel his relationships to the land at Big Rock. He also wrote about Blackfoot mathematizing in general and navigation specifically.

Sterenberg and McDonnell (2010) concluded:

> What holds promise for us is the potential for viewing Western and Indigenous mathematics as having complementary strengths. Recognizing the strengths of each type of mathematics could maximize mathematical learning. To date, very little has been done to intertwine these knowledge systems and the reciprocity of cultural strengths in Indigenous and Western mathematics is not fully understood. (p. 21)

Sterenberg and McDonnell are paraphrasing two-eyed seeing (Hatcher et al., 2009) as a goal and outcome for culture-based school mathematics for reconciliation. A growing literature on cross-cultural mathematics featured in this monograph supports intertwining EAM Platonic content with Indigenous mathematizing.

However, Sterenberg and McDonnell's (2010) three-strand intertwining model could certainly be much more effective for maximizing Indigenous students' achievement in school mathematics if the three strands were: Indigenous mathematizing, *EAM cultural content*, and EAM Platonist content. Such a three-strand braiding model holds promise for developing curricula, units, lessons, and activities that minimize culture clashes more than most projects mentioned in this monograph, and therefore would maximize student achievement even further. Alternatively, a medicine-wheel format with four directions would incorporate: Blackfoot mathematizing, EAM Platonist content, EAM cultural content, and personal mathematics.

8.3. University of Alaska Project

The Alaskan political-social reality is very similar to the Canadian context described in section 2.1, except in Alaska there has been no apology or reconciliation for the disastrous colonization of Alaskan Natives, including the Yup′ik Nation. Due to poverty, racism, and marginalization similar to Canada's, student alienation was high and achievement was particularly low in school mathematics. Alaskan schools were continuing to colonize their Indigenous students, especially in rural areas.

This was the main educational issue that motivated the creation of a small, Yup′ik teacher's research group, associated with the University Alaska Fairbanks (UAF) in the late 1980s (section 5). The group called themselves "Ciulistet," a Yup′ik word meaning leaders. They collaborated extensively with people in their villages so their culture-based school mathematics would have strong local roots and become a powerful movement for change throughout southwestern Alaska (Lipka, Mohatt, & The Ciulistet Group, 1998). Outside of Alaska, their cross-cultural innovation gave hope and direction to others.[xii]

"Two-way learning occurred, and both Western and Yup′ik systems were valued" (Lipka, 1994, pp. 15-16). The group's action research and ethnographic school-based results led them to realize "the potential of using their culture and language as a means to change the culture of schooling" (p. 14). Ciulistet worked closely with the Alaska Native Knowledge Network (ANKN, 2016), with whom many newsletters and booklets were published. Most of these resources are now archived on the Math in a Cultural Context website (MCC, 2016).

Ciulistet's and the ANKN's policy was to negotiate what school mathematics will be, one school jurisdiction at a time. They altered the colonial established power-imbalanced relationship between each Yup′ik community and its school mathematics (Lipka et al., 1998; Lipka et al., 2005) The status quo became more like two-way learning – cross-cultural school mathematics (Lipka, Yanez, Andrew-Irhke & Adam, 2009). Ciulistet's capacity development within Yup′ik communities put participating schools on a pathway to decolonization.

This policy's success (described in detail just below) helped establish at the UAF the MCC project, an elementary science curriculum supplement (MCC, 2016). It evolved further due to the continued extensive collaboration among: (1) Yup′ik Elders, knowledge holders, teachers, school consultants, and rural communities in southwest Alaska; (2) non-Indigenous Alaskan school districts, teachers, and consultants; (3) academic mathematics educators, education researchers, and mathematicians; and (4) the U.S. Department of Education and the National Science Foundation for funding. The success of MCC is indebted to this loose informal *political* alignment (section 10.2).

Currently, MCC (2016) has produced 10 modules (e.g., Picking Berries, Designing Patterns, Kayak Design, and Salmon Fishing) that supplement the Alaskan mathematics curriculum for Grades 1 to 7. They are intended for Indigenous and non-Indigenous students in rural and urban schools. The modules include: "DVD clips of teachers' implementing exemplary lessons, written case studies, a *Guide to Implementing MCC*, literacy activities and stories that develop cultural, mathematical, and contextual connections for students" (MCC, 2016, website p. "About MCC"). The Yup′ik language appears in the materials enough to warrant having a Yup′ik glossary at the end of each module. By the way, Yup′ik means "real person." But the proportional appearance of the Yup′ik language must be negotiated in each community: "…[S]ome want schooling to represent their culture while other groups want schooling to teach the dominant society's 'secret language' – the language of power" (Lipka, 1994, p. 22).

Most often an evaluation of R&D projects does not occur due to a depletion of human and financial resources that all went into completing the project. In other cases, the evidence of success comprises testimonials and the number of teachers and/or schools using the materials produced. A much higher standard was established by the pre-MCC group (Ciulistet), which seamlessly morphed into the MCC in 2005. Their evaluation's mixed methods approach included both quantitative and ethnographic studies. The former provided a highly appropriate and reliable big picture of what happened, of special interest to administrators and funding agencies.

The extensive quantitative studies covered five years (2001 to 2005); collected pre- and post-test scores on EAM Platonist content tests, which yielded student gain scores from about 3,000 students in 15 schools (Lipka & Adams, 2004; Lipka et al., 2005). Treatment classrooms (where MCC modules were taught) were paired with control classrooms (conventional school mathematics) in a way that established a rigorous quasi-experimental research design. The studies established the following results: (1) the treatment groups' gain scores were statistically significantly greater than the control groups' gain scores; (2) "Although the urban treatment group gained the most from this curriculum, the most important finding is that the rural treatment group outperformed the rural control group" significantly (Lipka & Adams, 2004, p. 3); and (3) Lipka and Adams concluded, "[I]t shows that the treatment effect on Yup'ik students narrows the long-standing academic gap when comparing [the Yup′ik treatment] group's and the Yup'ik control group's relative performance against the urban control group" (p. 3).

The ethnographic side of the evaluation of MCC was comprised of four case studies (*Journal of American Indian Education*, 2005, Vol. 44, No. 3). They support the quantitative findings and offer rich detail to help explain those findings and to pinpoint complexities in what occurred in the treatment classrooms. These complexities are very useful to mathematics educators and researchers involved in culture-based education of Indigenous students; whether producing teaching materials, teaching with them, conducting teacher professional development programs, or introducing them into teacher education. The MCC case studies included a wide spectrum of teachers, grades, modules, and proportion of Yup′ik students in a classroom (between zero to 100 percent).

Factors that were found to interact with the MCC cross-cultural teaching materials and to influence student academic achievement include *the extent to which teachers*: (1) fostered interpersonal relationships (teacher-student and student-student); (2) based their mathematics lessons on local culture and people, including students experienced in the cultural topic, when feasible; (3) ensured students took ownership of their own personal contributions to group knowledge building (e.g., when reaching a consensus); (4) judiciously balanced an expert-apprentice model and an interactive model based on equality – something like camping spots of dialogue, or kiskinaumatowin (Swampy Cree) (section 6.4); (5) planned experiential coming-to-know activities, and determined how many will occur outside the school; (6) created a harmonious, comfortable, respectful classroom environment; (7) used problem-centred, inquiry-oriented methods; and (8) got students to articulate their reasons behind their ideas and answers.

What a teacher does, of course, makes all the difference to the success of a culture-based module. For this reason, professional development programs that begin with a transformative culture immersion experience, especially for non-Indigenous teachers, should be a prime consideration (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, 2017; Fyhn et al., 2011; Lunney Borden et al., 2017; Michell et al., 2008).

Belczewski (2009, p. 197) summarized her “White teacher” learning moments acquired when she developed her own lesson plans and teaching materials for First Nations students: “I have come to realize that for my contribution to go above tokenistic culturally relevant content, it is not only the content that must be decolonized but also myself.”

Currently, MCC is collaboratively expanding its student assessment item bank “to integrate authentic cultural practice and mathematical knowledge” (Lipka et al., 2013; p. 130). An exemplar Grades 3-6 question is illustrated by a fish rack question (p. 140):

“For every 2 King Salmon there are 3 Reds.”

(A diagram shows two large King salmon lying head to tail, and immediately below, three small Red salmon head to tail, in order to visually represent the text’s meaning; i.e., both sets of salmon are the same length.)

“If a fish rack holds 6 King Salmon, how many Red Salmon would the fish rack hold?”

There is no tokenism, insensitivity, or neo-colonialism here. Like Fyhn (2013), each question goes through a rigorous R&D process.

8.4. University of Hawaiʻi Project

I begin this section with a synopsis of Native Hawaiians’ political-social context pertinent to school mathematics. Hawaiians at Waimea Bay, Kauaʻi Island, discovered Captain Cook and crew on their beach in 1778; their first encounter with a White person (Caucasoid). A year later on Hawaiʻi Island, cross-cultural misunderstandings escalated at Kealakekua Bay; Captain Cook had overstayed his welcome; a spear ended his life. The crew fled in their ship.

A strong Hawaiian leader, Kamehameha, realizing the technological prowess of Captain Cook’s culture and anticipating more of his kind would arrive eventually, he forcefully unified all the islands into one strong Royal Kingdom by 1795, well in time to welcome the first missionaries in 1820.

Commercial traders and whalers became an increasingly permanent fixture in the 19th century, bringing with them modern technology and foreign diseases that decimated the Native

Hawaiian population. American businesses flourished in co-existence with the Kingdom of Hawaiʻi, until friction heighted over the authority of Queen Liliʻuakalani's sustainable development policies versus American business interests. It was settled by the U.S. Marines in 1893, who invaded Oʻahu, dethroned the Queen, overthrew the monarchy, and gave power to the business groups who soon helped secure Hawaii's territorial status with the United States in 1898.

A familiar story of racism, marginalization, and oppression ensued, thereby colonizing most Native Hawaiians into poverty without a treaty to protect their interests. The invasion of the Kingdom remains an important issue of recent history for most Native Hawaiians today. Confirmation of statehood in 1959 encouraged a smoldering Indigenous sovereignty movement to go public in the 1960s. This movement initiated the rebirth of the Native Hawaiian language, culture, wisdom, and identity, which continues today.

The initial success of this Indigenous renaissance was proudly celebrated in 1985 when a replica of an ancient double-hulled Pacific Island canoe, the Hōkūleʻa, sailed around the Pacific as far as Aotearoa New Zealand and back to Hawaiʻi (Finney, 1994), thereby "rekindling the Pacific Island tradition of non-instrument way-finding techniques that include celestial navigation" (Furuto, 2014, p. 113). Invigorated by this success, a major circumnavigation voyage of Hōkūleʻa began in 2013 to return in 2017, in which Furuto has already participated for one leg of its journey. This worldwide event has spawned notable interest and pride in Indigenous wisdom of Native Hawaiians and Pacific Islanders. It has become "a vehicle to bridge [I]ndigenous models of mathematics at the local and global levels" (Furuto, 2014, p. 113, spelling corrected).

The Hōkūleʻa voyages, a state treasure to be sure, function as one of many contexts with which to embrace cross-cultural EAM. This is the central interest of the Ethnomathematics and STEM (science, technology, engineering and mathematics) Institute (ESTEMI). This group's diverse projects began in about 2008 at the University of Hawaiʻi at West Oʻahu and is now also housed at the University of Hawaiʻi at Mānoa. Anchored in college/university-level courses and programs, the Institute offers ethnomathematics pedagogies that fit culturally responsive and place-based agendas. Ethnomathematics is defined as D'Ambrosio (2006) did, "the intersection of culture, historical traditions, sociocultural roots and mathematics" (Furuto, 2014, p. 112) (section 7.2).

ESTEMI has been incredibly successful at increasing student enrolment in undergraduate mathematics courses (by about 70% each year recently), and successful at almost doubling the overall passing rate in these courses, to a level “far above” the average for all campuses in Hawaiʻi (Furuto, 2014, p. 118).

But there is much more to ESTEMI than undergraduate ethnomathematics. It emphasizes school level international examinations. For instance, Furuto (2014, p. 111) draws attention to the 2009 PISA mathematics results that show U.S. 15 year-olds scoring 487 on average, which is below the average score (496) among all participating countries.

The Hawaiian educational context of schools features a multicultural diversity of students greater than in any other U.S. state. This multicultural feature makes mathematics classes much more challenging for teaching and learning. ESTEMI participates fully with K-12 teaching practices “informed by cultural knowledge possessed by [university] students” (Furuto, 2014, p. 110); and with Mathematics Common Core State Standards (MCCSS), federal standards, and benchmarks. ESTEMI’s ultimate goal is to strengthen the STEM pipeline from K-12 to college mathematics, and then on to a career and community readiness. The pipeline’s strength arises from school and post-secondary mathematics courses being attractive throughout students’ education (Furuto, 2017).

ESTEMI gives great attention to school teachers’ professional development that features cultural immersions in communities throughout Hawaiʻi. Furuto (2014) describes the experience this way:

> Through an orientation series of professional development workshops, and a 1-week summer institute, educators from across the State of Hawaiʻi and [other] Pacific [Islands] design and implement mathematics lesson plans grounded in ethnic, historical and cultural diversities of our island homes. The resulting research and practicum-based textbook is used by current educators to supplement curriculum, and future teachers as training material aligned with MCCSS. (p. 115)

Compared to other major R&D projects, the Institute appears to reach a more equitable balance between its focus on EAM Platonist outcomes and its attention to Indigenous perspectives, due to: its cultural immersions, its priority on Native Hawaiian values, its attention to Native Hawaiian vocabulary, and the university-level courses that extend that Indigenous focus further, even assessing university students on this cultural content (Furuto, 2013a, 2017). This balance characterizes a *cross-cultural* project.

The textbooks mentioned in the quotation above refer to Furuto (2012, 2013b), which include some of the module lesson plans found on the ESTEMI (2016) website. These modules are aimed at four levels of education: elementary (8 modules), middle school (7), high school (18), college/university (7 on the website and 3 others in Furuto, 2012). These relate specifically to some facet of Native Hawaiian or other Polynesian cultures. Some attention is given to Indigenous languages, more so in upper-level modules. To produce the modules, university students conduct research in a Polynesian community of choice and study its history, its language, and/or literature (Furuto, 2017). Each year more modules become available because of students who enrol in "Math 241" at the university and who choose to develop a module as a major assignment in the course.

Teachers and university instructors who implement the modules are given some support in assessing students with respect to the modules' Indigenous cultural content (Furuto, 2017), but the overall emphasis is primarily on meeting a specific mathematics curriculum and MCCSS's general standards. Both focus entirely on EAM Platonist content.

8.5. Algonquins of Pikwakanagan First Nation Project

Beatty and Blair (2015) embarked on a long-term goal of decolonizing schools by way of community-focussed R&D projects designed to "permeate" the school curriculum with Indigenous perspectives. These projects will address "the needs of students from the Algonquins[xiii] of Pikwakanagan First Nation who attend a provincially funded school" (p. 22) located close to their reserve and to Pembroke, Ontario.

Beatty and Blair (2015, p. 4) described these projects as culturally responsive education, by which they mean "aligning instruction with the cultural paradigms and lived experience of students." The projects are also founded upon "a theoretical framework" of ethnomathematics, which differs from D'Ambrosio's meaning of ethnomathematics (section 7.2) by fully embracing EAM cultural content (sections 4.5 & 7.1). In the words of the authors: "From the ethnomathematics perspective, school mathematics is one of many diverse mathematical practices and is no more or less important than mathematical practices that have originated in other cultures" (p. 4). Two such cultural knowledge systems were identified: the mathematics knowledge system found in the Ontario curriculum, and "the mathematics inherent in Indigenous cultural practices" (p. 4). The project adds evidence supporting a critical analysis of school mathematics in an era of reconciliation.

Specifically, Beatty and Blair (2015) report on teaching "number sense, spatial reasoning, and patterning and algebraic reasoning" (p. 9) by engaging a Grade 2 class (20% Algonquin and 80% non-Indigenous students) in a year-long unit on traditional Algonquin looming. Glass beads and thread were substituted for dyed porcupine quills and sinew. Appropriation of Pikwakanagan culture was avoided by assuring that the "core research team was comprised of cultural insiders and outsiders" (p. 5): two Algonquin teachers, three non-Indigenous teachers, two university academics, and an Algonquin knowledge holder who taught the looming technique and pattern designing to the core research team, and then to the Grade 2 students.

The Pikwakanagan community became involved by offering guidance on what should happen, and feedback on what was happening, in the classroom from time to time. This occurred according to a cyclical collaboration model that specified the following sequence: "consult, plan, teach, reflect, and share" (Beatty & Blair, 2015, p. 7). The team's first consultation meeting heard guidance from three Elders. One spoke of her struggle to maintain her Algonquin self-identity; "*[T]hey couldn't let us be who we are and just teach us*" (p. 7, emphasis added). Another Elder repeated the wisdom of his father, "[T]ake the best of [Algonquin culture] and the best of the cultures that are out there and marry the two of them together and make them work for you" (p. 7); advice associated with hybridized knowledge (Enyedy et al., 2011; Lipka et al., 2007; Jannok Nutti, 2013) and two-eyed seeing (Hatcher et al., 2009). The Elders' initial guidance was seen as a request of the research team to help the community regain its culture that has almost been obliterated by Canadian society.

Parents and grandparents also offered guidance and feedback at several subsequent consultation sessions throughout the year, and whenever they visited the classroom or attended evening school events that showed off their children's accomplishments. Community feedback was "overwhelmingly positive, and community members expressed pride in the mathematical thinking the children were demonstrating" (p. 19). This is key evidence indicating a successful cross-cultural mathematics project, because what their project did was to bring community members into the school to focus on mathematics instruction, which was a first for everyone involved. This outcome is phenomenal in itself.

Importantly, at the end of the year the research team was asked by community members to include more Algonquin language in the mathematics instruction (section 6.4). It became an objective for the ensuing year of Beatty and Blair's long-term project.

Classroom looming activities were videotaped and then analyzed by the core research team to identify "students' mathematical thinking and the cultural connections" (Beatty and Blair, 2015, p. 12). The mathematical thinking was described in rich enough detail to give me the impression of having watched some of the videos. For instance, drawing upon diagrams and an interview with a student named Lexie, Beatty and Blair stated:

> When analyzing their looming patterns, students coordinated their perceptions of multiple patterns that extended vertically and horizontally as well as diagonally, and *incorporated spatial reasoning with algebraic reasoning*. Lexie, for example, recognized her pattern core within each column (blue, pink, purple) but also identified the larger core made up of three columns. …The students' reasoning drew upon both their growing understanding of the complexity of patterns and how these related to the spatial relationships created within the grid on the loom. (p. 18, emphasis added)

The Algonquin knowledge holder taught the cultural connections of looming "effectively, passionately, and authentically" (p. 10), yet evidence of Algonquin stories, songs, language, values, and protocols appropriate for Grade 2 students were not mentioned by Beatty and Blair. However, they did point out that features of Algonquin culture were brought into the classroom in a way that privileged Algonquin culture "alongside the dominant society's pedagogy and content" (p. 19). Thus, a more equitable sharing of power was achieved.

A key element of the teacher's pedagogy was "the numerous opportunities students had to communicate their ideas to one another" (p. 18). I interpret this as exemplifying both kiskinaumatowin (Swampy Cree for teaching each other when learning is interactive and based on equality; section 6.4) and child-centred learning (Anderson, Comay & Chiarotto, 2017). In short, it sounds like two-eyed seeing (section 8.2) applied to pedagogy. Not surprising then, the experience of looming was "more cognitively demanding for students than mathematical instruction that prioritizes algorithmic memorization" (p. 20). Real life is always messier and more challenging than conventional school lessons.

Beatty and Blair (2015) not only witnessed and documented Pikwakanagan students changing their views of what it means to do mathematics, but "this was also important for the non-Native students who gained greater insights into the culture of their classmates, and who extended their own mathematical thinking" (p. 21). In other words, this Grade 2 classroom evolved into a microcosm of reconciliation and increased academic achievement.

8.6. Sámi Projects

Two independent R&D projects with Sámi Indigenous people too place in Norway and Sweden. Each makes a unique contribution to identifying innovative taken-for-granted notions about school mathematics that improve student achievement. These positive features will guide other projects. But first, a word about the political-social context of the Indigenous Sámi Nation, living in nine distinct geographic regions (with even more dialects) across northern Europe, occupying land in Norway, Sweden, Finland, and the Kola Peninsula of Russia. Specific political-social details differ from region to region, but for my purposes here, the Norwegian Sámi group will be represented.

The following is a succinct account of Sápmelaččat (the Sámi people), in broad brush strokes. The original Sápmelaččat first occupied Sápmi (the land occupied by Sápmelaččat) as the ice retreated over 10,000 years ago. They have had consistent contact with Caucasoids over the past 1,100 years, mostly with the mining industry. “The mining process has many side effects which greatly contradict and undermine Sámi world views, ideals, and rights” (Sommer, 2016, website quotation). Sápmi has had European settlers arrive, in varying numbers, over the past 700 years. As a result, the Sámi have gone through periods of devastating colonization, similar to what the Turtle Island Indigenous peoples experienced (Partida, 2016), for example: loss of land, treated as inferior humans, and pressured to convert to the Lutheran faith during the 1600s. The final assault on their culture lasted between the early 1800s and the 1960s when various governments instigated boarding schools to assimilate Sámi children into the dominant culture. Resilience and determination of Sápmelaččat have preserved some features of their culture in spite of attempts to eradicate them. Due to the Sámi demands for education reform and for a degree of sovereignty in the 1960s, the Norwegian Sámi Council was established, and then replaced by the Norwegian Sámi Parliament on the authority of Norway’s 1987 Sámi Act, which stipulated its limited responsibilities and powers. The first session was convened in October 1989.

Today, there are two unofficial groups: Boazosápmelaččat (Reindeer Sámi) and Mearrasápmelaččat (Sámi by the sea). The latter have been closer to, and in longer contact, with Europeans.

8.6.1. Norway

Inspired by Lipka and colleagues’ (2005) Math in a Cultural Context project, Fyhn (2009) began her research program at Norway’s University of the Artic to develop “culturally congruent Sámi mathematics” lessons (Fyhn et al., 2011, p. 191); backed by the authority of Norway’s Sámi Language Act, and in collaboration with a very experienced Sámi mathematics teacher and school principal. The expression “culturally congruent Sámi mathematics” means that elements of Sámi culture appear in the school mathematics courses. This applies to Sámi immersion schools or to both Sámi and non-Sámi students in regular Norwegian schools.

Ideas, activities, and language from a Sámi local culture become the basis for teaching examples of Sámi mathematizing. In that context, teaching Euro-American mathematics (EAM) shows Sámi students another (foreign) culture's way of understanding the Sámi ideas, activities, and language; in other words, a cross-cultural or a two-way (Lipka et al., 2009) pedagogy. Because Fyhn forged relationships with Sámi students, teachers, and community members over the years; she has developed a facility with sámegiella (the Sámi language) and a deep understanding of Sámi worldviews.

Fyhn's R&D project explicitly expresses a pluralist notion of mathematics, although EAM's cultural features are not discussed. The Sámi school mathematics curriculum has always been written in Norwegian until 2010 when the Sámi Parliament funded its translation into sámegiella. The translation process was, and continues to be, a quagmire similar to the Māori experience (section 5). Fyhn and colleges' R&D project (2011, p. 199) gave detailed attention to sámegiella, carefully analyzing its translation into either Bokmål ("book" Norwegian) or Nynorsk (a Norwegian dialect based on people's language of the countryside). In Norway, all curricula, textbooks, and examinations are published in both Bokmål and Nynorsk. Currently, high-stake examinations offer a Sámi version, which has raised serious problems concerning its equivalence with the Bokmål and Nynorsk versions (Fyhn, 2013).

Fyhn and colleagues' (2011) culturally congruent Sámi mathematics lessons pay specific attention to language-laden cognition. For instance, the ratios 2:1 and 1:2 are a challenge to learn because in everyday sámegiella, "bealli" means half. But the word

> is a richer term than just half, because it also is used in different contexts in describing the ratios 2:1 and 1:2. …A translation creates an unclear meaning or a misleading expression. Common ways of expression in Sámi are beali unnit and beli eanet. Unnit means 'less' and eanet means 'more.' [In a back-translation] beali unnit means 'a half less,' while beali eanet means 'a half more'; a precise Sámi term sounds odd when translated. The Sámi understanding is that beali eanet means 'twice.' (p. 195).

English or French speaking teachers will be familiar with this type of student confusion over their idiosyncratic everyday language and the technical language found in the culture of EAM.

Another example of culturally congruent Sámi mathematics is evident in the introduction to International System of Units. Sámi body-based measuring units and their accompanying Sámi names are explored. Students learn the Sámi processes and key terms, and they also become acquainted with peripheral concepts that connect with: the appropriate contexts of use, the associated cultural values, and the worldview perspectives that wholistically accompany the mathematizing process. The culture-based rationality and linguistics of Sámi proportional reasoning are learned prior to teaching the Euro-American-based rationality and linguistics of school mathematics. Similarly, in an introduction to the study of Euro-American geometry, students learn Sámi perspectives and language concerning what we call angles and triangles.

The Norwegian Sámi R&D project also explores “culturally valid” assessment items (Lipka & Adams, 2004; Lunney Borden et al., 2017; Nelson-Barber & Trumbull, 2007) that will help Sámi students “show what they know and can do” (Fyhn, 2013, p. 356), for both Platonist content and Sámi mathematizing. Four Sámi values served as a framework for writing culturally valid test items: reasonableness, cooperation with nature, respect for Sámi traditional culture, and treating Sámi mathematizing as a verb. These values explicitly contextualize both Platonist and Sámi test items.

Fyhn’s R&D project attracted the attention of colleagues at other Norwegian universities, who in 2015 initiated a much expanded R&D project, Local Culture for Understanding Mathematics and Science (LOCUMS, 2016). The project is dedicated to culturally responsive teaching/learning/assessing in secondary school mathematics and science, involving place-based knowledge for the purpose of strengthening Sámi and non-Sámi students’ cultural self-identities and academic achievement, thereby reducing the school drop-out rate. For Sámi students in Sámi schools, the aim is to go beyond the Norwegian curricula translated into sámegiella and develop, as much as feasible, a school mathematics program contextualized within a Sámi worldview, language, and culture; much like the Māori independent total-immersion schools in Aotearoa New Zealand (section 5). For other Norwegian schools, however, a more cross-cultural and less bicultural approach seems appropriate. Central to LOCUMS is the challenge of creating modules that will enhance Norwegian school mathematics *and* science culturally.

8.6.2. Sweden

Across the border from Norway, and with similar concerns and goals as Fyhn (2013), Jannok Nutti (2010, 2013) independently carried out a two-part R&D project with Sámi teachers in a Swedish Sámi school for children between the ages of five and 12. Jannok Nutti is of Sámi ancestry. Her agenda is the transformation of school mathematics in Sámi schools.

The first part of her project included an action-research professional development program that supported six Sámi teachers in learning their culture in greater depth through seminars, lectures, and contacts with knowledge holders (Jannok Nutti, 2010). There was no mention of Elders being involved. To prepare teachers to be agents of transforming school mathematics, she guided them in designing and implementing *culture-based* EAM Platonist content along with "ethnomathematics content" (Jannok Nutti, 2013, p. 57).

Her term "culture-based" signifies her intention of moving beyond a simple translation of the national mathematics curriculum into sámegiella (Sámi language), to one that reflects Sámi ontology, epistemology, and axiology. Anticipated student outcomes included a balance between their strengthened Sámi self-identities and their achievement in EAM Platonist content. Jannok Nutti's plan for the six teachers was to instill in them a sense of empowerment so they would feel confident in the role of change agent in Sámi school mathematics.

Key Sámi cultural values were emphasized, such as iešbirgejeaddji (being independent) and birget (to manage). "The concept birget implies managing to survive and becoming financially self-sufficient, which requires a need for knowledge and skills that enable independence" (Jannok Nutti, 2013, p. 61). These implications associated with birget define some of the Sámi term's *peripheral concepts*. As previously stated, scholarly publications should include important peripheral concepts whenever introducing key Indigenous terms, as Jannok Nutti did. It would even be better to add a back-translation, when appropriate.

Jannok Nutti encouraged the teachers to incorporate Sámi cultural values into both EAM Platonist content and Sámi classroom interactions. These teachers were expected to analyze Sámi artifacts, processes, or ideas to find examples of Bishop's (1988b, 1990) six fundamental activities (Jannok Nutti, 2013):

> [M]athematics as a cultural knowledge according to Bishop (1988b) is developed on the basis of six universal activities: *counting*, locating, *measuring*, designing, playing, and explaining. More specifically, counting and knowledge of counting; locating and how to encode and navigate the natural environment; measuring and how to measure including measurement units and methods; designing and how to design items, artifacts, and technology; playing and how to play including games and activities; and searching for and explaining a theory or connecting pattern. (p. 60, emphasis added)

Various Sámi people use a range of body-based units when measuring. When Boazosápmelaččat (Sámi people who work with reindeer) are counting the number of reindeer in a herd, "different names for reindeer herds are based on the approximate number of animals" (Jannok Nutti, 2013, p. 61). Unlike Standard Average European (SAE) languages such as Swedish, many other languages worldwide have different names for numerals depending on the type of thing being counted (section 4.2). In contrast, a SAE number has a *universal* name regardless of what is being counted. This generalizability value exemplifies EAM cultural content that could be taught explicitly.

Part two of Jannok Nutti's (2013) project investigated the teachers' perceptions of their participation in part one, and then she assessed the classroom activities the teachers created. Assessment criteria included how closely their activities reflected local Sámi ontology, epistemology, and axiology.

From extensive teacher interviews, Jannok Nutti (2013) discovered the following:

> During the study the teachers changed from a problem-focused perspective to a possibility-focused culture-based teaching perspective characterised by a self-empowered Indigenous teacher role, as a result of which they started to act as agents for Indigenous school change. The concept of "decolonisation" was visible in the teachers' narratives. The teachers' newly developed knowledge about the ethnomathematical research field seemed to enhance their work with Indigenous culture-based mathematics teaching. (p. 57)

These are promising results.

The teacher-designed activities in Jannok Nutti's (2013) study were documented by: photos, video recordings, Jannok Nutti's (2010) research entries, the teachers' written plans, and their professional notes. The degree to which the activities represented Sámi ontology, epistemology, axiology, and ideologies was determined by a four-category assessment rubric, which could be adapted to suit other researchers' culturally appropriate teaching materials. Jannok Nutti (2013) adapted hers from Banks' (2004) assessment of multicultural teaching. This decision reflected her wish to emphasize the heterogeneity among Sámi groups within Sweden and Norway, as well as Indigenous groups elsewhere (Jannok Nutti, May 12, 2016, personal communication). Her four categories are:

1. Sámi cultural thematic work with ethnomathematical content.
2. Multicultural school mathematics with Sámi cultural elements.
3. Sámi intercultural mathematics teaching.
4. Sámi intercultural education based on Sámi ontology and epistemology (p. 63)

The term "intercultural" indicates a "transfer of traditions and knowledge between different cultural groups" (p. 63) – Sámi and EAM, in Jannok Nutti's case. Because the meaning of "intercultural" epitomizes conversations to the depth of ontology, epistemology, and axiology, I interpret it as meaning a "camping spot of dialogue" (Vickers, 2007, p. 592) type of interaction (section 6.1).

In category #1, EAM Platonist content and Sámi *general* themes (e.g., reindeer management) would appear almost independently, but Sámi content (e.g., traditional ways of counting reindeer along with davvisámegiella [the northern Sámi language]) would not appear. Students would only use a Platonist counting process to count reindeer in a herd, completely ignoring Sámi mathematizing. This tends to happen with *contextualized* teaching materials, but not with cross-cultural ones. The problematic meaning of "ethnomathematics content" in category #1 is discussed in sections 7.2 and 9.1.

In category #2, EAM Platonist content is integrated into a *specific* everyday activity (e.g., baking or cooking a specific Sámi dish, or fishing for a specific fish species), but not quite in a way that follows traditional Sámi ways. Jannok Nutti (2013) offered the example of using numbers to record measurements in a Sámi recipe for black pudding. The authentic local Sámi way is to use qualitative directions, such as "a pinch of" (p. 65). Consequently, the classroom activity was actually a false representation of the local community's Sámi culture. As a result, Platonist content was imposed on the Sámi activity. Many Sámi students might not see themselves in the activity and would lose interest accordingly.

Category # 3 represents a cross-cultural activity. "The intention of the culture-based mathematics activities was *to reconstruct school mathematics* by creating a learning environment grounded in both school mathematics [Platonist content] and Sámi culture-based knowledge" (p. 63). The conventional power imbalance between the curriculum and a Sámi community moves closer towards a balance in category # 3. If the reindeer counting activity described just above were modified to teach the traditional way of counting reindeer, along with some relevant Sámi vocabulary, then the Sámi cultural elements of counting could then be compared with Euro-American cultural elements of counting (e.g., its generalizability or its convention of using base 10 rather than base 5). A comparison tends to identify some EAM cultural content, which is usually appropriate for students beyond primary grades.

Notably, this illustration of what a teacher *could* do exemplifies cross-cultural teaching materials that entwine: EAM Platonist content; EAM cultural content; and an Indigenous artifact, process, or idea; in short, *cross-cultural EAM*.

Category # 4 features a “social action approach” (Jannok Nutti, 2013, p. 66), in which students “become engaged in social and political issues” (p. 66) related to Sámi equitable treatment in Swedish society, and onward to an eventual negotiated sovereignty. This category is certainly appropriate in high schools but not expected in primary grades, of course.

The six teachers’ activities fell within the first two categories, which was explained by Jannok Nutti (2013, p. 68): “The teachers wished to implement Sámi culture-based mathematics teaching, but felt that they lacked the knowledge and time to implement Sámi culture-based teaching.” This result could be mistaken as a negative outcome. We must realize that transformational changes take time; sometimes measured in years, maybe decades, or even generations. Thus, we need to conceptualize teachers’ efforts as a journey. Perhaps the four-category scheme needs fine-tuning to capture a teacher’s journey into Jannok Nutti’s category # 3 for non-Indigenous and Indigenous teachers alike, along with forays into category # 4 led by enriched cross-cultural activities. Jannok Nutti certainly did acknowledge the six teachers’ accomplishments:

> After the implementation of Sámi culture-based mathematics activities the teachers still faced the previously described external obstacles, but those that initially seemed to prevent them from adapting to culture-based mathematics teaching no longer stopped them from starting to implement culture-based mathematics activities. The teachers’ work with Sámi culture-based mathematics lessons demonstrated their competence in dealing with the described challenges. In addition, *the challenges were now viewed as opportunities*. (p. 68, emphasis added)

8.7. Mi′kmaw First Nation Projects

“Mi′kmaw communities in Nova Scotia have a unique jurisdictional agreement with the Government of Canada that gives them control over their education system and collective bargaining power” (Lunney Borden, in Sterenberg et al., 2010). This type of sovereignty facilitates innovation in mathematics education.

Show Me Your Math (SMYM, 2016) in Atlantic Canada engages Mi′kmaw students in exploring their Indigenous communities mathematically. The SMYM fair is, in part, a wonderful out-of-school educational-social event that brings students together once a year. At home, students take a traditional or contemporary artifact, process, or idea found in their community (e.g., a hockey rink markings, archery, commercial pizza production, beadwork, snowshoes, and birch bark biting) and learn about Mi′kmaw traditional and contemporary mathematizing, well enough to analyze it in terms of Platonist content. The everyday contemporary mathematizing is a type of mathematics-in-action (section 7.1).

The SMYM website (http://showmeyourmath.ca) has short videos to introduce, explain, and illustrate the project. A major conceptual feature is the website's superordinate definition of mathematics, which lends itself to cross-cultural EAM. A video featuring an Elder and some students establishes Bishop's (1988b) six foundational processes (section 4.1.3) as the answer to "What is mathematics?" This approach: (1) emphasizes a verb-based definition that makes common sense to Indigenous students especially, (2) uses vocabulary to which students can easily relate, and (3) subtly frames the notion of mathematics as culturally pluralistic. The definition lends credibility to internal human dimension to EAM's cultural content (section 4.5).

Of particular interest to R&D project researchers, the SMYM project evolved out of a familiar strategy by which Lunney Borden in 2005 asked Elders for an artifact or process that represented Mi′kmaw mathematizing; after which she reworked the artifact or process into content found in the region's Platonist curriculum; followed by her composing contextualized teaching materials for volunteer teachers to try out (Lunney Borden et al., 2017). Not satisfied with the results, she removed herself from this strategy by organizing a direct consultation between students and Elders or knowledge holders. After their consultation, students create a display to show others what they learned about both EAM Platonist content and Mi′kmaw mathematizing. This version of a cross-cultural project became Show Me Your Math in 2007.

Lunney Borden's "Indigenist" research methodology (Rigney, 1999) was anchored in "challenging hegemony and overcoming oppression" and was guided by the "interrelated principles of resistance, political integrity, and privileging Indigenous voices" (Lunney Borden, 2013, pp. 5-6). This methodology harmonizes with her research program: "*How can curricula and pedagogy be transformed to support Mi'kmaw students as they negotiate their position between Aboriginal and school-based concepts of mathematics?*" (p. 7, emphasis original); that is, between Mi′kmaw mathematizing and EAM Platonist content.

As a non-Indigenous researcher who undertakes studies with Indigenous people, Lunney Borden (2013, p. 8) outlined her personal background and relationships with the Mi′kmaw First Nation in order for readers to understand this significant contextual feature of her cross-cultural R&D. She immersed herself in the Mi′kmaw culture during her 10 years as a teacher and administrator in one Mi′kmaw school. She confronted and deeply reflected upon the complexities of teaching school mathematics from a Mi′kmaw perspective, aided considerably by a functional literacy in Míkmawísimk (the Mi′kmaw language).

Her commitment is evidenced by her enrolling in a PhD program culminating in her thesis entitled “Transforming Mathematics Education for Mi′kmaw Students through Mawikinutimatimk,” a Mi′kmaw word meaning “coming together to learn together” (Lunney Borden, 2010, 2013, p. 9). Lunney Borden’s ultimate agenda is to work towards the sovereignty of Mi′kmaq (the Mi′kmaw people). She is explicitly conscious of the present power imbalance between the mathematics curriculum and Mi′kmaw communities. She views SMYM as a mechanism to reduce this imbalance, in particular; and she embraces a decolonizing agenda for Mi′kmaw schools, in general (Lunney Borden et al., 2017).

Some Mi′kmaw schools are bi-cultural (e.g., an Eskasoni school on Cape Breton Island) due to the community’s special efforts to revive Míkmawísimk there, as is happening on some reserves across the Maritime Provinces (Kinew, 2015, p. 182).

Lunney Borden (March 28, 2016, personal communication) pointed out a unique advantage of collaborating with Mi′kmaw schools: “[T]he majority of teachers are Mi′kmaq and from the communities in which they teach, so they knew the community context well, and they had ideas about the kinds of things they wanted to do.” Moreover, it is fairly easy to find Elders and knowledge holders to offer guidance and support.

Similar to the MCC project in Alaska, SMYM’s capacity development within Mi′kmaw communities is on her agenda for the decolonization of Mi′kmaw schools; as is her allegiance to ethnomathematics expressed in “the significance of making ethnomathematical connections for students” (Lunney Borden, 2015, p. 758).

At first, Lunney Borden’s allegiance to ethnomathematics seemed to exclude EAM cultural content; an exclusion both noticeable in her earlier descriptions of SMYM, and detrimental to her decolonizing agenda for school mathematics (section 7.2). I appreciate her ambivalence about a truncated EAM school mathematics; an ambivalence that she apparently worked through.

Upon reflection, Lunney Borden (2013, pp. 10-11) summarized four key capabilities with which teachers should be conversant so they can better help Mi′kmaw students make meaningful personal connections to EAM content. These are paraphrased here. Two of them (#2 and #3) definitely address EAM cultural content that Platonist-culture-blind ethnomathematics purposefully ignores.

1. Learn *from* students’ Indigenous language

2. Point out to students the *value* differences (i.e., differences in axiological presuppositions) between Indigenous mathematizing and school mathematics, by emphasizing cross-cultural understanding and EAM cultural content that gives a cultural context to the Platonist content in school mathematics
3. Attend to diverse ways of learning among students (e.g., recurrent learning strengths, Aikenhead et al., 2014, Appendix D) and ways of knowing between the culture of EAM and the local Indigenous culture (e.g., Aikenhead & Michell, 2011, pp. 63-98)
4. Make explicit relationship connections that are physical, intellectual, emotional, and spiritual.

Within SMYM, important relationship dynamics are responsible for renegotiating Platonist school mathematics into cross-cultural school mathematics (i.e., "D'Ambrosio mathematics;" section 7.2). The Mi′kmaw community becomes a valid source for school mathematics content, which establishes a shared authority with the mathematics curriculum (i.e., from a power imbalance towards a more balanced relationship). Students can then enter a discussion about the Mi′kmaw community's knowledge being privileged alongside the textbook's Platonist content, in a coexisting way. Students' roles expand from being just classroom mathematics students to becoming *researchers* of Mi′kmaw mathematizing, *constructors* of EAM Platonist content, and *disseminators* of both (Lunney Borden et al., 2017).

A sampling of evidence for this comes from systematically crafted "storylines" (i.e., short narratives) that capture how these new dynamics play out; for example, intergenerational (grandparent-grandson) instruction-investigation into how to cut circles in a tanned hide to make sacred drums. Another example is two Grade 12 students learning the Mi′kmaw way of replicating an 8-point star design in beadwork; a design of sacred significance to Mi′kmaq (the Mi′kmaw people) and to other First Nations as well. The students compared that process with the Euro-American mathematizing found in the geometry section of their textbook that produced the same design. By doing so, these two Grade 12 students illustrated cross-cultural learning in action.

Unlike a number of researchers cited in this monograph, the two students recognized that the Mi′kmaw beadwork mathematizing occurred with perfect results "*without*" the bead worker knowing any Euro-American geometry (Lunney Borden et al., 2017; the students' actual word). Remarkably, the two young women, after 11 years of schooling, had not been assimilated into a Platonist belief system, which claims that the universalist Platonist geometry was situated or embedded in the Mi′kmaw 8-point star beadwork mathematizing all along. It seems to me that SMYM encourages students' capabilities to be independent thinkers and problem solvers.

Lunney Borden's (2013) decolonization agenda for SMYM and the evidence of success provided by Lunney Borden and colleagues (2017), matches her enriched and explicit attention to EAM cultural content.

Other evidence shows that as students forge stronger relationships with Mi′kmaw mathematizing, they tend to improve their relationship with school EAM (Lunney Borden and colleagues, 2017). This helps explain why Indigenous students' mathematics and science achievement increases significantly, on average, when the school subject is taught enhanced with Indigenous perspectives (Banks & Banks, 2012; Fowler, 2012; Lipka, et al., 2005; Keane, 2008; Meaney et al., 2012; Nichol & Robinson, 2000; Perso, 2012; Richards et al., 2008; Sakiestewa-Gilbert, 2011; U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education, 2008). All of this occurs in synchrony with the ever changing role of teachers: as knowledge holders of EAM Platonist content, as sources of information and ideas, as facilitators, as mentor-guides, and as learners.

How exactly do most students analyze Mi'kmaw artifacts, processes, or ideas in order to produce a SMYM display? One answer is a summary of earlier discussions on a transformation process that includes superimposing, deconstructing, and reconstructing. Students would: need mental images from EAM Platonist content in the first place; superimpose an image (or images) on the artifact, process, or idea in a best-fit fashion; deconstruct the artifact, process, or idea, stripping it of its Mi'kmaw peripheral concepts; and finally, reconstruct it to fit the parameters of the culture of school mathematics (i.e., school EAM). Thus, students will need to know specific EAM Platonist content with which to *transform* an Indigenous artifact or process into a SMYM display. Hockey rinks and archery, for instance, were transformed into geometric concepts and vectors, respectively. SMYM's need-to-know feature offers substantial motivation for students.

But students will respond to the challenge in varying degrees. Students whose "worldview harmonizes with" the *quantitative* worldview endemic to the culture of Euro-American mathematics usually require little guidance (Aikenhead, 2007, 2006, p. 29). However, SMYM is less engaging in varying degrees for other students (section 9.3), depending on how severe they experience a culture clash with quantitative, abstract, and hypothetical thinking in the context of their everyday world (section 3.3).

In Show Me Your Math, students, not researchers, create materials to teach others about Mi′kmaw and Euro-American mathematizing. This occurs at schools, but for highly successful students, at the annual SMYM fair as well. Understandably their displays are not professional grade resources teachers can use. But Lunney Borden and Wagner (2016) initiated a related and more broadly based R&D project, “Mawkinumasultinej: Let's Learn Together!” which grew out of a few excellent SMYM displays. The researchers consulted with more Mi′kmaw Elders, learned the Mi′kmaw mathematizing processes in depth, and then designed a series of *cross-curriculum* and *cross-cultural inquiry* modules that emphasize both EAM and Mi′kmaw cultures. Topics include: Kataq/Eels (including a study of Mi′kmaw inherent treaty rights), Quill Boxes (honouring an Elder), and Birch Bark Biting – with direct analogues to the Grades 5-7 geometry curriculum (Lunney Borden, 2015). These modules are designed according to collaborative classroom inquiry pedagogies. The inquiry projects contain support resources for teachers, including a link to an extensive assessment rubric.

Similar to schools across Canada, most non-Mi′kmaw schools do not find SMYM especially attractive at this time, in spite of the strong expert support they would receive from the well-organized Mi′kmaw communities in Atlantic Canada. Based on my experience with enhancing school science with Indigenous perspectives, I would suggest that a cascade of reasons for this low interest would begin with the absence of Mi′kmaw perspectives explicitly in the content domains of the mathematics curriculum. Other reasons would likely include: a paucity of resources, few teacher professional development opportunities, the political stance to maintain current dogma about mathematics (i.e., purely Platonist content only) found in the general public who have been assimilated or indoctrinated by years of exposure to the dogma, and lastly the pervasive racism that exists in the underbelly of Canadian society (Battiste, 2002; Government of Alberta, 2010; Ralston Saul, 2014; St. Denis, 2004; Truth and Reconciliation Commission, 2016b).

Optimism, however, resides in the knowledge that Canadian Ministries of Education are currently taking reconciliation very seriously.

8.8. University of Alberta Project

Significantly, the research and R&D projects explored above created a respectful space for Indigenous representation, voice, and collaboration. Many demonstrated that the academy’s conventional research methodologies can make room for Indigenous interests in school mathematics. Other R&D projects were explicitly influenced by, or drew upon, an Indigenous research methodology (sections 8.2, 8.5, 8.6, & 8.7).

The University of Alberta project, conducted by three academics, Donald, Glanfield, and Sterenberg (2011); went even further by immersing their project in a purely Indigenous methodological approach described by Kovach (2009). What took place was a shift in research perspectives, from a mainly literacy tradition to a mainly oral tradition that celebrates relationships. Donald is of First Nations ancestry, Glanfield is of Métis ancestry, and Sterenberg is Euro-Canadian.

Donald and colleagues' (2011, p. 73) "culturally relational" Indigenous inquiry paradigm emerged when Eagle Flight Band Councillors (named for anonymity) contacted them for help in improving student performance on provincial school mathematics tests. The research team decided against being the experts who would come to the reserve and solve their problem, as the academy is prone to do. Instead, the research team invited the whole Eagle Flight community to engage in research with them. Most members accepted. Working together, they defined what their community-based research project would be: to explore "the ways in which community members, children in school, school staff, and school and community leadership come to develop a shared understanding of mathematics" (p. 77). This became a four-year project at this K-9 reserve school, in which teachers individually created their own personal teaching techniques and materials as a result of Donald and colleagues' collaborative action research in teacher development.

Because this is a novel approach compared with Euro-American paradigms of research, more detail is shared here about Donald and colleagues' project in their own words; more so than would normally be expected.

The first year unfolded with meetings among community members, teachers, and staff, directed at the task of developing:

> an understanding of the ways in which children in the community know mathematics. …The teachers identified the need to know what their children could do mathematically so, together with the teachers, we designed a variety of assessment strategies such as performance-based tasks and interviews to develop an understanding of the way in which the children in their classrooms think about mathematical ideas. (p. 77)

During the second year, the research team:

> worked alongside three staff-identified lead teachers as [they] engaged children in mathematical interviews and collected data on children's thinking through video-

> recordings. The recordings were analyzed by this team of teacher/researchers and then we shared video clips and preliminary results with the rest of the school staff. (p. 77)

In the third year, "the teachers of the entire school decided to engage their children in paper-and-pencil and performance task assessments. These assessments were designed by the researchers and the initial lead teachers, and then administered by all teachers" (p. 77).

The fourth year began by all staff analyzing the results of the previous year-end assessment. Then:

> [T]eachers are using what they are learning about children in their classes to inform planning and classroom practices. Specifically, they are focusing on mathematical vocabulary development and teaching mathematics with manipulatives. Assessments of children's thinking are ongoing and data collected on these assessments is shared with us. During staff in-service meetings, our conversations about what teachers are learning from children when children are asked to explain their mathematical thinking are recorded as data. Insights into the ways in which teachers are learning inform our ongoing work together. (p. 77)

Donald and colleagues' (2011) culturally relational approach harmonizes strongly with Indigenous perspectives. It is ontologically, epistemologically, and axiologically student- and community-centred. For instance, it is wholistically organized and profoundly anchored in relationships. Teachers' and students' knowledge was not expressed in written lesson plans or modules for public consumption. Instead it was captured in action, for example, in the teachers': (1) active listening skills (mentored by the researchers); (2) revisions to lessons that took into account differing unique ways students had resolved a problem based on their Indigenous worldview; and (3) videotaped interactions with students in the classroom. Within a culturally relational research stance, knowledge is action while action is knowledge; an instance of Diné (Navajo) paradoxical reasoning (Maryboy et al., 2006) or a non-deterministic "paradox of reason" (Sriraman, 2005, p. 101).

"As educators and researchers, …we are seeking to honour meaningful engagements with Indigenous philosophies and knowledge systems as they are understood and lived by all in relation" (Donald et al., 2011, p. 80).

Any R&D project that explicitly invests in relationships, and in the Indigenous participants' understanding of their own culture, will certainly be successful. For non-Indigenous researchers, however, a prerequisite is necessary. Their degree of grounding a project in

Indigenous methodologies must correlate with their experiential understanding of an Indigenous community or communities, along with the Indigenous human-resource support found in those communities (Aikenhead, 2002a).

The result is a unique, wholistic, place-based, cross-cultural school mathematics. For the University of Alberta project, EAM Platonist content was present because the community's focus was definitely on raising test scores. But likely the Platonist content taught did not have its usual neo-colonial influence, but instead a decolonizing influence; an issue worth investigating in the future. Cross-cultural strategies were present and completely in tune with the local Indigenous worldview, although explicit attention to EAM cultural content was not made apparent.

8.9. Conclusion

The Eagle Flight First Nation was superbly served. Each project discussed in section 8 attended to different clusters of pedagogical factors and chose different content features to emphasize when they developed their version of culture-based school mathematics. Their decisions necessarily involved making trade-offs among a complexity of factors and emphases that could help attain their goals for Indigenous students.

So it was with Donald and colleagues (2011). The knowledge/action that they can share with teachers in other reserve schools is limited. The trade-off between a four-year in-depth project with a few teachers and a four-year in-depth project to produce and disseminate effective teaching materials to many teachers, is a trade-off all researchers make. The potential number of Indigenous students who benefit from a project is but one criterion in the complexity of criteria present in designing a R&D project. Most important, however, much is gained from the *diversity* of research and development studies described in this monograph.

9. Correcting Lingering Impediments to Student Success

European mathematics is offered to [I]ndigenous people around the world, “gift-like”, as a passport to success and future development. …The promotion of mathematics may be ethnocentric, but is it not, we believe, offered with malice. The bearers are probably unaware of the inherent dangers as the receivers.

Bill Barton and Uenuku Fairhall (1995, p. 1)

Fasheh (2012) agrees, but from the perspective of Western civilization’s powerful tools (e.g., schools and student grades) constructed with a blueprint of universalism. “This led to ‘evangelising’ the world through education, which was done mainly through people with good intentions but who carried the virus of one and only one path for learning” (p. 100).

Throughout this monograph, I too have acknowledged the dangers to Indigenous cultures when children study “European mathematics” (Platonist school mathematics). And while I also agree with Barton and Fairhall’s (1995) “no malice” and Fasheh’s “good intentions” assertions, these assertions strike me as red herrings to be ignored, because the dangers of which they speak actually emerge from a different source: masking power with the innocence of claiming no malice intended or claiming good intentions; not from a malevolent intent itself. The obstacles Indigenous students face are more subtle, systemic, and implicit ones. The various species of masking power with innocence noted in this monograph maintain the neo-colonial impediments to Indigenous students’ success in school.

These impediments tacitly undermine the quality of teaching materials and pedagogy because the impediments cause Indigenous students to feel their culture is not respected as it should be (Doolittle, 2006; Meaney, 2002). In the lived reality of Indigenous students, educators’ intentions, benevolent or malevolent, are simply irrelevant. Student reaction will be: tolerance, reluctance, resistance, or rejection; depending on the student.

Accordingly, section 9 answers the following questions: What are the conventional taken-for-granted notions that impede student achievement? Which of these conventional notions continue to be held by many innovators who have enhanced school mathematics culturally? What practices should be avoided? Exactly how do researchers or teachers “see” school mathematics content “embedded” in an Indigenous artifact or everyday activity?

Some of the answers have been discussed previously in general terms. In order to give greater depth and clarity to them, this section explores them concretely in the context of projects described in section 8. The issues are organized around four general topics: the altar of Platonist content, myth blindness, projectionism, and misinterpretations of quantified assessment and evaluation.

9.1. The Altar of Platonist Content

The dynamics of Platonist content extends its neo-colonial power into mathematics teaching and R&D projects, where some culture-based innovations to school mathematics have been sacrificed on the altar of Platonist content. This content deserves serious attention, to be sure. But the current *excessive* emphasis on Platonist content has created the following cycle: (1) myths about Platonist content, (2) beliefs in those myths, (3) social power bestowed upon those beliefs, and (4) privileges gained by that social power (sections 3.2, 4.1, & 4.2.1). Unless broken, this cycle will repeat itself for generations to come. Now is the time to break the cycle. The altar's ideology assumes "that our current [mathematics education] system is the ideal and that students who do not aspire to that goal are 'at risk'. …'If you're not like us, then you are deficient'" (Meyer, 2003; quoted in Russell, 2010, p. 42).

The altar of Platonist content, "the hegemony of academic mathematics" (Skovsmose 2012, p. 351), comes in direct conflict with educators' efforts to decolonize school mathematics. Classroom incidents of this were identified by Russell and Chernoff (2013) who showed how teachers' Platonist beliefs and Euro-American worldviews disrupted Indigenous students' engagement in activities linked to Indigenous cultures. An exclusionary Platonist content presence occurs at the expense of Indigenous perspectives.

For instance, the MCC project (section 8.3) appears to assume that Indigenous students' exposure to authentic Yup′ik culture is sufficient to strengthen their cultural self-identities, without having that cultural content assessed by its modules' assessment regime. If it were assessed, however, it would certainly send a clearer message about the importance of Yup′ik mathematizing. In addition, it would tend to open up class discussions comparing features of the Yup′ik culture with the culture of Euro-American mathematics (EAM), just as Fyhn (section 8.6.1) and Lunney Borden (section 8.7) described.

This important task seems left to teachers to complete. Some would certainly do it, but many may not. In four ethnographic case studies (*Journal of American Indian Education*, 2005, No. 3), teachers' attention was only on improving EAM Platonist content outcomes. If "two-way learning" (Lipka, 1994, p. 15) is to reach its full potential, students must experience two-way assessment. The proportion of each culture (Indigenous and EAM) does not have to be *equal*, but Indigenous perspectives need to be noticeable beyond tokenism; in a word, *equitable*.

For example, the mathematics problem – "If a fish rack holds 6 King Salmon, how many Red Salmon would the fish rack hold?" – is all about EAM Platonist reasoning (see the end of section 8.3). Nothing about Yup′ik mathematizing appears, such as: "What steps does a person go through to dry salmon on a fish rack? In your answer, include the protocols to follow." A repetition of such absences of Yup′ik mathematizing from student assessment suggests that MCC is singularly focused on EAM Platonist content outcomes, thereby largely ignoring any student assessment of EAM cultural content and content related to Indigenous perspectives. The altar has been effective here.

An exclusionary reliance on EAM Platonist examinations is fraught with dangers to Indigenous students' cultural self-identities (Doolittle & Glanfield, 2007; Barton & Fairhall, 1995; Fyhn et al., 2016; McMurchy-Pilkington & Trinick, 2002; Nasir, 2002). From an Indigenous viewpoint, this indicates a degree of disrespect (Sterenberg, 2013a). In such situations, Elders and Indigenous teachers can experience "a threat towards their culture and language" (Fyhn et al., 2011, p. 191). The issue is this: "How and to what extent can Sámi mathematizing be included in school mathematics" (Fyhn et al., 2011, p. 190; Jannok Nutti, 2013). "Insofar as the Sámi curriculum claims that the teaching should focus on Sámi values, the [national] examination needs to reflect some Sámi values" (Fyhn, 2013, p. 364). If not, school mathematics will continue its neo-colonizing, systemic discrimination that sustains a political-social power imbalance between the mathematics curriculum and Indigenous communities.

But the issue is even more complex. "Many Sámi parents believe that their children will succeed better in their future education if they use national textbooks in mathematics" (Fyhn et al., 2016, p. 415). For most Indigenous communities, decolonizing school mathematics means collaborating with parents to maximize the innovation's sustainability in that local community.

Describing the Algonquins of Pikwakanagan First Nation Project (section 8.5), Beatty and Blair (2015) state:

> As a group, we watched and analyzed video recordings of in-class lessons to identify the mathematical thinking that surfaced during classroom conversations. This process of co-analysis involving Algonquin and non-Native teachers and university faculty ensured that our focus was both on the students' mathematical thinking and *the cultural connections*. (p. 12, emphasis added)

Ample evidence was presented for claiming Platonist mathematical thinking by students, but little specific evidence, other than assurances, was given for students' making connections to Algonquin mathematizing. A reader could wrongly form the opinion that the project's Platonist content had marginalized features of Algonquin culture. As questioned earlier, where is the Algonquin language, values, spirituality, and intergenerational stories that supplement the Algonquin looming project and convey aspects of the community's culture?

Perhaps such evidence does exist for both the Yup′ik and Algonquin projects, but consciously or unconsciously, the authors excluded it from their manuscripts submitted for review and publication. Like Kovach (2009, p. 28), I could conclude "that Western epistemological privilege pervades the academy." Is this another privilege blindness enacted by refereed journal Editors and reviewers? Mathematics education researchers need to resolve this dominance by the Platonist altar.

The University of Hawai′i R&D projects appear to express an ambivalence over the role of Native Hawaiian mathematizing (section 8.4). On the one hand, the word "mathematics" refers to Native Hawaiian mathematizing: "This textbook draws on the unique pedagogies, values, and *mathematics of our sacred islands*" (Furuto, 2013b, p. ix, emphasis added), and "The *mathematical ideas of the peoples of the Pacific region* have often been overlooked, particularly in…mathematics. Every cultural group develops its own ways and styles of explaining, understanding, and coping with their environment" (Furuto, 2013a, p. 39). The concept here is certainly pluralism of mathematics and a legitimacy of Polynesian mathematizing.

On the other hand, EAM Platonist content is clearly central to Furuto's (2014, p. 112) allegiance to ethnomathematics (section 7.2). Indigenous mathematizing is acknowledged (especially its values), yet not specifically in comparison to values found in Euro-American mathematizing (section 4.5). This ambivalence is likely caused by unconsciously moving between two different meanings of the word "mathematics" – its superordinate and subordinate meanings (section 4.1). Without making that distinction, conversations will be laced with confusion and misconceptions (section 6.3); thereby causing the relationship between Polynesian and Euro-American mathematizing to be ambiguous and unbalanced. A balanced relationship epitomizes cross-cultural teaching, which includes camping spots of dialogue (Vickers, 2007, p. 592) as a key pedagogical strategy to maximize transparency for Indigenous and non-Indigenous students.

To repeat a comment from section 7.1: A better R&D project would not treat Indigenous practices, artifacts, and ideas *merely* as cultural objects to be encountered with a EAM Platonist lens. In effect, Native Hawaiian mathematizing seems to have been subordinated with respect to EAM Platonist content, rather than being two equal but different mathematical knowledge systems, in accordance with Furuto's pluralist belief. In other words, its superordinate meaning is wrongly equated with Platonist content, which is actually a subordinate mathematics as is Polynesian mathematizing. This is the flaw in ethnomathematics (section 7.2).

However, in some undergraduate mathematics courses at the University of Hawai′i, Polynesian mathematizing is certainly an important explicit feature. As noted in section 8.4, compared to other major R&D projects, the ESTEP Institute at the university level appears to reach a more equitable balance between its focus on EAM Platonist outcomes and its attention to Indigenous perspectives.

Yet the altar of Platonist content, which severely marginalizes Indigenous perspectives, shows up in otherwise excellent lesson-plan modules developed for undergraduate students (section 8.4). For example, the undergraduate module "Vectors and Navigating a Voyaging Canoe" (ESTEMI, 2016) details Polynesian mathematizing but does not acknowledge it as mathematizing:

> Polynesians used a conceptual star compass to determine direction through the rising and setting of different stars. During the day, close observation of the sun, steady wind patterns and ocean swells were used to navigate. …In order to find land, the navigators looked for cloud and swell patterns, as well as for birds that usually fly up to 20 to 30 miles from land. (p. 1)

In the following passage, is Polynesian mathematizing dismissed and replaced by EAM Platonist content by the author?

> The Polynesian navigators had an *intuitive sense of math*. Math was needed to estimate speed and time, and to observe the different angles of the stars, planets, and sun. As demonstrated in this lesson plan, math was also needed to understand the different forces that altered the path of the voyaging canoes. (p. 2, emphasis added)

If there be any doubt that it has been marginalized, consider the module's conclusion: "Even though the early navigators never took a math course in their entire lives, they were natural mathematicians" (p. 4). As discussed in section 6.3, Platonist power has been masked by an intended compliment that privileges Platonist content at the expense of Polynesian mathematizing (Garroutte, 1999). Subtly, the power imbalance has shifted further towards the colonizers in this module, it would seem; another example of the altar's exclusionary dynamics.

Culturally sensitive editing of the lesson-plan modules is needed for three reasons: to complement the culturally responsive feature of this ESTEMI module with Polynesian mathematizing and EAM cultural content; to challenge the imbalance of social power between Native Hawaiians and the University of Hawaiʻi's ethnomathematics curriculum; and to be a model for producing decolonizing teaching materials. All three reasons speak to ESTEMI's espoused goal of social justice.

As Neel (2008) discovered in his Haida Gwaii study (section 8.1.1), not all Indigenous mathematizing is necessarily amenable to a transformation process (superimposing-deconstructing-reconstructing). If we cannot make the process age-appropriate and transparent to students (e.g., "This is how mathematicians would understand your Indigenous activity according to their worldview…"), then we should not proceed with that specific process. If the relationship between the two knowledge systems is forced; for instance, by pretending that EAM Platonist content was present in Indigenous mathematizing to begin with; the module becomes superficial and contributes to the neo-colonial teaching materials already circulating in classrooms.

By experiencing cross-cultural teaching materials, all students should come away with at least an explicit appreciation of both the local Indigenous worldview and the worldview endemic to EAM. Both are powerful in their own way. Both coexist. The ESTEMI project likely meets this culturally responsive standard for some students in some circumstances.

Most mathematics educators acknowledge the tensions between, on the one hand, student achievement in Platonist content (the intellectual tradition of thinking), and on the other, achievement in strengthening Indigenous students' cultural self-identities (the wisdom tradition of thinking, reflecting, doing, living, and being). As mention in many studies, most Indigenous families wish for their children's success at acquiring school mathematics accreditation in order to do well as adults in the dominant society (Bang & Medin, 2010; Beatty & Blair, 2015; Jannok Nutti, 2013; Lipka, 1994; Lunney Borden, 2013; Meaney, 2001; Neel, 2008; Neel & Fettes, 2010; Pelletier et al., 2013), but usually not at the cost of students losing their Indigenous values and ways of thinking. "In the world in which we now live, it is easy to develop mind to the exclusion of all else. As teachers and learners we must find ways to develop all those other aspects of the persons, even within mathematics" (Doolittle & Glanfield, 2007, p. 29).

The Alaskan Ciulistet and MCC projects (section 8.3) counselled mathematics educators to find a consensus within each community on a balance between the intellectual and wisdom traditions of understanding. Others, like Jannok Nutti (2013), discovered a range of opinion on this issue among Sámi mathematics teachers, and this distribution of opinion changed in both directions as teachers experienced culture-based teaching (section 8.6.2). On the one hand, the teachers were concerned about their responsibility to their pupils' success in future education in the Swedish municipal compulsory school system. On the other hand, they were committed to "providing [students] with specific Sámi culture-based knowledge" (p. 67). What made the Sámi teachers' conflict even worse was the fact that their desire to prepare students for future courses actually corresponded to the Sámi values of iešbirgejeaddji (to become independent) and birget (to manage one's life towards survival and financial self-sufficiency). In other words, the teachers faced a wrenching internal conflict within their Sámi values of iešbirgejeaddji and birget, which tore the teachers towards two conflicting outcomes – more versus less attention to Sámi culture.

First and foremost, I am convinced that all teachers have their students' best interests at heart. Their vision of what is best arises from either a compromise among many innovative views to be considered, or a complete adherence to the status quo curriculum. Reasonable pragmatism has been a ubiquitous teacher criterion in the research literature concerning how teachers determine what they will do.

The wisdom of reaching a consensus over a curriculum balance seems like a more rational strategy than blindly submitting to the coercion by corporate-economic-political institutions that dismiss the development of Indigenous self-identities (Aikenhead & Sutherland, 2015) in their quest to increase the population of students in the science, technology, engineering and mathematics (STEM) pipeline (Weinstein, Blades & Gleason, 2016). STEM tends to worship at the altar of Platonist content. Lunney Borden and Wiseman (2016) show how we can manipulate STEM to support the inclusion of Indigenous mathematizing in school mathematics.

The STEM student enrolment crisis is mainly a smoke and mirrors creation by corporate-economic-political institutions (Charette, 2013; Lunney Borden & Wiseman, 2016) (section 3.2). If a country wants to augment its international economic competitiveness, which is the rationale for a STEM pipeline policy (i.e., proselytizing the altar of Platonist content), pushing more students into *conventional* school mathematics and science classes – the STEM pipeline – will not resolve the crisis; it will only make it worse by continuing the current trend in declining student enrolment in STEM subjects.

Three different clusters of evidence concerning the STEM pipeline are mentioned here. First, as I documented in section 1, attention to Indigenous self-identities when teaching mathematics augments both Indigenous and non-Indigenous students' Platonist content achievement, thus encouraging *more* students to continue on in the STEM pipeline.

Secondly, a 16-year longitudinal study conducted by the U.S. Office of Technology Assessment (Frederick, 1991) summarized in Aikenhead (2006, pp. 25-26), began with four million grade 10 students, 18 percent of whom expressed an interest in continuing toward university STEM courses. Of these interested students, 19 percent lost interest during high school (i.e., they moved out of the pipeline). Then, during university undergraduate programs, 39 percent of first year STEM students moved out of the pipeline during their undergraduate program; *twice* the proportion of high school students. These quantitative data support in-depth qualitative research that concluded: *the crisis or problem of qualified students moving out of the STEM pipeline resides almost entirely with universities and colleges, and not hardly with high schools* (Aikenhead, 2006). To make a real difference, the corporate-economic-political institutions advocating STEM, and indirectly the altar of Platonist content, need to visit university and college STEM departments because that is where the problem obviously lies.

Thirdly, the longitudinal study also found a 42 percent loss in the number of students interested in the STEM pipeline between high school graduation and first-year university. These data are partly explained by an in-depth Oxford University study which discovered that highly capable A-level science and mathematics students, particularly young women and minority students, switched out of STEM courses as soon as they received their high school credentials, because characteristics of STEM classes were perceived as "socially sterile, impersonal, frustrating, intellectually boring, and/or dismissive of students' life-worlds and career goals" (Aikenhead, 2006, p. 26). In short, the altar of Platonist content helped *discourage* them from studying further.

The altar of Platonist content is often subtle. For instance, it is invisible at work in the project title "Math in a Cultural Context" (section 8.3) depicts school mathematics as not having a cultural context of its own (i.e., no EAM cultural content). The MCC project's title implies that only Indigenous mathematizing constitutes a cultural context, not "Math;" therefore, EAM Platonist content must be culture-free, but it can be placed in the context of Indigenous mathematizing. "Math" is not identified with its own cultural features or cultural context. This suggests that Platonist content is not a human construction anchored in Euro-American cultures. Bishop (1988b), Ernest (1991), Greer and Mukhopadhyay (2012), and Skovsmose (2016), among many others cited in this monograph, would disagree.

In addition, *school* mathematics is cultural because it operates according to a school's culture, which places the subject on a pedestal of authority not to be questioned (sections 3.2 & 4.1). Interestingly, Lipka and colleagues (2005) state:

> Although we use the concept "culturally based curriculum and pedagogy," it is, in fact, a misnomer. *All curricula are culture based*. The key question is on whose culture is it based? In most [I]ndigenous contexts in the U.S., curriculum has been based on and imposed by the majority culture. (p. 3, emphasis added and spelling corrected)

These researchers appear to say that mathematics curriculum and pedagogy are constructed by education committees and by teachers who have an allegiance to U.S. mainstream culture. But the authors either purposefully or subconsciously omit the cultural status of the conventional mathematics content itself; assuming it to be acultural, as their MCC project title suggests.

Bishop's (1988b) pluralism implies the superordinate designation to the term "mathematics," and it causes *all* subordinate mathematics, such as EAM, to exist within a cultural context. Thus, a true pluralist stance helps to decolonize the political-social power imbalance between Indigenous communities and the mathematics curriculum. However, the altar of Platonist content opposes it, through the process of cognitive imperialism (Battiste, 1986; Kuokkanen, 2006); thereby giving authority to the systemic neo-colonialism of Platonist school mathematics (Ernest, 2016a). It certainly is the case that the Emperor (the altar of Platonist content) has no clothes (a rational claim to sole authority). Let us drop the pretence that he does.

In the context of cross-cultural school mathematics, such as MCC, it appears that a 19th century Platonist belief about school mathematics, manifested through the altar of Platonist content, is well beyond its "best before date." *Now is an appropriate time to replace it with* a 21st century, pluralist, cultural understanding of school mathematics with both its Euro-American cultural and Platonist components, along with another culture's mathematizing.

9.2. Myth Blindness

Lipka and colleagues (2005, p. 1) reaffirm that "bringing local knowledge into American Indian/Alaska Native education requires reversing historic power imbalances that continue to separate school knowledge from community knowledge." But how far do the MCC, ESTEI and Algonquian projects go in reversing historic power relations? Not as far as they could, due to a pervasive blind spot.

As I emphatically stated in sections 8.3, 8.4, and 8.5, the MCC, ESTEI and Algonquian projects *do not* embody privilege-blindness, described in section 1 as not knowing or not caring that personal privilege and power have accrued at the expense of another group. But many mathematics educators do appear to suffer from a different kind of privilege-blindness: mathematics myth blindness.

Their school subject's privileged status rests on the myth that its Platonist content is acultural, universalist, value-free, objective in its use, and non-ideological (sections 4.1.1 & 4.2). (This position correlates with a D'Ambrosio ethnomathematics viewpoint; section 7.2.) Because these mythical characteristics deny the pluralism of mathematics, the myth indirectly denies an equitable legitimacy of Yup′ik, Native Hawaiian, and Algonquian mathematizing. MCC, ESTEMI, and the Algonquian projects own a conundrum. Their myth masquerades as an acceptable view of school mathematics to the detriment of many Indigenous and non-Indigenous students. The myth is allowed to continue as Platonist dogma and is firmly ensconced in such standards as the National Council of Teachers of Mathematics (NCTM, 2000), and Alaska's and Hawai′i's Mathematics Common Core State Standards (MCCSS).

The dogma issue is certainly not one of semantics because it can seriously influence Indigenous students' negativity, as Doolittle (2006) observed (worth repeating one last time), "Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures" (p. 20). The degree to which Doolittle's position is not held by Indigenous participants involved in MCC's or ESTEMI's development could indicate the extent to which internalized colonization has occurred. "We can also colonize ourselves in our very own language if we are not aware of the subtle and more insidious forms of colonization and assimilation some of which we may have internalized as parts of our thinking" (Kuokkanen, 2006, p. 3). The dogma issue is certainly not simply about the name "Math in a Cultural Context" discussed in section 9.1. Nor is the dogma simply a matter of someone's "vision of mathematics" (Fyhn, 2013, p. 355), because that vision masks power with innocence by impeding the decolonization of school mathematics.

The conundrum remains: popular dogma versus legitimate interests of Indigenous and non-Indigenous students. How do we resolve it? – By explicitly teaching both *Euro-American* mathematics (EAM) cultural content and Yup′ik or Hawaiian mathematizing. EAM cultural content may not be age suitable for the intellectual development of Grade 2 students in the Algonquian project.

This much needed EAM cultural content has a "myth-busting" quality aimed at the nature of Platonist content and its influence on society. As exemplified throughout this monograph, EAM cultural content comprises (sections 4.5 & 7.1): ideologies (formerly implicit in school mathematics); values (formerly suppressed); presuppositions (formerly suppressed); the multi-national history of pluralist mathematics (generally ignored); Euro-American cultural artifacts, activities, and ideas explicitly or implicitly related to Platonist content (formerly relegated to pedagogy, away from the nature of Platonist mathematics); explicit or implicit influences of Platonist mathematics on a society (formerly suppressed); and the limitations and consequences of a Platonist belief (formerly censored). For mathematics educators wanting to make their subject inviting and accessible to students who are anxious or bored over studying school mathematics (Stoet et al., 2016), EAM cultural content can act as a bridge between, on the one hand, students' everyday non-Indigenous or Indigenous cultures, and on the other, Platonist content; if and only if Platonist content is treated as another culture's way of mathematizing; a very powerful way indeed.

In ESTEMI's case, which also involves university mathematics courses and programs, only a modicum of change is needed to alter a university's Platonist dogma-laden curriculum standards. The resulting cross-cultural curriculum will better represent the academic subject as a human endeavour (a stated goal of ESTEMI), thereby further reducing the culture clashes experienced by many Indigenous and non-Indigenous university students studying in the culture of EAM at the University of Hawai′i.

9.3. Projectionism

Euro-American people often assume that their knowledge expressed in their SAE (Standard Average European) language will have a direct equivalent in an Indigenous language. Many mathematics teachers and researchers also tend to believe the naïve implication that when transforming an Indigenous activity into school mathematics content, nothing important gets lost in that transformation (sections 4.4 & 6.3). These assumptions and beliefs occur because teachers and researchers: (1) do not recognize the unconscious act of projecting their Euro-American presuppositions, images, and concepts onto Indigenous artifacts, activities or ideas; and (2) do not understand a mechanistic process of transformation (e.g., superimposing, deconstructing, and reconstructing). In other words, they are blind to this projectionism problem that many Indigenous students intuitively experience daily, in concrete ways and often with negative effects.

As Einstein pointed out (section 6.3), projectionism happens in diverse contexts; many with positive consequences. Positive projections include making inferences in an attempt to understand something in greater depth, exemplified by scientific investigations and by inferences made by mathematics teachers and researchers. But some projections have negative consequences, revealed here by critically analyzing R&D projects to gain a deeper understanding of how subliminal the negative consequences can be.

In the following quote, Neel and Fettes (2010) infer that EAM Platonist content can implicitly exist in a Haida artist's mind without the artist having learned that content in the first place.

> Somehow the artist has a sense as to what shape will be most appealing to the eye of an observer. The art may *integrate mathematical concepts* such as symmetry, congruency, and transformations without the artist implicitly or intentionally knowing that they are doing it. Thus a significant pedagogical challenge is to connect such *implicit* mathematical understanding with the *explicit* ways in which mathematics is presented in classrooms. (p. 46, emphases added)

Clearly, the authors are projecting EAM Platonist content onto the artist's Indigenous mathematizing and calling it "implicit mathematical understanding." Whereas the artist's intuitive reasoning is Haida mathematizing that needs to be understood from a Haida point of view in the Haida language or a back-translation thereof. The effect of this projecting is to express disrespect for Haida mathematizing by replacing it with EAM Platonist content. Whether intended or not, the detrimental effect on many Indigenous students is the same (Doolittle, 2006; Meaney, 2002). Neel and Fettes' pedagogical challenge is first to learn what the Haida artist understands, rather than projecting their personal understanding onto the artist. Only then can authors transform Haida mathematizing into a mathematics classroom activity to teach EAM without appropriation and marginalization occurring (sections 4.4 & 6.2).

In documents composed by Alaska's MCC researchers, a key technical expression, "embedded mathematics," was never overtly defined in the many articles I examined. In Lipka and colleagues (2013), one reads:

- "…the embedded mathematics in [an Elder's] everyday activity…" (p. 130)
- "…to become familiar with mathematically embedded processes of body proportional measuring…" (p. 139)

In Figure 1 of Lipka and colleagues' article (p. 132), a Venn diagram shows three overlapping circles: "Culture & Context," "Math Content Knowledge," and "Pedagogy." The expression "embedded mathematics" is located in the Venn diagram's "Culture & Context" circle. It is excluded from the "Math Content Knowledge" circle. Thus, embedded mathematics and mathematics content knowledge are logically different but somehow related concepts: "The mathematics embedded in everyday activities *relate directly to* ratios and proportional thinking" (p. 132, emphasis added). What is the language-laden cognition that both separates them and relates them? What is the relationship between the two? An answer should be found in a definition of "embedded mathematics." If defined, it would have clarified a type of superimposing-deconstructing-reconstructing process carried out in MCC. However, projectionism seemed to exclude the thought of defining "embedded mathematics."

Yet, two answers were indirectly suggested in Lipka and Andrew-Irhke (2009):

- "…we are increasingly able to understand the mathematical threads *woven into* authentic cultural knowledge and practices" (p. 8, emphasis added)
- "It's in the construction of the border patterns on these parkas that the math *is revealed*." (p. 9, emphasis added)

These two metaphors ("woven into" and "is revealed") suggest a relationship. Accordingly, the mathematics educator/researcher's role seems to be: (1) observe Platonist mathematical threads, copy them into a MCC module, and then teach them to students; or (2) wait until the Platonist mathematics are revealed, and then copy them into a MCC module. In other words, the process that the mathematics educator/researcher goes through is a passive one, not an active one.

When the process is passive, the educator or researcher has no responsibility in the process. It is not them, it is what is observed or revealed.

But when the process is an active one, which it surely is (Skovsmose, 2016), the educator or researcher bears responsibility for their actions. These actions can be interrogated with critical questions. Unconsciously or not, any move *away from* responsible transparency in order to avoid such responsibility exemplifies masking projectionist power with opaque responsibility. "Mathematics-based actions easily appear to be conducted in an ethical vacuum" (Skovsmose, 2016, p. 15).

In the context of Māori mathematics education, Meaney (2002, p. 170, spelling corrected) warned, "[C]hoosing activities from the experiences of [I]ndigenous students can result in the original purpose of the activity becoming lost or denigrated through the concentration on the Western mathematical idea 'seen' to be embedded in it."

A cross-cultural solution for MCC and other mathematics educators is to be equitable in a pluralist way by recognizing Euro-American mathematics as a cultural-laden human construction just as Yupʼik mathematizing is. From this equitable perspective, we can more clearly answer the question: How are the two related? – Through the transformation process that strips peripheral concepts belonging to a local Indigenous culture and then adds a different set of peripheral concepts belonging to a Euro-American mathematics culture (sections 4.4 & 6). Mathematics teachers or researchers actively transform an Indigenous artifact, process or idea into a Euro-American Platonist concept or process, for the intended wellbeing of Yupʼik students. These are students who want to achieve in mathematics and walk in both worlds – the Indigenous world embracing values prominent in a "subsistence economy" based on survival, and the dominant Euro-American globalized world embracing values of progress defined by a "capitalistic/market economies" (Lipka et al., 2013, p. 136).

In summary, the concept of embedded mathematics is not transparent. It is purely a cultural projection. According to anthropologist Hall (1976, p. 164): "Cultural projection always has been a stumbling block on the path to better understanding. Yet progress in getting rid of cultural projection has been slow." The concept of embedded mathematics also exemplifies a "conceptual pitfall" that Garroutte (1999, p. 107) of Cherokee ancestry would call "proto-mathematics," which she pointed out privileges school mathematics at the expense of Indigenous mathematizing. Similarly, the conundrum mentioned in section 9.2 can be seen as acting pluralistically with Yupʼik people's mathematizing but not with one's own understanding of EAM Platonist content.

An explicit transformation process (e.g., superimposing, deconstructing, and reconstructing) will confirm for Indigenous students that: (1) their culture's artifacts, activities, or ideas remain unchanged in spite of any superimposition process; (2) those artifacts, activities, or ideas continue to possess their cultural integrity and power in an Indigenous context, even after the deconstruction process by researchers or teachers (Garroutte, 1999); and (3) Indigenous students will *add* another culture's perspective (a Euro-American perspective) to the students' intellectual world. Transparency prevents the neo-colonial claim that the student's Indigenous culture already contained those "foreign" Platonist mental forms in the first place. Garroutte forcefully advocated such transparency in order to achieve authoritative equality between the two ways of mathematizing; thus meeting MCC's affirmed political-social aim: "bringing local knowledge into

American Indian/Alaska Native education requires reversing historic power relations that continue to separate school knowledge from community knowledge" (Lipka et al., 2005, p. 1).

As noted above, these fundamental problems do not cause a project to fail, but they keep the project from reaching its full potential for benefiting more Indigenous students.

These problems naturally infiltrate modules and lesson plans in teaching materials, because their developers have been taught: (1) the dogma and myths of Platonism that facilitate projectionism, and (2) the historic, self-serving, subjective bifurcation of Euro-American mathematizing into formal versus informal discourse (section 4.2.1). These problems became unquestioned epistemic presuppositions. One of many examples will illustrate the problems concretely.

As described in section 8.4, University of Hawai′i undergraduate students were given an optional course assignment: To develop a short mathematics unit for others to teach. In an interesting and engaging teaching unit for Grades 9-12, Fractal Landscapes (ESTEMI, 2016), teachers are informed that "Fractals are patterns found in nature and [are] part of everyday life" (p. 2), a statement with which Einstein (1930) would disagree (section 3.3): "It seems that the human mind has first to construct forms independently, before we can find them in things" (quoted in Director, 2006, p. 116). As well, the module's statement seems to profess the Platonist ontological presupposition that mathematics concepts, such as fractals, are *discovered in nature* (section 4.1.1).

Abbott (2013) provided evidence that fractals were invented by humans; by Benoît Mandelbrot in 1975, to be specific. Mandelbrot's computer-assisted mathematics research stood on the shoulders of other mathematicians' work. In the process of generating a method to accurately estimate the length of the coastline of Britain, he generated a new class of mathematically derived shapes that could *mimic* irregularities found in nature, for example, the shape of a cloud. He gave the name "fractal" to a particular set of solutions to some of his equations.[xiv] This set mimics a variety of patterns observed in nature (i.e., projected *onto* nature). Therefore, the claim (above) "That fractals are patterns found in nature" is in error by suggesting they were discovered by mathematicians[xv]. Instead, *fractals are human-generated patterns that approximate shapes perceived in nature*. They were first invented by mathematicians and can now be projected onto naturally occurring patterns *in a best-fit way*, just as students do when studying Fractal Landscapes.

Students spend about one or two weeks constructing mathematical fractals in different interesting ways. Simultaneously, of course, students are constructing *mental* forms or images of fractals. A cultural extension to the module is suggested for students: "Assess fractals in Hawaii using native or endemic plants on land or in the ocean through observation and comparison to fractal examples" (p. 2), a statement possibly congruent with Einstein's idea, depending on a student's interpretation of "assess."

On the last page of the module, the developer poses a rhetorical question: "*Where do we find fractals?*" (p. 10, emphasis added). Directly below the question are two distinct answers, side by side: "In Nature" and "In Math." Below the phrase "In Nature" are four excellent photos of a leafless tree, river patterns taken from space, a hurricane cloud taken from space, and a spiral galaxy. And below these photos, we read, "*Natural* fractals include branching…" (p. 10, emphasis added), but fractals are *not* natural. Below the phrase "In Math" are three computer-generated fractal patterns, plus a mathematical equation that generates fractals. Those are purely mathematical fractals.

On the one hand, this page seems to invite students, in effect, *to project their mental images* of fractals onto photographs of nature, in order to "see" fractals in those photos (i.e., to recognize fractal-like shapes in a best-fit process). This is what the cultural extension activity required students to do ("Access fractals in Hawai′i by…"). But transparency (i.e., explicit wording) is missing. Students need to be told outright, "Superimpose (or project) your mental images of fractals onto photographs of nature." Accordingly, what would students probably learn? – That's what mathematicians and scientists do; and so should I. Einstein would be pleased. For Indigenous students, *cultural transparency of the colonizer's culture is the goal*.

On the other hand, the module claims that some fractals are in nature (embedded in nature?). Einstein would not be pleased. Overall, the module Fractal Landscapes is very unclear on this point. As mentioned before, this is not a matter of semantics; it concerns articulating accurate and transparent messages to Indigenous students in a way that promotes their comfort level with the teaching materials, which for most students leads to higher achievement in understanding Platonist content. Thus, the module is missing a crucial point: lessons should include EAM cultural content in an age-appropriate way in order to be more transparent to Indigenous students. A teacher possessing a cultural belief about school mathematics could easily modify the module so it reflected a *cultural* application of mathematics (section 4.1.1); that is, culture-based school mathematics.

As noted in section 4.1.2, students hear a common rationale for why they should learn mathematics: because “mathematics is all around you.” We are now in a position to take our analysis of the expression to a deeper level, in order to explore its projectionism to discover the expression is only a half-truth.

The rationale obviously applies to aspects of EAM such as arithmetic and the ideology of quantification; both of which are recognized by a *cultural belief* about mathematics, but only arithmetic by a *Platonist belief* (Ernest, 1988).

However, if we consider the mathematics curriculum content in its abstract formulation, the statement only makes sense to a minority of high school students, about “24 percent” of non-Indigenous students (OECD, 2016, p. 362). These are students whose worldviews and self-identities resonate with their mathematics teacher’s (Cobern, 2000).

Full disclosure: I belonged to this group. We easily made a connection between school mathematics (i.e., “the study of number, quantity, shape, and space and their interrelationships by using a specialized notation;” a denotative definition by Collins English Dictionary) and our world around us if required to, because we knew the mathematical patterns to superimpose on an artifact, process, or idea in our everyday surroundings. Most of us would agree with such Platonist sentiments as “Mathematics is the music of reason” (James Joseph Sylvester [1814-1897] English mathematician) or “Mathematics is the language of the universe” (Galileo). Mathematics *will be* all around us, the minority who have learned mathematical concepts and images meaningfully enough to project them onto our surroundings and see a good fit between an artifact or event and our deep understanding of mathematics.

A few other smart students will acquire as much EAM Platonist content as possible, motivated by an external need, such as wanting to be a good student. They are challenged by mathematics’ ideologies and decontextualized abstractions. However, a large majority of students, will most likely play Fatima’s rules (Aikenhead, 2006, p. 28) by going through the motions of algorithmic memorization and hoping that is enough for them to pass the course and receive a credential. For this large majority of students, mathematics is *not*, and will never be, “all around them.” How discouraged they must feel by being told that the content they try to learn in mathematics classes is all around them in the everyday world, but they do not see it.

A much different context for projectionism, researchers and teachers making inferences about students, is found in mathematics classrooms. For example, in their Algonquins of Pikwakanagan project (section 8.5), Beatty and Blair (2015) watched and listened to Grade 2 students engaged in traditional looming while talking to each other very naturally. This is legitimate raw data from which to infer what the students were thinking. On a cautionary note, however, we need to be mindful of limitations when making inferences because the process involves projecting concepts or images, according to Einstein. Because Beatty and Blair did offer very credible inferences, their project is a productive one to critically analyze.

Is the curriculum's abstract Platonist content the appropriate image for superimposing on the actions and words of Grade 2 students? Is the student's thinking that abstract? Or do their actions and words serve as a concrete precursor to constructing or co-constructing an abstract concept in a higher grade? An answer is found in Beatty and Blair's (2015) theoretical framework:

> [I]ncorporating Indigenous epistemologies and perspectives aligns with other researchers…who advocate placing less emphasis on numbers and algorithms, and more emphasis on spatial reasoning, geometry and measurement, which are more relevant within Indigenous cultures than number or arithmetic. This shift in emphasis is mirrored in current mathematics education foci, which have shifted to the importance of spatial reasoning…and algebraic reasoning through patterns…as foundational for the development of mathematical thinking. (p. 5)

Patterns lend themselves to more accurate projection by researchers and teachers onto students' actions and words. This age group is notably different, and therefore projectionist thinking must be adapted to a primary grade context as Beatty and Blair did.

Trustworthy projectionist reasoning was also demonstrated by Warren and Miller (2013) in research with primary Australian Indigenous students in 15 schools. The researchers developed visual patterns to assess student progress in learning mathematics and learning English in an innovative program "representations, oral language and engagements in mathematics" (p. 151). Their conclusions point out:

> the importance of an oral language approach to teaching [the innovative program], and in particular ensuring students have a range of experiences with the language of mathematics, and the mapping of this language onto [students' home language] within contexts that are meaningful for these students. In addition, the use of a range of different representations during this process is important. The results also show that a pattern and structure approach to mathematical learning does assist these students to understand a range of mathematical concepts. (p. 168)

In higher grades, the transition to spatial and algebraic reasoning has promise, but with the objective of diminishing culture clashes still needs to be part of that instruction. A critical question remains: Are Indigenous students being assimilated into a Euro-American epistemology (cognitive imperialism) by aiming for "development of mathematics thinking" for all students when we know that a sizable majority's worldview and cultural self-identity do not harmonize at all with a mathematical worldview. Or will the aim be two-eyed seeing (Hatcher et al., 2009) (section 8.2)?

When matching the degree of convergence between the curriculum content and a student's actions and words, how valid is the process when an observer who thinks in noun-based ways infers students' verb-based thinking? This was an issue investigated by Donald and colleagues (2011) (section 8.8). Notably, Beatty and Blair's (2015) work is trustworthy due to their collaboration with Indigenous teachers on their core research team.

Rather than frame our projections on students' thinking in terms of a binary: "yes their thinking matches the curriculum content, or no it does not;" we can describe students' mathematical thinking in terms of degrees of "*working towards* an abstract concept or process." Students' individualities challenges a standard one-size-fits-all mapping of student progress, however.

Donald and colleagues (2011) reduced projectionism in their inferencing. They took pains to listen intently to students and discern the *students' ways of formulating* a solution to an activity or problem, in a process that made students feel they are related to that solution. In short, students drew upon their Indigenous culture to mathematize their way to a useful solution to an activity or problem recognized as important to the culture of Euro-American mathematics (EAM). Donald and colleagues trained teachers to conduct spontaneous interviews with students, and to develop deep active-listening skills that placed teachers in the position of learning how students mathematize in the context of one particular problem or activity.

I am intrigued with the confluence of this situation and the Māori word "tuakana," a teacher who learns from students (section 6.4). The word pairs with "teina," a student who teaches their teacher. This appears to be happening during the student interviews conducted first by Donald and colleagues, and later by teachers mentored to conduct such interviewing. Although time consuming (Alrø & Johnsen-Høines, 2016), this approach to teaching/learning seems most suited to bridging across two frameworks: the curriculum's *Euro-American cultural framework* and the students' *Indigenous cultural framework.*

9.4. Misinterpretations of Quantified Assessment

This section deals with three separate issues about misinterpreting standardized test results, especially when they deal with national, provincial or state standards; or with international tests that compare countries' mathematics education systems. In general, the ideology of quantification is explored here by critically analyzing: (1) the recent Programme for International Student Assessment (PISA) results produced by the Organisation for Economic Co-operation and Development (OECD), (2) an earlier *Scientific American* quantitative and qualitative evaluation study based on the Third International Mathematics and Science Study (TIMSS), and (3) the validity of one of three PISA sections acclaimed for its everyday relevance to students. To make the discussion concrete, the University of Hawai'i's project is chosen as a context (Furuto, 2013b, 2014), but the implications apply to all educators, administrators, and politicians.

In section 3.3, a distinction was made between *intellectually* knowing how to calculate an average, and having the *wisdom* of knowing when it is appropriate or inappropriate to do so. An equivalent type of discussion occurs here with real life examples.

9.4.1. PISA Results

ESTEMI (section 8.4) reacted to the international assessment results from PISA by drawing attention to the U.S. average score being below the collective average of all participating countries. However, nothing was mentioned about the plethora of evidence that PISA is essentially a political project masquerading as an educational tool (Sjøberg, 2015). Is this masking power with naivety? A short critical analysis of the PISA instrument and its data reveals several illusions, all of specific interest to EAM educators who assess or defend projects that include Indigenous perspectives. (For an in-depth critique of PISA, see Sjøberg, 2016.)

Furuto (2014) insightfully highlighted a key finding in the PISA 2012 report on the 2009 PISA results: "[A]mong data on students, community, and institutional factors that could help explain differences in mathematics performance…access to *equitable and quality mathematics resources* is key to attaining academic success" (p. 111, emphasis added). In other words, the students' average score is only one factor among several that could be used to assess the quality of a mathematics education system.

Therefore, we should ask: Can a one-dimensional evaluation (students' average scores) be trusted to judge the quality of a country's or a state's complex mathematics education system? Furuto's quote above suggests not; and a resounding "no" comes from a number of other researchers (e.g., CIEB, 2015; Gibbs & Fox, 1999; Parkin, 2015; Serder & Jakobsson, 2015; Sjøberg, 2015; Sriraman, 2016). For instance, the ranking for the U.S. in the PISA 2012 report on the 2009 test would have changed dramatically if two additional major factors – systemic equity and efficiency – were taken into consideration, according to the Centre on International Education Benchmarking (CIEB, 2015). Social equity in an educational jurisdiction is, in part, "measured by the gap between immigrants and non-immigrants…between rich and poor children" (Parkin, 2015, p. 1). Equity also encompasses "the distribution of resources within the education system" (p. 1), such as the *quality of mathematics resources* (e.g., culture-based school mathematics projects); and the support offered to teachers, including appropriate professional development programs. The second factor, efficiency, arises from the fact that financial support for education competes with many other social needs. A criterion for a better quality education is "spending that is efficient as opposed to high" (Parkin, 2015, p. 2); in other words, what outcomes accrue per unit of financial expenditure.

CIEB (2015, website quotation) ranked countries based on three factors: (1) mathematics *performance*; "average rank on all three sections of PISA;" (2) *equity*; "percent of variation in mathematics performance explained by socio-economic status;" and (3) *efficiency*; "spending per secondary student, U.S. dollars 2011." CIEB integrated these three factors. One result was the substantial increase in the trustworthiness of the country's rankings. (In the language of statistics, by analogy, an analysis of covariance is superior to an analysis of variance, all other factors being equivalent.)

A second result of CIEB's analysis was the completely different rankings for each country. For example, the PISA 2012 test report (OECD, 2013) ranked Finland, Estonia, and Canada at 11, 12, and 13 (respectively) on the basis of student performance alone. These rankings were behind many East Asian countries. The much more sophisticated three-factor analysis resulted in Finland, Estonia, and Canada being tied for the *top* PISA rantings overall. Which analysis should be used, PISA's or CIEB's? At best, the decision would be a rational one; at worst, entirely political. One reason politicians and the general public would favour PISA's simpler analysis is the consequence of being able to blame mathematics teachers for low student performance, even though both cultural attitudes towards equity and government fiscal decisions influence greatly what teachers can accomplish.

Notably, however, when any measure of achievement focuses on Indigenous versus non-Indigenous students, the resulting consistent large gap is simply unjustifiable on ethical and economic grounds (Parkin, 2015).

Furuto (2014) was correct to question a one-dimensional measure of differences in countries' mathematics education systems. But she could have either simply ignored a political instrument's incursion into mathematics education, or publicly critiqued its incursion by demanding a more sophisticated and valid CIEB type of analysis.

9.4.2. Scientific American's Investigation into TIMSS

A second and related issue is the interpretation of TIMMS data. Even though the instrument had more validity problems than today's PISA, the misinterpretations exposed here are the same for both TIMMS and PISA.

Two science reporters from *Scientific American*, Gibbs and Fox (1999), investigated the "low ranking of American teenagers" (p. 87) on TIMMS. The low rankings had caused a national political crisis at the time, mostly at the expense of mathematics and science teachers. Gibbs and Fox interviewed a wide sample of educational experts and completed three case studies of typical high school mathematics and science teaching: a high school in Texas (representing a below-average TIMMS score), in Saskatchewan (above-average score), and in Sweden (the highest scoring country).

Among political leaders and the general public, Gibbs and Fox (1999) discovered a complete lack of understanding that the testing was actually an elaborate polling project with concomitant statistical confidence limits. Naturally, if countries with different scores have overlapping confidence limits, they were clustered together as being tied in their ranking. Political leaders and the general public were masking the power of Platonist content with ignorance.

I illustrate this by using the U.S. PISA 2012 data (OECD, 2013). The U.S. average score was 481 with a ranking of 29. Its score was a statistical tie with nine other countries whose results ranged from 489 (ranked 23) to 477 (ranked 32). In this context, therefore, the number 29 is no different than the numbers between and including 23 to 32[xvi]. Interpreting differences in scores is tricky, but there is more.

Although the U.S. average score (481) is *statistically significantly different* from the OECD average score (494) as Furuto (2014) correctly pointed out, there is another critical question to pose: Is the difference between 481 and 494 *economically and educationally* *significant*? How much would it actually cost to bring the U.S. average score up to 494? In what ways and to what extent must the culture of U.S. education change in doing so? Is it worth it for a measly 13 points (on a scale over 600 points) on a test of questionable trustworthiness? A singularly narrow focus on quantitative data's statistical significance masks power with statistical calculations. Are mathematics educators masking power with naivety? Have they been coerced at the altar of Platonist content? This is only one of many ways that the ideology of quantification asserts itself in mathematics-in-action (section 4.5 & 7.1). This issue is an example of EAM cultural content.

Gibbs and Fox (1999) concluded that differences in scores are not really large enough to be concerned about it. In their article, "The False Crisis in Science Education," the smoke-and-mirrors of politically embellished score differences between the average for all countries and the U.S. average created a false crisis. Their article's title could easily have read "The False Crisis in Mathematics Education" if it were not for the publication's name.

Do these quantitative data reflect Gibbs and Fox's (1999) three case studies, one in each of three countries? No at all. In fact the quantitative data hide a second false crisis from view. By using systematically acquired qualitative data, we can usually expose what gets lost when we rely on quantitative data alone. In their Texas case study, Gibbs and Fox characterized school mathematics and science as memorizing definitions or formulas, copying notes from teachers, and following procedures hoping to get the right answer. Innovative courses introduced by teachers were popular with students, but the school administration curtailed them in order to cater to "state-required courses aimed at college-bound kids" (p. 90) – the STEM pipeline – to maintain standards similar to the ESTMEI project's EAM Platonist content standards. The Texas school acted as a screening device for university STEM interests and educational "standards" that conventionally marginalize most Indigenous students (section 3.2).

In the top-scoring, Swedish, typical high school, Gibbs and Fox (1999) discovered students "quieter and more attentive than their Texas peers. However, like the Texas school, science classes were a paragon of traditional instruction" (p. 90). Although the content and labs appeared sophisticated in their technology and topics, in the labs students only mimicked the teacher's demonstrations; followed by taking notes. During the teacher's class presentations, a catechism of "'Any questions?' Silence. 'Okay on to…'" (p. 90) was observed. This apparently was somniferous, judged by the observation of yawning students. The Swedish school's pedagogy was not all that different from the Texas school's pedagogy.

In Saskatchewan, Gibb and Fox (1999) found a different ethos pertinent to the mathematics classes. Teachers tended to rely more on open-ended assignments and to talk about the social issues raised by science, technology, and the associated mathematics. Students were observed learning from the curriculum on a need-to-know basis. Students asked teachers such questions as, “How do you know that?” (p. 91). The following exchange was observed:

> “What’s this useful for?” one fellow asks.
>
> “I’ll give you some electrical engineering problems that use rational functions,” the teacher promises. “How’s that?”
>
> “Cool,” the boy says. (p. 91).

What second false crisis in U.S. mathematics and science education arose from these qualitative data?

> The false crisis [manufactured by embellishing the significance of small differences between student averages from different countries] masks the sad truth that the vast majority of students are taught [content] that is utterly irrelevant to their lives – and that scientists [mathematicians] are a major part of the problem” (Gibb & Fox, 1999, p. 92).

In other words, *quantitative* representations of a country’s EAM Platonist education system (i.e., in the form of international test scores) tend to be far removed from what actually transpires in that country’s classrooms, as indicated in systematic *qualitative* case studies. These types of standardized test scores suffer a serious validity problem, no matter how statistically reliable[xvii] they are. When looking at such test results, therefore, our critical vigilance needs to be asserted in order to avoid masking power with systemic naivety.

Organizations that conduct large-scale testing *intellectually* know how to calculate averages and ranks. Mathematics educators, school administrators, and politicians must have the *wisdom* of knowing how appropriately valid it is to compare those averages; and if so, to proceed with critical vigilance; and if not, educate colleagues about their masking power with naivety. Are mathematics teachers being sacrificed on the alar of Platonist content?

9.4.3. Validity of One PISA Section

This validity issue relates directly to writing cross-cultural teaching materials or lesson plans relevant to students. In one of the three sections of the PISA test, each question begins with a short passage called a “backstory,” which is like a newspaper item relevant to a 15-year-old. Students answer questions based on the backstory.

Serder and Jakobsson's (2015) research asked: How relevant are those backstories to the 15-year-olds who take the test? Although their research was on the PISA science test, the issue is equivalent for the PISA mathematics test. Serder and Jakobsson's results added to the evidence undermining the trustworthiness and cultural validity of PISA. Many students in the study positioned "themselves as being different from and opposed to the fictional pictured students who appear in the backstories of the test" (p. 833). This happened because of: the academic language uttered by the fictional students; the elite "little scientist" images conveyed by them; and their inclination to conduct experiments at home, to which some students (out of a sample of 71) reacted, "Why bother so incredibly much?" (p. 848). In other words, many students could not see themselves reflected in the backstories, and their engagement suffered accordingly.

A general feature of students' notion of relevance seems to be the degree to which they see themselves in a real or hypothetical situation (Jannok Nutti, 2013; Lunney Borden, 2013; Neel, 2008; Sterenberg, 2013a). Writers of cross-cultural modules for students need to conduct a preliminary study to ensure that their students see themselves reflected in the writer's materials.

PISA claimed it monitored students' attainment of scientific knowledge. But for its section with backstories, PISA was closer to monitoring students' scientific self-identities. That was not what the instrument intended to measure, thus making the backstory section of PISA invalid. Serder and Jakobsson's conclusion sums up the problem: "This study adheres to research that advises caution in not over-interpreting the PISA results and stresses that understanding students' 'knowledge' about science [or mathematics] is much more complex than what is communicated by the international assessment organizations" (p. 834).

9.4.4. Closing Notes

When discussing standardized test results and describing their limitations, Larson (2017, website quotation), President of the National Council of Teachers of Mathematics (NCTM), wrote, "Despite these limitations, there is still much we can learn. …At the elementary and middle levels, the evidence suggests the U.S. is making progress in mathematics teaching and learning." Success was attributed to the articulation of various national and international organizations' standards that drive local mathematics curriculum and instruction; an attribution solely based on test data that have limitations. Larson, (2017) decried the high school test results without using the word "crisis." However, he rationalized the poor results on: (1) a lack of "rigorous, and focused standards across all classrooms" common to "high-performing countries;" and (2) "structural obstacles, such as a narrow curricular emphasis on procedures that lack relevance and meaning to students, lack of student engagement… .” The first point highlights a one-size-fits-all homogeneity. The second point mentions student relevance and meaning making, but is silent on students seeing themselves in a mathematics-related situation.

Larson (2017) concluded, “Clear themes can be found in the data…eliminate structural obstacles that stand in the way of the learning of each and every student; and substantially reform high school mathematics curriculum and instruction.” My closing note is that the high sounding rhetoric of the status quo inhibits school mathematics’ move towards a 21st century reconciliation.

9.5. Conclusion

I encourage researchers and teachers to consider their next educationally significant step by doing whatever is possible to correct lingering impediments to Indigenous students’ success. Those successes tend to encourage non-Indigenous students’ engagement and successes.

The Platonist ideology of quantification demands that the outcomes of schooling be commodified so that achievement can be measured numerically (Ernest, 2016a; Fasheh, 2012). This quantified worth of students, teachers, programs, and educational jurisdictions is so simplistic it immeasurably distorts reality (section 9.4), in spite of the quantification’s false aura of objectivity (Aikenhead, 2008). Simply put, political expediency trumps quality education – defined as “the human dimensions of knowing” (Ernest, 2016a, p. 53). Even worse, the allocation of a government’s “resources for testing is the main argument to justify math contents” in curricula (D’Ambrosio, 2016, p. 33). What is easy to test may not be ethically defensible.

How will Indigenous students benefit? A pluralist cross-cultural understanding of Euro-American school mathematics lessens their reluctance or resistance to understand relevant EAM Platonist content, for three reasons: (1) because that relevant content is contextualized in a Euro-American culture, and consequently their participation is with a familiar culture-to-culture transition (rather than from a cultural context to an *acultural* mathematics context); (2) because some of that content has been associated respectfully with their own culture’s integrity; and (3) because students are encouraged to continue their appreciation of their cultural artifacts, activities, ideas, and language; all of which helps strengthen students’ cultural self-identities, which in turn augments their achievement.

10. Mathematics Curricula Revisited

Reconciliation is not an event. It's something that needs to enter into the way we do things.

John Ralston Saul (2014) *The Comeback* (p. 260)

In conventional mathematics classrooms, most teachers conscientiously try to cover the legally mandated, highly abstract, and heavily overcrowded curriculum (section 2.4). The curriculum's apologists expect students will later apply the curriculum's content in being savvy citizens and competent employees. For the most part, these curriculum advocates are irresponsibly naïve.

> There is no reason to expect students who have learned isolated strategies and atomised content throughout school to be able to combine, separate, or integrate what they have learned in "real" and demanding situations. Mathematical activities need to be open enough, not only for students to formulate strategies but to formulate their own meaning. (Boaler, 1993, p. 16)

Moreover, those activities must be sufficiently realistic and relevant that students can see themselves involved in them, even vicariously (Jannok Nutti, 2013; Lunney Borden, 2013; Neel, 2008; Sterenberg, 2013a) (section 9.4.3).

Personal meaning making, "coming to know" (Cajete, 2000, p. 110), occurs when students connect with the *cultural aspects* of Platonist mathematics and its many roles in society – mathematics-in-action (sections 4.5 & 7.1). This pathway to learning reduces culture clashes for most students, but especially for Indigenous students (sections 3.3 & 6.4). They respond very successfully to cross-cultural Euro-American mathematics (EAM) in which their culture's mathematizing is respectfully included as an experiential connection to another way of mathematizing. Their non-Indigenous classmates also tend to benefit. Reconciliation enters into this way of doing things in mathematics classrooms (Richards et al., 2008; Rickard, 2005).

In a research study concerning what Indigenous parents, teachers and students say about improving student learning outcomes, a mathematics education graduate student, Kukahiko, stated, "I think that part of my research challenges the colonial, Eurocentric mess of mathematics. I think people need to see that. I think *the people making those curricula decisions* need to see that" (SIDRU, 2014, p. 62, emphasis added). When speaking about a group of creative, intelligent, mathematics teachers, Kukahiko lamented, "[T]hey also trained in mathematics in a Western way, so they are looking at the universe through a Western mathematics lens" (p. 62); that is, a Platonist lens.

She mentioned an example of traditional Hawaiian mathematics that could easily be included in Hawai′i's school mathematics curriculum. When people sail a double-hulled Hōkūle′a-like canoe (section 8.4), their counting system is founded on base-4, rather than EAM's base-10. Hawaiian students, in the context of culturally revered double-hulled canoes, could easily be engaged with mathematics' number bases at a younger age than currently introduced in mainstream education. As illustrated earlier, graph quadrants can also engage young Indigenous students whose cultures affirm Turtle Island medicine wheels (section 8 introduction). Numerous other examples emerged from the R&D projects and research discussed in this monograph. Curriculum committees have been challenged by such feedback over the years, but to little or no avail.

Some researchers expressed the goal of influencing the established curriculum through a natural groundswell of success (Beatty & Blair, 2015; Lipka, 1994; Lipka et al., 2005; Lunney Borden, 2013; Jannok Nutti, 2013). In the end, however, even their documented successes have proven impotent in the face of a Platonist curriculum entrenchment, except for encouraging token changes (section 2.4).

In section 10, major conclusions about mathematics curricula are reached, pertaining to: (1) the National Council for Teachers of Mathematics (NCTM) as representing similar professional institutions that have brought us to our present 19th century-like situation, (2) school mathematics educators and researchers, and (3) all stakeholders in provincial, state, national, and territorial mathematics curricula.

10.1. NCTM

Located in the U.S., the NCTM is a particularly authoritative source of standards for school mathematics in both the U.S. and Canada. It has disseminated its influence on curriculum and pedagogy mainly through its publication *Principles and Standards for School Mathematics* (NCTM, 2000). Upon scrutiny, however, does this publication pass the trustworthiness test (section 9.4.1)?

Its Equity Principle section, referring to "poor or minority students" (p. 368), states: "All students should have a common foundation of challenging mathematics, whether those students will enter the workplace after high school or pursue further study in mathematics and science" (p.

368). Two points demand comment. First, by referring to “all students,” the NCTM appears to have a loose grasp on research concerning student diversity and the large proportion of students (76 percent; OECD, 2016, p. 362) whose worldviews and cultural self-identities are, to varying degrees, at odds with the worldview conveyed by Platonist school mathematics (sections 3.3 & 8.7). Secondly, as mentioned in section 9.4.4, the NCTM’s inflexible “common foundation of challenging mathematics” for all students to learn suggests a homogeneity perspective and leads to an unintended hidden agenda of exclusion (Pais, 2012; Skovsmose, 2012) (sections 3.2, 4.1.1, & 9.1).

The NCTM’s Learning Principle section concludes, in contradiction to most research cited in this monograph:

> Many teachers have found that if they teach mathematics in ways similar to those advanced in *Principles and Standards* – for example, by approaching traditional topics in ways that emphasize conceptual understanding and problem solving – many apparently uninterested students can become quite engaged. (p. 372)

The document seems to offer a tabula rasa, melting-pot model of student learning with no hint of including Indigenous mathematizing. And who is blamed if students do not respond favourably to the melting-pot model? – Students themselves:

> Too many students disengage from school mathematics, which creates a serious problem not only for their teachers but also for a society that increasingly depends on a quantitatively literate citizenry. Students may become uninvolved for various reasons. Many, for example, find it difficult to sustain the motivation and effort required to learn what can be a challenging school subject. They may find the subject as taught to be uninteresting and irrelevant. (p. 371)

If not the students, then blame “the attitudes and actions of adults who have influence with students” (p. 371). But never blame the value-vacant, culture-supressed, highly abstract, 19th century subject matter that comprises the majority of mathematics content championed by the NCTM, especially for grades 5-12. The astute reader will have noticed a political subtext in the above quotations from *Principles and Standards*: NCTM supports a competent citizenry screened by an elitist school subject (section 3.2), but student attitudes and bad parenting get in the way.

Recently, however, NCTM President Larson (2016a, website quotation) advised that the NCTM Board had reacted favourably to a position statement written jointly by the National Council of Supervisors of Mathematics together with an organization named TODOS:

Mathematics for ALL[xviii]. The joint position statement, "Mathematics Education through the Lens of Social Justice" (http://www.todos-math.org/socialjustice), argued for a curriculum renewal defined by a social justice stance: "fair and equitable teaching practices, high expectations for all students, access to rich, *rigorous*, and *relevant* mathematics, and strong family/community relationships" (Larson, 2016a, emphasis added). In the past, the term "rigorous" has been a code word for a wealth of Platonist abstractions displayed in an overcrowded curriculum. And does "relevant" mean superficially contextualized in the accoutrements of a neighbourhood culture, such as counting candies in a piñata or calculating probabilities of different types of candies? Throughout the eight-page position statement, the word "culture" only appears once, as an item in the list "race, class, culture, language, and gender." American Indians are ignored. NCTM's public interest in a lens of social justice is undeniable, but its authenticity leaves something to be desired.

Larson (2016a, website) admitted: "We recognize that much of our work has focused on standards, curriculum, instructional practices, and assessment, and that we have too often addressed these issues in decontextualized ways that have frequently ignored the experiences and realities of children's lives." And "High school mathematics has not changed substantially in my lifetime" (Larson, 2016b, website). In a different unit of measurement – generations – high school mathematics has not changed substantially for six or seven generations, for reasons discussed in sections 3.2, 4.1, and 4.2.

There is now an intention to reframe the NCTM Equity Principle (quoted just above) to one of "Access, Equity *and Empowerment*" (Larson, 2016a, website, original emphasis). A 2018 publication is promised, which has a tentative title "*Rehumanizing Mathematics Teaching and Learning for Students Who Are Latina and Black*."[xix] In a later message, Larson (2016b, website, emphasis added) stated: "This new publication will…include guiding principles such as access, equity, and *empowerment*; and [will] define math *curricular pathways* leading to college pathways and career readiness, as well as active participation in our democratic society."

Will "empowerment" mean a more equitable political-social power balance between the mathematics curriculum and minority cultures? Or will it mean gaining high school credentials for passing Platonist mathematics courses made attractive by a token of contextualizing Platonist mathematics content for a minority group? Will "curricular pathways" mean a separate curriculum for minority students, an idea rejected in Larson's (2017, website quotation) message, "These structural obstacles, such as…tracking of students into low-level mathematics courses, must be discarded”? Or will it mean a 21st century school mathematics curriculum developed with a social justice theme similar to reconciliation?

On this issue President Larson’s (2016b) choice of vocabulary delivered an optimistic outlook, but at the same time, a hint that not much will change fundamentally:

> Today, it seems as if nearly everyone agrees that high school mathematics needs to change. For far too long *high school mathematics has not worked for far too many students*: too many students leave high school unprepared for college or a career, particularly a STEM career; too many students do not see how math is useful in their lives; too many students leave high school without an affinity for doing math. (website quotation, emphasis added)

Interestingly, Larson’s observation that “high school mathematics has not worked for far too many students” is shared by Skovsmose and Greer (2012). But while the NCTM (2000) generally blamed students, teachers, and parents, Skovsmose and Greer (2012, p. 383) concluded, “For too many people, their experience of school mathematics is personally, emotionally, and intellectually dehumanizing. It does not have to be like that.” I wonder if the anticipated innovation “re-humanizing mathematics teaching and learning for students who are Latino/a and Black” will expand to all students. If not, Skovsmose and Greer’s description of school mathematics as “dehumanizing” will continue to have validity.

I expect an attentive reader would understand Larson’s (2016b) passage quoted just above as an admission of guilt that the NCTM’s version of school mathematics has failed at its mission: ample rigorous mathematics for all. Greer and Mukhopadhyay (2012) would certainly take exception to NCTM’s mission.

> “All students must have a solid grounding in mathematics to function effectively in today’s world.” Really? Think about people you know. Aren’t there many who do not have a solid grounding (assuming that means roughly what we take it to mean) in mathematics that are living full and productive lives? Isn’t it offensive to tell such people that they are dysfunctional? “Mathematics for all” has a fine equitable sound to it, but neither theoretically, nor in terms of the actual situation for students, can it bear scrutiny. (p. 239-240)

The NCTM’s mission was also challenged by Skovsmose and Greer’s (2012, p. 379) simile: “[A]ll children should learn considerable formal mathematics (up to symbolic algebra, for example), which is rather like having every child in a country taught to play a sport with the aim of producing a top-class national team.”

A different challenge to NCTM's mission showed how its pronouncement that mathematics competence leads to being a "full citizen" (Pais, 2012, p. 51), is both fraudulent and causes school mathematics to act in an exclusionary way (see section 3.2 for Pais' critical analysis).

Other academic researchers, however, seem to have been swayed by the NCTM's "rhetoric and slogan systems, couched in the discourse of propaganda and advertising" (Skovsmose & Greer, 2012, p. 383), or perhaps these other researchers are unconsciously asserting NCTM'S Platonist stance. For instance, Neel and Fettes (2010, p. 49) wrote: "It [NCTM] also advocates the need to learn and teach mathematics as a part of cultural heritage and for life," which Neel and Fettes interpreted as calling for a paradigm shift in pedagogy. But here is the actual full quotation from the NCTM (2000): "*Mathematics as a part of cultural heritage*. Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement" (p. 4, original emphasis). NCTM is certainly anchored to a supremacist version of Platonist school mathematics (section 4.1.1). Indigenous mathematizing is completely ignored in NCTM's (2000) curriculum standards, and Indigenous students do not appear in NCTM's future with it innovation of access, equity, and empowerment.

An authoritarian ideology of NCTM, as well as federal, and state standards in the U.S., are daunting, to be sure. For instance, the University of Hawaiʹi renewed its undergraduate mathematics program so Polynesian students could see themselves in it (section 8.4). The response was dramatic (Furuto, 2014). But its undergraduate renewal did not filter down to influence Hawaiʹi's school mathematics curriculum explicitly, in spite of the plethora of mathematics units developed through the Ethnomathematics STEM Institute (ESTEMI). Similarly, the University of Alaska Fairbanks made inroads at the local level with Ciulistet and Math in a Cultural Context (MCC), but their successes seem more like manoeuvering around the state curriculum by partially ignoring it or "supplementing" it (Lipka et al., 2013) (section 8.3). No evidence was provided by either project to show a change in their State curriculum as a result of their extensive, high quality, R&D projects.

NCTM's notice of a rhetorical change in policy is very positive. However, significantly more acknowledgements and changes are needed to reach NCTM's goal of access, equity, and empowerment in response to the country's racism. The same goal (access, equity, and empowerment) is envisioned through a lens of reconciliation in this monograph in response to Canada's vicious colonization, residential schools, and lingering personal and systemic racism (sections 2.1-2.3). The word “racism” does not appear in any of Larson’s NCTM messages. NCTM’s credibility as a 21st century education leader suffers accordingly.

10.2. Educators and Researchers

Culture-based school mathematics for reconciliation that aims for high quality student learning of Platonist content *relevant to* most citizens takes more time for students to achieve than does vocabulary and “algorithmic” memorization (Beatty & Blair, 2015, p. 20). As mentioned in section 2.4, this classroom reality seems mostly ignored by mathematics curriculum developers who are often politically forced to cram so much extraneous EAM Platonist content into the curriculum. As a result, algorithmic memorization is forced on teachers’ pedagogy. For example, this famous quote among many from Lockhart’s (2009) *A Mathematician's Lament*, captures the problem precisely:

> No mathematician in the world would bother making these senseless distinctions: 2 1/2 is a "mixed number" while 5/2 is an "improper fraction." They're EQUAL for crying out loud. They are the exact same numbers and have the exact same properties. Who uses such words outside of fourth grade?

“How often is our answer to the student’s question of ‘Why do I have to know this?’ based upon the superior intellect that abounds from knowing more rather than the relevance to the individual student?” (Russell, 2010, p. 40). “When will I ever use this?...We need to reconnect students with mathematics by engaging students in the ‘why’.” (Matthews, 2015, website quotation). D’Ambrosio (2016, p. 33) described much of the mathematics curriculum as “uninteresting, obsolete, and useless.”

In his book *The Math Myth: And Other STEM Delusions*, political scientist Hacker (2016) argues that: (1) arithmetic (numeracy) is sufficient mathematical preparation for most people, (2) algebra in high school unnecessarily lowers the country’s high school dropout rates, and (3) more complex mathematics such as trigonometry is uncalled for. He advocates two curriculum pathways. First, a challenging and stimulating “citizen mathematics” for the large majority of mathematics students (i.e., “76 percent” in OECD countries; OECD, 2016, p. 362)[xx]. It would function as the main high school curriculum credential for graduation. Secondly, an optional, enriched, elitist pathway for the 24 percent, that is, those mathematics-oriented students wanting a series of courses that resemble the International Baccalaureate program, to prepare for STEM-related postsecondary programs and employment opportunities.

Two pathways for cross-cultural EAM school mathematics must be designed to maximize student mobility between the two, as students' needs dictate, in order to mitigate the negative consequences of streaming (Jorgensen, 2016). Thus, considerable flexibility must be built into the curriculum.

Commiserating with mathematics teachers, Hacker points out how the current curriculum frustrates their efforts to show students the utility, beauty, and fun of doing mathematics. But Hacker is not alone. Mathematician Newman (1956) wrote:

> The most painful thing about mathematics is how far away you are from being able to use it after you have learned it. … [I]n mathematics it is possible to acquire an impressive amount of information as to theorems and methods and yet be totally incapable of solving the simplest problem. (p. 1978)

"Not everyone needs to know about triangular numbers, no matter how *cool* they are" (Russell, 2010, p. 36, original emphasis).

Excessive vocabulary and algorithmic memorization serves as mythical armour for fighting against curriculum reform. A curriculum crowded with both essential and non-essential content *masks power with innocence* on a grand scale. One crucial task is to identify and purge non-essential content. Almost any adult with a science, engineering, or medicine background who has coached or home-schooled their child in EAM Platonist content would most likely make a worthwhile contribution to the task. Wolfram (2012), mathematician turned entrepreneur, emphatically supported this position:

> The users of mathematics are perhaps 95% not classified as mathematicians, they are scientists or engineers or artists or architects, many different areas. So they are not really mathematicians in their own right, and yet we have mathematicians setting most of the curricula. So they have a rather narrow view in many cases of what people actually need in this compulsory subject that we push everyone into doing. (p. 2)

For example, rather than memorize the algorithms for solving algebraic equations as a mathematician-defined curriculum invariably requires, students could learn how to think algebraically by building spreadsheets that have personal interest to them. They would learn even more if they taught what they had learned to classmates or presented it at a community mathematics festival (section 8.7). Let the students do the algebraic thinking and their computer the calculating. That is what happens in the real world of business and other types of work.

Russell (2016) presents a case study of a mathematics educator for whom school mathematics from elementary school through to university courses inculcated the belief that there was only one correct answer and one correct way to arrive at it, even though her personal experience contradicted this dogma. The case study revealed that even after being concerned with the achievement gap between Indigenous and non-Indigenous students, and even after interacting with Indigenous educators over issues in mathematics education, a deep understanding of pluralist mathematics continued to be foreign. An epiphany occurred when learning that a base-10 number system was not universally used by all cultures. If ever there was a reason not to entrust the selection of content of cross-cultural EAM entirely to mathematicians and mathematics educators, Russell’s case study removes all doubt. Collaboration, rather than consultation, tends to be a remedy.

Even more entrenched in Platonist mathematical dogma, however, are the absolutist mathematicians, educators, and general public. Ernest (1991) described them as discounting any importance to the achievement gap, as well as vehemently resisting any cultural dimension to the mathematics curriculum (section 10.3). Emboldened by their security of exercising political power, they dismiss contrary systematic evidence and rational arguments by the response, “Over my dead body” (Russell, 2016, p. 31).

As mentioned above, an absolutist’s crammed curriculum perpetuates the myth that the curriculum is rigorous, even though the concomitant algorithmic memorization is far from rigorous in terms of meaningful learning – the type of experiential learning expected by the Indigenous “coming to know” (Cajete, 2000, p.110). Even worse, an absolutist’s curriculum maintains the neo-colonial treatment of Indigenous communities. An unconscious, systemic race-based policy perhaps?

For example, systemic racism inherent in certain bureaucratic rules that govern how to write a mathematics curriculum can often dismiss reasonable consultative feedback from Indigenous educators. Saskatchewan’s current curriculum (section 2.4) is a case in point. Although one of its four general goals was altered to “mathematics as a human endeavour” as a result of consultations during its development, rules forbade other advice to be incorporated; for instance, the idea to include “stories” along with descriptions of Indigenous perspectives in specific curriculum outcomes and indicators (Russell, 2016, p. 47). The purpose of including stories was to emphasize

> the importance of valuing where knowledge is from, learning within the context of community, valuing the knowledge others bring to learning, valuing of alternative perspectives and approaches, and the notion that mathematics is developed to meet the needs of situations and determined by the time, place, and people involved. (p. 51)

This message about values was written into the preamble for the goal "mathematics as a human endeavour," but the message was forbidden in the outcomes and indicators – the part of the document most teachers regard to be *the* curriculum. As clearly indicated in section 8, Indigenous values belong to Indigenous mathematizing in cross-cultural EAM; just as Euro-American values belong to EAM cultural content, clearly indicated throughout section 4. Both must appear in a 21st century curriculum that resonates with reconciliation.

Curriculum and lesson-plan content has a strong ethical dimension to it (Boylan, 2016, p. 399): "[T]he choice in immediate and specific situations as to what curriculum content to include, or not to include, is an ethical choice not only for considerations of social justice but also because of how content may alienate or include learners."

The fear that culture-based school mathematics would reduce standards is contradicted by numerous research studies (Banks & Banks, 2012; Fowler, 2012; Lipka, et al., 2005; Keane, 2008; Meaney et al., 2012; Nichol & Robinson, 2000; Perso, 2012; Richards et al., 2008; Sakiestewa-Gilbert, 2011; U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education, 2008). Advocates for the Platonist status quo present no such systematically determined evidence to support their crowded curriculum (section 3.2). I join Andersson and Ravn (2012) when I ask: *Where are their data? Where is their rationale? How do they mask their power?*

A narrow focus on Platonist content achievement leads to: a deficit approach to school mathematics, a neo-colonial pedagogy, and a false assumption that learning mathematics is an acultural process (Bang & Medin, 2010, p. 1009; Russell, 2010). On the other hand, a cultural relevancy focus on EAM Platonist content harmonizes with the projects described in section 8.

A plan seems clear for a 21st century re-envisioned curriculum with much needed flexibility:

1. Base school mathematics content invariably on cultural practices; mostly mainstream cultural practice found in out-of-school mathematizing, along with equitable Indigenous mathematizing beyond tokenism, with which Elders and knowledge holders collaborate from the beginning of the re-envisioning project.

2. Strip conventional curricula of their extraneous content, screened by the cultural practice criterion, including advice from people such as and Hacker (2016), Lockhart (2009), Russell (2016), Wolfram (2012), and many others mentioned above.
3. Follow research findings about the placement of essential content in the curriculum. For example, Jorgensen (2016, p. 135) "has identified growth points as critical concepts that must be learned, but the order in which they are learned is of little consequence."

To augment the impact on participating schools by the project Show Me Your Math (SMYM) (section 8.7), Lunney Borden and colleagues (2017) advised strengthening the connections between students' SMYM projects and the mathematics curriculum outcomes by focusing on new strategies with students.

My advice is quite different. Focus instead on an Atlantic Canada *political project* to officially update the current Platonist curriculum to a cross-cultural EAM curriculum, leveraged by the SMYM project's achievements (e.g., class attendance increases, drop-out rates decrease, community members' testimonials, test results, etc.).

Such an agenda amplifies Lunney Borden's (2013) research program: "How can curricula and pedagogy be transformed to support Mi'kmaw students…?" (p. 7). Well worth repeating: *both* Mi′kmaw and non-Mi′kmaw students will benefit (Adams et al., 2005; Aikenhead & Michell, 2011; Beatty & Blair, 2015; Davison, 2002; Lipka et al., 2013; Nelson-Barber & Lipka, 2008; Rickard, 2005).

In most projects described in section 8, degrees of decolonization cohabited with revitalizing a marginalized people's culture. Not only did this promote reconciliation, it directly benefited Indigenous students' academic success through strengthening their cultural self-identities, while reducing the political-social power imbalance between the curriculum and local Indigenous communities. And in turn, a decolonized curriculum will help stem the cycle of low graduation rates, unemployment, poverty, and family dysfunctionality. Canada's economy will benefit immeasurably as a consequence (Cooper, 2012; Sharpe & Arsenault, 2009).

By producing teaching materials, the researchers and teachers described in section 8 furthered their decolonizing goal to a significant extent. But more must be done to scale-up their successes (Elmore, 1996) for the benefit all schools, especially for lower and upper secondary grades. This requires a totally revised curriculum.

Researchers and teachers need to work out an explicit plan to renew the government sanctioned curriculum. Such a plan would expand on the worn-out content/process binary of most curricula standards, by recognizing that reality is first and foremost about *context*; which necessitates a curriculum framework designed with a context-content-process triad. EAM cultural content and local Indigenous mathematizing are viable contexts that have led to greater student success.

In 2008, the Province of Saskatchewan renewed its science curriculum into a cross-cultural one, along with its production of a Grades 3-9 science textbook series that enhances school science with Indigenous perspectives (Aikenhead & Elliott, 2010). This happened as a result of a broadly based *political* project that involved a number of independent people diversely situated in various institutions. This informal coalition was initiated in 2005 soon after a group of high school science teachers participated in a province-funded, year-long, in-service program that ended with teachers understanding and accepting the benefits of culture-based teaching. But when asked what changes they would make the next year, all replied, "None;" for the simple reason that culture-based content was not explicitly in the science curriculum. The teachers had the best interests of their students at heart. Their criterion for best was determined by the outcomes and indicators found in the legally mandated science curriculum. Pervasive general goals in the curriculum did not count.

Among all Canadian Ministries of Education at this time, there is a renaissance of openness to enhancing school subjects with Indigenous perspectives – Indigenous ways of knowing, doing, living, and being. This policy renewal is the direct result of Canada's Truth and Reconciliation Commission's final report (2016a, b). The time is right for substantially reworking mathematics curricula *as an act of reconciliation*. One example is the newly established group "Revisioning Regaining Reconciling School Mathematics" in Saskatchewan.

A government sanctioned, cross-cultural, EAM curriculum gives teachers the political power, legal authority, and much encouragement to work towards reclaiming school mathematics with culture-based content related to Platonist content (Lipka, 1994). The challenge to updating a Platonist mathematics curriculum can be seen as an opportunity to mount a political project. Any curriculum is certainly a political document. Thus, it needs *political* action to get revisions started. The literature cited in this monograph supports such a renewal for the benefit of all students.

A concrete first step towards forming a political project could follow the lead of the School of Indigenous Knowledges, Charles Darwin University, Australia. The School organized a two-day workshop between Indigenous Yolŋu educational consultants and Balanda (non-Indigenous) academics to give voice to Yolŋu knowledge holders involved in education (Maths in Aboriginal

Communities Project, 2007). A similar well-advertised event with public impact on education stakeholders could be a catalyst for a broader coalition to renew a mathematics curriculum.

"As human culture changes, so too does some of what we believe we know with certainty. Our ideas and standards of truth…have changed over the history and development of mathematics" (Ernest, 2016b, p. 390).

Where are the agendas for creating an up-to-date, Indigenous cross-cultural, Euro-American mathematics curriculum so Indigenous students can see themselves reflected in it? Because each educational jurisdiction worldwide has its unique political-social context with a unique group of local "movers and shakers," a political strategy must be mapped out locally by a committed diverse group of innovators, visionaries, Indigenous educators, Elders, selected university academics, and persons with connections to political-social power (Aikenhead, 2002b).

In Saskatchewan, a formal coalition, "Revisioning Reclaiming Reconciling School Mathematics," is currently taking shape. We have an internal group beginning to form, and we have established a national and international external group of advisors, whose substantial academic contributions already appear in this monograph.

10.3. The Opposition

"People and institutions within mainstream mathematics education too often collude with the political establishment by wilfully remaining oblivious of the social and political contexts outside their self-constructed cage" (Greer & Skovsmose, 2012, p. 6). "There are powerful forces at work keeping cultural domination and institutional racism in place, for it serves the interests of capital and the politically powerful" (Ernest, 1991, p. 268). "Against the background of the global interconnectedness of mathematics as a discipline, the struggle to counteract the rigid homogenisation of school mathematics worldwide is part of a larger struggle for cultural diversity" (Greer & Mukhopadhyay, 2012, p. 244).

I shall first recap from earlier discussions. The decolonization of mathematics education will be seen as a direct assault on:

1. the mythical armour of rigor that envelops a Platonist curriculum (sections 4 & 10.3)
2. those who subscribe to an absolutist philosophy that sides with "Plato's World of Ideas" and who demands a rigid Platonist curriculum (sections 4.1.1, 4.2.1, 4.4, 6.2, & 6.4)
3. the "supremacist position" of many professional mathematics organizations (e.g., the NCTM), and some mathematicians and other enthusiasts who are enamoured with

transmitting the crowning achievement of the human intellect (pure abstract mathematics) to all students (section 4.1.1)

4. nationalistic economists who champion corporate profits and global competitiveness, and in turn, finance the intrusion of their STEM movement into school mathematics and school science curricula worldwide (sections 3.2 & 9.1)
5. certain government policy-setting education committees: who side with nationalistic economists, who reject reconciliation, and who resort to manipulative ploys to obstruct an innovation, such as a culture-based mathematics curriculum, as illustrated in a case study of the U.S. National Mathematics Advisory Panel (Greer, 2012) and
6. the Western globalization impulse to colonize nations economically with Platonist mathematics as one weapon of choice (sections 4.2 & 4.4).

In this list I detect a Platonist "ideology of certainty" (Greer & Mukhopadhyay, 2016, p. 169), in a world comprised of varying degrees of uncertainty. "[M]athematics can offer a tantalizing illusion of certainty" (Greer & Skovsmose, 2012, p. 6).

Einstein (1921, website quotation) summed up the situation succinctly: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." By drawing upon mathematics, the physical sciences aimed to produce "an exact picture of the material world. Göbel's Incompleteness Theorems in mathematics (Stanford Encyclopedia of Philosophy, 2015) and theoretical physics in the 20th century have proven "that aim is unattainable" (Bronowski, 1973, p. 353). Bronowski explained that "errors are inextricably bound up with the nature of human knowledge" (p. 360). These two quotations from Bronowski come from his chapter entitled "Knowledge or Certainty," a phrase meant to define a continuum between the two. Certainty is attained at the expense of knowledge, a point of view that resonates with Einstein's quotation.

Ernest (2016b) explored and clarified "the problem of certainty in mathematics" (p. 379) in greater detail, by giving attention to "how beliefs in the certainty of mathematics have been constructed, both historically and individually, among learners. For it is the experience of school mathematics, combined with external cultural influences, that develops and buttresses such beliefs" (p. 391). His analysis distinguished between:

1. *absolute certainty*: the unconditional certainty of absolutists (section 4.2.1), "existing independently of humankind, free from the processes of change and decay that characterize the lived world" (p. 384); in other words, Plato's World of Ideas; and

2. *qualified certainty*: “circumscribed by the limits of human knowing… [and] consistent with social constructivism” (p. 391).

Harmonizing with the positions expressed by Einstein (1921) and Bronowski (1973), Ernest (2016b, p. 390) holds that “as our knowledge has become better founded and we learn more about its basis, we come to realize that the absolutist view is an idealization, a myth.”

One epistemic implication for Platonists is clear: their claim to pure knowledge with certainty is either an oxymoron or a tantalizing illusion of certainty (sections 4.2 & 11.1). “When people believe that they have absolute knowledge, with no test in reality, this is how they behave” in political-social contexts (Bronowski, 1973, p. 374); a context in which mathematics education certainly exists. Bronowski described such behaviour as arrogant, dogmatic, and ignorant.

Mathematician Newman (1956) stated:

> The difficulty [in explaining the subject of mathematics] arises not only from the abstract character of the subject but also from the generality and lack of content of its propositions. It is hard to know what you are talking about in mathematics, yet no one questions the validity of what you say. There is no other realm of discourse half so queer. (p. 1614)

This monograph seriously questions the validity of what Platonist school mathematics says. Similarly, some students, especially Indigenous students, have confronted its political-social power and have reacted intuitively as if it were a deceptive scam to avoid.

Much more than philosophical tension exists between an agenda to decolonize the school mathematics curriculum and an agenda to support the Platonist’s status quo. It is raw political-social power; the kind that can sweep ethical accountability under the carpet and can rely on pervasive racism in the underbelly of Canadian society (Battiste, 2002; Government of Alberta, 2010; St. Denis, 2004). It is an apparition from Canada’s 19th and 20th century nation building achieved at the devastating expense of Turtle Island’s civilizations of Indigenous peoples.

The poignancy in the final report of Canada’s Truth and Reconciliation Commission is its sharp focus on decolonization through reconciliation. Stakeholders of the mathematics curriculum, especially Ministries of Education and their curriculum writers, have an ethical choice:

- Support reconciliation by renewing the curriculum to enhance it with Indigenous perspectives exemplified by the decolonizing projects described in this monograph, or
- Avoid this act of reconciliation and thereby support a 19th century political-social policy, again at the expense of Indigenous students, families, and communities.

If we do not decolonize our curricula, we reject reconciliation.

11. Conclusion

No such thing in the Indyun way as gettin' wise. Gettin' wisdom. Wisdom's a path you decide to take'n follow, not someplace you get to.

Richard Wagamese (1994) *Keeper'n Me* (p. 189)

This monograph's Preface listed seven general questions. By responding to them, many significant ideas arose, including: the foundational importance of paying attention to culture clashes; challenges faced by people moving back and forth between the culture of Euro-American mathematics (EAM) and Indigenous cultures; pluralism; fundamental beliefs about mathematics; the cultural application of mathematics (distinguished from its scientific application); EAM Platonist content as an intellectual tradition of understanding; EAM cultural content as a wisdom tradition of understanding; the function of language-laden cognition; cognitive imperialism; camping spots of dialogue; the power of cultural self-identities; a notable flaw in ethnomathematics; the systemic neo-colonialism inhabiting a Platonist mathematics curriculum; Platonist content taught as Euro-American cultural practice, with the content's essential-nonessential status judged accordingly; the close interpenetrating relationship between reconciliation and Platonist or EAM school mathematics; and the inspiration of innovative mathematics researchers and teachers who are pioneering a new future for school mathematics.

The monograph began by introducing the crucial concept "masking power with innocence" (McKinley, 2001, p. 74). The concept expanded along the way to masking power with: convention, ignorance, racism, Platonic innocence, a rhetorical sleight of hand, an intended compliment, opaque responsibility, systemic naivety, statistical calculations, and feigned mythical innocence of academic rigor. Self-critical analyses on these themes will continue to liberate Euro-American minds from outdated policies for, and approaches to, school mathematics.

For example[xxi], Barwell (2013) reminds us that

> the development of mathematics has made possible the industrial-technological economic system that has led directly (e.g., through the oil industry) and indirectly (e.g., through population growth) to climate change. As D'Ambrosio (2010, p. 51) says, we have a responsibility, as mathematics educators, "to question the role of mathematics and mathematics education in arriving at the present global predicaments of mankind." (p. 6)

Accordingly, through a critical analysis of evidence-based practice in conventional and innovative school mathematics, this monograph has rationalized a 21st century renewal of Canadian mathematics curricula to serve the diverse interests and aspirations of all students, non-

Indigenous and Indigenous. This curricular transformation is summarized here in one sentence with several interlinking elements. The monograph argues for:

- a pluralist mathematics perspective;
- based on a cultural understanding of the mathematics taught in schools;
- developed within an Indigenous cross-cultural framework of respect, collaboration, and reconciliation,
- expressed by a curriculum in which age-appropriate EAM content is chosen mainly on its relevance to cultural practices found in both Canadian mainstream culture[xxii] and to a non-tokenistic degree, cultural practices in local Indigenous communities;
- flexible enough to embrace diverse approaches, such as
  - analytical and wholistic ways of thinking,
  - intellectual- and wisdom-based understandings, as well as
  - the ideologies and values embedded in the culture of Euro-American mathematics and in local Indigenous mathematizing;
- implemented within a local, culturally responsive or place-based pedagogy;
- mindful of students' diverse recurrent learning strengths and home-based languages;
- taught by teachers supported by:
  - effective cultural immersion experiences designed and run by Indigenous Elders and knowledge holders,
  - documents that explain the cultural nature of mathematics; that is, the content suppressed historically by 19th century writers of the first set of school mathematics curricula, who embraced an elitist, absolutist and acultural perspective on education, and rejected an alternative perspective proposed at the time: an expediently relevant culture-based education; and
  - teaching materials collaboratively developed with Indigenous Elders and knowledge holders by: people who are experienced in society's use of Platonist content, mathematics educators conversant with the research, and teachers experienced at teaching culture-based school mathematics.

11.1. From Tolerance and Inclusion to Dialogue and Collaboration

About two thousand years ago, Euclidian geometry was heralded by ancient Greeks as being consistent with Platonic philosophy and therefore, the only framework with which to understand the world. Just two hundred years ago, however, Guass, Schweikart, Reimann, Lobachevsky, and Minkowski helped invent non-Euclidian geometries (Director, 2006). A *universalist* geometry gave way to *pluralist* geometries, and humanity *intellectually* evolved as a result. Einstein's theory of general relativity is but one outcome.

Currently in the domain of education, the Platonist's *universalist* notion of mathematics is giving way to a contemporary *pluralist* notion of mathematics (the superordinate sense of the word) (section 4). Existing outside the domain of philosophy, this change reverberates worldwide within the political-social contexts of countries that are home to Indigenous cultures.

A pluralist notion of mathematics (its superordinate meaning) generally adheres to Bishop's (1988b) two coherent positions (section 4.1.3): (1) mathematics is a symbolic technology for building a relationship between humans and their environment; and (2) mathematics is: counting, measuring, locating, designing, playing, and explaining. Indigenous perspectives on building relationships between themselves and their place in Mother Earth (e.g., Indigenous mathematizing) are being ***tolerated*** and ***included*** in some classrooms today; where "learning environments leverage knowledge associated with everyday [cultural] experiences to support subject matter learning" (Bang & Medin, 2010, p. 1014).

Not only does a pluralist view of mathematics acknowledge the mathematizing carried out by Indigenous cultures, it identifies school mathematics as an expression of Euro-American cultures. In short, EAM embraces cultural content and Platonist content (section 4.3 & 4.5). An example of its cultural content would be to introduce proportional reasoning by asking students to judge whether it is more accurate to compare two measurements by differences or by proportion, given a specific everyday context such as sports teams' standings. Or in the context of climate change, students can decide which of the following two proportions makes sense to use in various contexts: either Canada's overall energy consumption compared to the world's consumption (0.016), or Canada's per capita consumption (almost the largest in the world)?

EAM cultural content has been supressed by a Platonist belief, a situation that marginalizes many students (Fowler, 2012; Nasir et al., 2008; Nichol & Robinson, 2000; Stoet et al., 2016). It ensures that Indigenous students' high school graduation rates will not reach full potential. And by what authority? – A self-composed social licence to narrowly define the subject matter of school mathematics, based on: its allegiance to an ancient Greek philosophical presupposition, its application to the Eurocentric sciences, and its arbitrary adoption of axiomatics (section 4.2.1). Einstein (1921) described this process of fabricating the subject matter's definition thusly:

> …[a] new departure in mathematics which is known by the name of mathematical logic or "Axiomatics." The progress achieved by axiomatics consists in its having neatly separated the logical-formal from its…intuitive content; according to axiomatics the logical-formal alone forms the subject-matter of mathematics, which is not concerned with the intuitive or other content associated with the logical-formal. (website quotation)

In the meantime, Ernest (1991) perceived the process of defining the subject matter as a rhetorical sleight of hand (section 4.2.1). This social licence was acceptable to a 19th century social era involved in colonizing Indigenous people through a residential school system that ensured low graduation rates for Indigenous students (section 2.1). Is that social licence still acceptable today for school mathematics?

For students coping with any degree of mathematics anxiety (Fowler, 2012; Nasir et al., 2008; Stoet et al., 2016), EAM cultural content and local Indigenous mathematizing serve as bridges between those students and a renewed school mathematics curriculum enhanced in cross-cultural ways. Culture clashes are certainly reduced (sections 3.3 & 8).

Because ethnomathematics does not recognize EAM's cultural status (section 7.2), significant portions of EAM cultural content are missing in R&D projects associated with ethnomathematics (sections 9.1 & 9.2). Many of these projects adhere to the ideology of a formal mathematics discourse constructed around axiomatics (Einstein, 1921; Ernest, 1991). I believe this is more than sufficient reason to retire ethnomathematics from mathematics education, unless a pluralist perspective on school mathematics could be agreed upon by D'Ambrosio's followers whereby Euro-American cultures are included in Neel's (2008, p. 23, emphasis added) definition: "Ethnomathematics has been identified as the study of mathematics that takes into *consideration the culture in which mathematics arises*."

What does the future hold? Will instances of masking power with innocence be rooted out of school mathematics? Will neo-colonial power imbalances be renegotiated between Indigenous communities and EAM curriculum developers? Will privilege-savvy be an aim of teacher education? Will Platonist curricula evolve into culture-based curricula? Will humanity eventually evolve as a result, but rather than evolve just intellectually, this time in terms of *wisdom*, accrued from newly reconciled relationships between Indigenous and non-Indigenous peoples?

Doolittle and Glanfield (2007, p. 29) point out an Indigenous axiological concept of wellbeing: the "balance among mind, spirituality, body, and emotion." An Indigenous ontology reveals a dependent, relational spiritual universe, with its web of interrelationships and responsibilities between humans and "the greater-than-human natural world" (Lowan-Trudeau, 2015, p. 653), from which flows the gold standard of sustainability. "[I]t's not just a question of how Western society can help Indigenous people, but how Indigenous people can help Western society" (Doolittle & Glanfield, p. 29). Doolittle and Glanfield are speaking of camping spots of ***dialogue*** and a commitment to ***collaboration***. "Respect is more than tolerance and inclusion – it requires dialogue and collaboration" (8Ways, 2012, p. 4).

At his very final concert, Canadian rock star Gord Downie (2016) of the Tragically Hip, referring to reconciliation, implored: "And we got a be a country that has taken 100 years to figure out what the hell went on out there. But it isn't cool. And everybody knows that. It was really really bad. But we're going to figure it out. You're going to figure it out." (13:40-14:07).

Dialogue and collaboration is needed to resolve a school's balance between teaching EAM and developing Indigenous cultural self-identities. "[S]ome mathematics is required for those who want to succeed in the society that has risen around us. But to be truly successful as Indigenous people, we must find a balance" (Doolittle & Glanfield, 2007, p. 29). Teachers of cross-cultural EAM can certainly help Indigenous communities reach a consensus on that balance (Bang & Medin, 2010; Beatty & Blair, 2015; Lunney Borden, 2013; Meaney, 2001, 2002; Neel & Fettes, 2010; Jannok Nutti, 2013; Pelletier et al., 2013) by strengthening students' Indigenous self-identities, and by firmly insisting that students achieve an understanding of EAM Platonist and cultural content. Just as we treat Indigenous mathematizing as cultural practice, we need to teach Euro-American mathematics as the Euro-American cultural practice it is.

The conceptualization of mathematics taught in schools is now open for renegotiation. In this context, we must ask: What is mathematics? In 1868, Japanese people called it "Yousan" (section 4.2). A contemporary translation is "Euro-American mathematics." Accordingly, a more contemporary curriculum is now required, given the advent of reconciliation. Evidence-based practice described in this monograph supports culture-based school mathematics for reconciliation.

Four major factors affect Indigenous student achievement: the curriculum; various strategies of instruction; culturally valid assessment; and interpersonal relationships such as a teacher's respect, firmness, personal warmth, and a sense of caring. Experienced together, these can lead Indigenous students directly to a rewarding occupation in the dominant society.

When mitigating the dominant culture's political-social power imbalance with Indigenous communities, there will be an exchange of precious gifts:

- from the non-Indigenous people, a practical *intellectual gift* of well-deserved Euro-American mathematics credentials;
- from the Indigenous people, *a gift of wisdom* "to chart the way to a more sustainable society and a more meaningful way of life" (Kinew, 2015, p. 266).

In the words of knowledge keeper Richard Wagamese (2016, p. 36): "ALL my relations. That means every person, just as it means every rock, mineral, blade of grass, and creature. We live because everything else does. If we were to choose collectively to live that teaching, the energy of our change of consciousness would heal each of us – and heal the planet."

11.2. Willy Alangui's Story

I conclude with a true story about mathematics-in-action (Salleh, 2006, website quotations). Mathematicians create highly complex sets of equations that model equally complex phenomena in the physical world, such as weather forecasting. So it was, with Willy Alangui, a professional mathematician and member of the Indigenous Kankanaey Nation in the Philippines. Large, tall, vertical rice-paddy terraces captured his interest. "I'm trying to understand how water is efficiently distributed in all the paddies," he told Salleh, a reporter from the Australia Broadcasting Corporation.

But every time he tested a revised model of water distribution, his model failed. A non-Indigenous mathematician may have given up perplexed. Willy, however, had two ways of seeing this problem (i.e., "two-eyed seeing;" Hatcher et al., 2009): the perspective of his non-Indigenous mathematics colleagues and his own Indigenous perspective. He decided to observe farmers working on the terrace and forge a relationship with them.

Willy discovered a key variable that determines how the Kankanaey rice irrigation system works: the ethic of cooperation. The community's social responsibility played a role when water was scarce. The farmers near the top of the terrace would lessen their intake so as to equally share water with those below. Social responsibility erratically interfered with Willy's mathematical modelling. Platonist equations are unable to deal with social responsibility.

This incident caused Willy to broaden his perspective as a mathematician. "My mathematics may be deficient. It's not the be all and end all of everything. It's just one way of looking at the world." Kankanaey mathematizing includes the value of social responsibility. How can mathematical equations incorporate an ethical value into mathematical modelling? Willy thought it may not be possible. He went on to describe Euro-American mathematics Platonist content as a “powerful” and “arrogant” field that marginalized other mathematical knowledge systems.

Willy’s story strongly suggests to me that cross-cultural school mathematics teaching materials should all include in their titles, “Mathematics: A Cultural Way of Knowing.”

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ENDNOTES

i Details differ regarding Métis and Inuit Nations, but the consequences are very similar.

ii Cynical equations are sets of mathematical equations that turn "a process of decision making into a calculation" (Sriraman, 2016, pp. 12-13), for example, modelling cost-benefit analysis.

iii Elder Dr. LeRoy Little Bear (Kainai Nation of the Blackfoot Confederacy) prefers spelling the word "wholistic," which avoids confounding the sacred "holy" with the secular "holistic." The colonizer's English spelling excludes the Indigenous meaning captured by the terms "wholistic" or "wholism." I follow Elder Little Bear's spelling convention in this monograph, out of respect for this eminent Elder, scholar, and mentor.

iv The "discovered or invented" debate is background history relevant only to understanding the philosophic history of the Platonist belief about the nature of mathematics. For introductory overviews, consult: Abbott (2013), Dekofsky (n.d.), Lamb (2011), NOVA (2015), Wolfram (2013), and Wikipedia's entry "The Unreasonable Effectiveness of Mathematics in the Natural Sciences": (https://en.wikipedia.org/wiki/The_Unreasonable_Effectiveness_of_Mathematics_in_the_Natural_Sciences).

v The philosophies of mathematics in relation to mathematics education are explored in general terms in Russell (2016, pp. 161-226).

vi In Ernest's (1991, pp. 259-260) own words:

Mathematical propositions and their proofs, the products of formal mathematical discourse, are admitted as legitimate mathematics. Mathematical invention, the practices of mathematicians and other products and processes of informal and professional mathematical discourse are not. Once the rules of demarcation of the discipline are established in this way, then it can legitimately be claimed that mathematics is neutral and value free. For in place of values there are rules which determine what is admissible. Preferences, choices, social implications and all other expressions of values are all eliminated by explicit and objective rules. In fact, the values lie behind the choice of the rules, making them virtually unchallengeable. For by legitimating only the formal level of discourse as mathematics, it relegates the issue of values to a realm that is, by definition, outside of mathematics.

vii Interestingly, Platonists ignore the fact that their binary is framed by Greek epistemology (Hall, 1976), thus casting their strategy as being culture-laden. Many world cultures, including Indigenous cultures, do not lend much credence to Greek epistemic binaries.

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viii Two examples are: (1) Discussing when it is appropriate or not to represent something with numbers. (2) Mathematics modelling is used by airline companies to calculate how many seats should be overbooked on a particular flight in order to maximize profits. (See section 4.5 for more examples.)

ix For a critical analysis of mathematics modelling in the context of mathematics education, see Jablonka and Gellert (2012, pp. 296-299).

x This introduction to section 7 is purposely silent on projects that address the internal and external human dimensions of mathematics-in-action, a subsection of EAM cultural content (section 4.5). But these topics are integrated into sections 7.1 and 7.2.

xi Haida Gwaii is an archipelago located off the northwest coast of British Columbia, not too far from Alaska. The colonizers named it the Queen Charlottes Islands.

xii In the late 1990s, I was strongly influenced and encouraged by Ciulistet's parallel group in science education led by Ray Barnhardt and the late Elder Oscar Kawagley. They inspired our development of the community-based cross-cultural science units *Rekindling Traditions* (Aikenhead, 2002a).

xiii Indigenous knowledges are always local knowledge. The Pikwàkanagàn First Nation has chosen to keep the settler-given name (Algonquin) to describe themselves, rather than follow the custom of other Algonkin or Algonkian communities.

xiv The history of mathematics matters. Had the Fractal Landscapes author known this history, they might have realized that fractal mathematical equations were created and named by humans to approximate a pattern perceived by humans when looking at nature. This perceived pattern itself was first a human construction of the mind, according to Einstein (1930). It would seem reasonable, therefore, to conclude that fractals were invented by humans; they are not abstract objects that comprise the concrete universe. The fact that fractal mathematics exists is not evidence that pure fractal patterns exist in nature. That would be a fallacy of circular reasoning. Both the equations and the perceptions are human constructions. See endnote iv.

xv This also illustrates the power of the altar of Platonist content (section 9.1) to manipulate historical accounts into myths to serve the interests of the altar.

xvi Here is a political-social context in which one can correctly state: 25 = 27.

xvii Statistical reliability refers to consistency of results from one occasion to another. It is a social science technical term. This meaning differs greatly from the media's meaning of a *reliable* *source*, which is informed and trustworthy. Potentially confusing is the fact that "trustworthy" is close to the statistician's meaning of valid – it actually measures what it claims to measure. Culturally valid student assessment is a central concern, for instance, when a national exam is simply translated into the local Indigenous language (Fyhn et al., 2011; Meaney et al., 2012; Nutti, 2013).

[xviii] TODOS: Mathematics for ALL is an international professional organization that advocates for equity and excellence in mathematics education for all students – in particular, Latina/o students (http://www.todos-math.org/).

[xix] I anticipate that the out-of-date spelling will be corrected to "Latino/a" or "Latina/o" before publication.

[xx] These OECD data, however, require a quality-control correction factor due to the percentage of students who drop out of the STEM pipeline by the time they graduate from high school. As mentioned in section 9.1, Frederick (1991) quoted data from the U.S. Office of Technology Assessment's 16-year longitudinal study that began with four million participants in Grade 10. The study determined that 19 percent dropped out of the STEM pipeline during high school, a correction factor of 0.81. Based on these corrected results, the proportion of Canadian, Saskatchewan, and American STEM-oriented students was 28, 26, and 31 percent (OECD, 2016, pp. 362, 447, & 362, respectively).

[xxi] For another poignant critique from a decolonizing, anti-oppressive, anti-racist education perspective focused on social justice, see Stavrou and Miller (in press).

[xxii] An example is found in section 10.2. It concerns teaching students to think algebraically while their computer does the tedious arithmetic. Then their thinking is transferred to abstract symbols and equations of Platonist math. The transfer was made easier by explicitly teaching some of the human features hidden in Platonist mathematics, such as its values (e.g., logical consistency) and assumptions (e.g., treat it as if it were objective).