A 21st Century Culture-Based Mathematics for the Majority of Students

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Philosophy of Mathematics Education Journal, No 37, 1-34

ABSTRACT

Connecting out-of-school mathematics with in-school mathematics is treated as a political agenda for liberating about a 70 percent majority of students (ages 12 to 18) from experiencing an unpleasant rite-of-passage into adulthood. They should be experiencing a measured common-sense preparation for it, informed by research in critical mathematics education. This requires some nuanced revisioning, revising and/or reconceptualizing:

a. the diversity of students' mathematics identities (from dichotomies and stereotyping to an evidence-based continuum), and identifying teachers' concomitant blind spots;
b. the nature of out-of-school mathematics (from a place to which one *applies* mathematics to a place in which one *learns* and applies mathematics);
c. the nature of in-school mathematics (from students assimilating Platonist mathematics to students acculturating math-in-use, math-in-action, the culture and nature of Platonist mathematics, and some abstract mathematics);
d. Platonist mathematics: either cultural or spiritual; the choice is yours.
e. the mathematics curriculum for the 70 percent majority (from an invisibly absolutist document to the political document it has always been);
f. the politics of school mathematics (from being taboo to enacting political and educational strategies, as well as engaging in politically defined research in mathematics education; and
g. specific political actions that have been successful in achieving almost all of the above, mostly in science education.

1 Introduction

One of the "three salient features of critical mathematics education …[is] the possible relationships between out-of-school mathematical practices and how mathematics might be contextualized in a school setting" (Greer & Skovsmose, 2012, pp. 15-16). Back in 1916, Dewey expressed his viewpoint, "Education through occupations consequently combines within itself more of the factors conducive to learning than any other method" (p. 309). In 1987, Resnick commented on the fallacy of imagining that a Platonist[1] school setting had much of a connection to out-of-school mathematical practices. Ernest (2016c) took stock of critical mathematics education's (CME) development since its beginnings in the early 1980s. CME has a solid base in university research into "the teaching and learning of mathematics with an emphasis on *real world contexts*" (Sriraman, 2016, p. ix, emphasis added).

[1] Platonism refers to a philosophy or an ideology of mathematics that is absolutist in keeping with Plato's philosophy of mathematics, or it refers to "mathematical fundamentalism" (Greer & Mukhopadhyay, 2016, p. 162)

Such a long history, yet so little headway has been accomplished in public school classrooms it seems. It is as if very few have answered the following ubiquitous question with action: "What mathematics is essential for students?" Perhaps it has been the wrong question to pose. Or perhaps its implication for creating change in mathematics classrooms precipitates a politically powerful taboo. "There are powerful forces at work keeping cultural domination and institutional racism in place, for it serves the interests of capital and the politically powerful" (Ernest, 1991, p. 268). Simeonov (2016) lamented, "[T]he effort to improve the general situation of mathematics in school has been in vain. We have to face this fact honestly and to consider all possibilities how to go on and not just to try the next effort in the same direction like all previous initiatives which have failed" (p. 439).

Although the CME literature does offer rich descriptions of Western mathematics' socio-politico-economic power, the literature seems sparse on *political* plans to challenge this power seriously by altering the school mathematics' landscape significantly for a majority of students, beginning on a small scale. This article identifies and illustrates specific political and educational strategies and their concomitant research programs that differ from previous initiatives.

A further exploration into CME leads to such dichotomies as *Platonist* school mathematics versus: *workplace, real world, functional, real-life education* or *culture-based* school mathematics – all associated with human activities outside of the school. Platonist mathematics, however, embraces "the antithesis of human activity – mechanical, detached, emotionless, value-free, and morally neutral" (Fyhn, Sara Eira, & Sriraman, 2011, p. 186).

However, before adopting one or a combination of these alternatives for school mathematics, both a *non-stereotype analysis* of students (section 2) and a scrutinizing look at mathematical preparation for adult life (section 3) are in order. The need to emphasize "non-stereotype analysis" arises from the question itself posed above, "What mathematics is essential for students?" Unfortunately, the question conveys a stereotype that all students are the same in terms of what mathematics they require. When the question is altered, we are in a position to recognize mathematics curricula as being political documents (section 4), thereby undertake reasonable political actions through focused research programs (section 4.3). This leads to a non-Platonist, culture-based, school mathematics: the culture of mathematics interacting with the mainstream culture of society; in short, $(\text{culture})^2$.

2 Student Diversity

Dichotomies such as "a math person" or "not a math person" (Boaler, 2013, website quote) can oversimplify an issue that the dichotomies structure (e.g., the issue of mathematics education). This article infuses other dichotomies into the mix in order to achieve ultimately a broader perspective on mathematics education for the purpose of deeper scrutiny.

Data collected by three major research studies, plus a professional public poll, are analyzed here in terms of students' affiliation with STEM subjects with an emphasis on mathematics. The three research studies are:

- PISA 2015 (OECD, 2016) mathematics assessment of 15-year-olds (Year 9 in Canada);
- U.S. Office of Technology Assessment's 16-year longitudinal study, beginning with four million Year 10 students following up to PhDs (Frederick, 1991); and
- a National Bureau of Economic Research STEM project (Card & Payne, 2017).

The PISA mathematics data showed that 35, 32, 38 percent of Canadian, the Province of Saskatchewan, and American Year 9 students expressed a STEM-oriented interest (OECD, 2016, pp. 362, 447, & 362, respectively). These figures can be transposed to Year 12 students by using the U.S. Office of Technology Assessment data (Frederick, 1991). First of all, 18 percent of their students expressed an interest in continuing toward university STEM courses. Of these interested students, 19 percent lost interest during high school (i.e., they selected non-STEM courses in Years 11 and/or 12). This left 81 percent of the original STEM group to continue their interest in STEM. By using this 81 percent conversion figure, the Year 9 PISA data were extrapolated to Year 12 data: 28, 26, and 31 percent, respectively. These are obviously approximations.

Card and Payne's (2017) research into a gender gap in STEM enrolments in the Province of Ontario, Canada, looked at the percentage of females and males entering university with the necessary high school courses to qualify as STEM ready. Their data showed 14.5 and 15.3 percent for females and males, respectively, were STEM ready. Together, and rounded off, that totals 30 percent. This independent result tends to confirm the trustworthiness of the extrapolated PISA results.

According to an AP-AOL News poll of 1,000 adult respondents (ages 20 to 39) conducted by Ipsos in the U.S.A., 23 percent stated that mathematics was their favourite subject (Ipsos, 2005). This gives further evidence in support of the results reported just above. According to the same poll, 37 percent stated that they "hated" mathematics.

It would seem that a *small minority* of Year 12 graduates tend to pursue a STEM trajectory, while a *sizable majority* do not. This simple dichotomy, however, ignores the fact that students' interest in mathematics classes can vacillate from time to time depending on such factors as the teacher, the mathematics topic being taught, the classroom social environment, past success, time of year, family encouragement, their social economic status, etc. An overarching factor, however, concerns the *extent to which* a student's mathematics self-identity harmonizes with a mathematician's (Aikenhead, 2017). A review of studies into students' mathematics self-identities by Heffernan, Peterson, Kaplan and Newton (2020), concluded: "mathematics identities motivate action and mathematics educators can influence students' mathematical identities" (p. 1).

Accordingly, Meyer and Aikenhead (2021a) devised a *heuristic* spectrum of student diversity for the purpose of discussing students' potentials and proficiencies. Towards the right end of the spectrum are students whose mathematics self-identities harmonize *to varying degrees* with their mathematics teacher's. "Math-oriented" students represent the extreme case of harmony. At the opposite extreme on the left side are math-phobic students who often develop serious psychological and physiological anxieties when forced to think mathematically (Ernest, 2018, 2019). Mathematics anxiety is a pervasive issue among students, teachers, and parents. This spectrum replaces the dichotomy non-STEM or STEM, and it encourages a realistic perspective (i.e., "degrees of"), while discouraging a stereotypic treatment of students. For example, it replaces the dichotomy that Simeonov (2016) challenges: gifted versus not gifted, as in, "If a certain gift is needed in order to master mathematics, then why should we teach all those people who do not possess this gift?" (p 443).

Meyer and Aikenhead's (2021a) partitioned their spectrum into six heuristic categories of students' degrees of disharmony/harmony with a mathematician's mathematics self-identity (Table 1). Fine distinctions between these six categories are not made in this article. A student's designation to a category must be flexible and tentative due to factors that cause a categorization to vacillate (e.g., see Henrik's interview data in section 3.1.4). If this spectrum were represented by a frequency distribution graph, it would be highly skewed to the left, as Table 1 roughly suggests by the width of its columns. Most importantly, however, the categories are certainly not to be used for streaming students.

Table 1. Six levels of Canada's high school graduates' mathematical identities derived from Year 9 PISA 2018 data (OECD, 2019, p. 105) and transformed to represent Year 12 graduates according to Frederick's (1991) data (section 2, above).

Student Category	math-phobic	math-shy	math-disinterested	math-interested	math-curious	math-oriented
Students in each PISA Proficiency Level	20%	24%	26%	20%	6%	4%
Aggregate Proportion	70%			30%		

Four implications are worth noting. First, broadly speaking from an anthropological perspective, math-oriented and math-curious students' experience learning (i.e., the cultural transmission-acquisition of) Western mathematics as *enculturation* – wanting to identify with it completely. This anthropological characterization of the term "enculturation" is a more finely tuned version of Bishop's (1988) concept, as explained in section 6. Math-phobic and math-shy students try to avoid the school's attempt at *assimilating* them into believing like a mathematician believes. These students usually succeed by playing "Fatima's Rules"[2] (Aikenhead, 2011) or dropping out of school. In between these two anthropological extremes is the process of *acculturation* (Aikenhead, 2015), in which students *selectively* modify some currently held ideas as a result of a positive influence from the mathematics taught to them. Important classroom nuances can lead to coercive acculturation, as explained in section 6.

Secondly, mathematics teachers will obviously fit the math-oriented or math-curious categories. That is one of their preferred ways of making sense out of their everyday world. Hence, when they were students in school, they likely did not experience much struggling with mathematics problems along with failing from time to time. Therefore, they have a tendency to assume that their way of making sense of the world is just common sense, a normal human tendency. When it comes to communicating with the math-disinterested, math-shy and math-phobic students, there is a lack of shared experience between the teacher and those students. The result is usually a teacher's blind spot that causes frustration for the students from not being able to understand their teacher sufficiently well. They do not see the world like the teacher does.

[2] Fatima's rules pertain to passing a course with the least amount of effort. The rules are a set of strategies to make it appear as if in-depth learning has occurred, when only the most specious, superficial and easily forgotten content has been learned. The rules were named after the student who informed a researcher how she and her friends passed their Year 11 chemistry course.

Thirdly, there is a tendency for mathematics teachers to misunderstand their students, hindered by this blind spot in the teacher's mindset. The blind spot is also responsible for: (a) a teacher's misunderstanding what type of mathematics course would harmonize with these students (e.g., culture-based, real-world, workplace, culture-based school mathematics)[3]; (b) a teacher's unreasonable expectations of these students (e.g., mathematics is all around you so you should learn to apply school mathematics to your everyday world[4]); and (c) a teacher's susceptibility to taking unreasonable pedagogical slogans seriously, such as Boaler's (2020), "All students can learn math to the highest levels" (p. 1), based on her dichotomy of growth versus fixed mindsets.

Fourthly, a humanities culture immersion for mathematics teachers could be helpful for appreciating other epitomes of human thought other than mathematics. Also, by mentoring their students on a yearly field trip into their community to learn what mathematics is actually used (not their hypothetical "could be used") in businesses, the trades, and the professions (Nichol, 2002).

3 Mathematical Preparation for Adult Life

Culture-based mathematics curriculum content exists beyond the confines of the non-STEM vs STEM dichotomy. It can be thought to exist in three dimensions:

1. *Mathematics-in-use.* Everyday encounters with Western mathematics: (a) in the culture of one's personal activities at home, and (b) in interactions with one's community culture, and (c) in the culture of one's employment, as an employee, employer, and/or having responsibilities for raising a family.
2. *Mathematics-in-action*.[5] Powerful interactions (explicit or implicit) between Western (Platonist) mathematics and society, for which there are political, economic, social and ethical consequences, both positive and negative. "Mathematics education counts in society. However, society does not necessarily count in mathematics education" (Andersson & Valero, 2016, p. 199).
3. *The culture and nature of Western (Platonist) mathematics*: (a) its general assumptions about reality (ontology); (b) its ideas about knowledge; how we know what we know, and the kind of knowledge this is (epistemology); (c) its values, ideologies, and aesthetics (axiology); and (d) its history.

Section 3 sets STEM-oriented mathematics to one side, leaving it to continue its politically privileged place in society, and to be meaningful and relevant to about 30 percent of high-school graduates. Section 3 addresses the approximately 70 percent non-STEM group's mathematics-in-use, exemplified by four domains: (a) real-world mathematics (Andrews, 2016; Stephan, Reinke & Cline, 2020), occupational mathematics (Andersson & Ravn, 2012), practitioner mathematics (Pais, 2012), and workplace mathematics for employers and employees (Nicol, 2002; Pais, 2012);

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[3] On the morning this sentence was composed, a member of our extended family read out her horoscope. The November 21, 2020, advice for those born under the Taurus sign read: "Practice is the best teacher. Besides, what *you* want to know cannot be learned theoretically. You're better for it, as the thick of *the task gives you not only knowledge but also fun and relationships*" (emphasis added, and no academic citation needed.) This horoscope provides an insight into some math-phobic, math-shy or math-disinterested minds. It also has pedagogical implications: Plan mathematics lessons accordingly – make it interesting and emphasize relationships.

[4] These students do not see the world through a mathematical lens; hence, mathematics is *not* all around them.

[5] This expression in the academy of critical mathematics education (e.g., Greer & Skovsmose, 2012) combines my first two hyphenated categories (mathematics-in-use and mathematics-in-action) under the rubric "mathematics in action" (p. 4). I have made two separate categories in order to help finetune curricular issues discussed in section 5.

and (b) mathematics associated specifically with the trades; (c) mathematics related to recreational or hobby activities; and (d) mathematics needed for specific problem-solving.

The following discussion draws upon some of the above finely tuned categories, and it properly excludes hypothetical examples that we math-oriented people are in the habit of superimposing on everyday life events, without releasing it.

3.1 Mathematics-in-Use

This discussion examines Platonist mathematics and other types of mathematics used by people in a country's *mainstream* culture. For mathematical systems used by a country's Indigenous cultures, for instance, see Aikenhead (2017) and Meyer & Aikenhead (2021b).

3.1.1 Self-Initiated Informal Survey

I challenge the reader to conduct an informal survey of a wide range of employees or relatives with whom you happen to interact over the next month, by asking them: "What mathematics do you use in your job?" Here is a sample of what I heard. "Nothing I was taught in high school. I've never needed to use imaginary numbers or to solve a quadratic equation," a physiotherapist. "I need to be competent at a Year 8 level of math,"[6] the general manager of a major car dealership in the third largest Canadian city. "We had to learn trigonometry to pass our certification exam, but we never use it on the job," plumber. "I have to put up displays in my store. There's geometry involved," a relative. A financial advisor told me, "I have all the algorithms on my computer. I haven't done an algebra calculation since high school, I'm not sure I could." This survey could also be a useful assignment for teacher candidates in a mathematics methods course.

In addition, be prepared to hear how much some of your interviewees hate mathematics. And finally, a Google search of "mathematics in the everyday world" will take you to STEM-related contexts or to implausible hypothetical business situations or home renovations; in other words, not the everyday world of the non-STEM citizen.

When discussing careers with students, the ethics of full disclosure requires you to make available to them documents entitled "High-Paying Jobs for People Who Hate Math" (e.g., Martin, 2014; Tencer, 2016). Some of the jobs listed include: guidance counselor; hearing aid specialist; camera operator for film or tv; occupational therapist; librarian; plumber; sonographer/ultrasound technician; information security analyst; compliance manager; lawyer; police officer; boilermaker; non-STEM postsecondary teachers; humanities school teachers; transportation vehicle, equipment, and systems inspector; technical writer; ship engineer; dental hygienist; elevators installer and repairer; and judges.

Research concerning out-of-school mathematics has investigated a variety of workplaces, including: investment-bank employees (Noss & Hoyles, 1996), commercial pilots (Hoyles, Noss, & Pozzi, 1999), and nurses (Hoyles, Noss, & Pozzi, 2001). This last occupation reveals particularly interesting results for mathematics educators, analyzed in section 3.1.2.

"Mathematical reasoning in the workplace can be quite different from mathematical reasoning found in school contexts" (Nicol, 2002, p. 293). Algorithms learned in mathematics classes "are often not the ones used by workers on the job" (p. 294). In short, decontextualized

[6] This comment speaks to the importance of mastery learning for Years 7-9, with content that survives the curriculum culling of obsolete and pedantic material; repeated throughout high school in different authentic contexts.

Platonist school mathematics is worlds apart from contextualized workplace mathematics. This is especially true for the math-disinterested, math-shy and math-phobic employers and employees, for whom it is a great challenge to apply pure abstract mathematics to real life situations.

Two examples detail this hiatus: (a) surgical ward nurses' mathematical and scientific understandings used in their workplace (section 3.1.2), and (b) teacher candidates in a preservice mathematics education program, interacting with mathematics contextualized in local businesses (section 3.1.3).

3.1.2 Hospital Ward Nurses

Ninety-five registered nurses' proficiency at proportional reasoning learned at school was measured by test items taken from standardized tests (Perlstein et al., 1979). The mean score for these pediatric nurses was 77 percent, with a range from 45 to 95 percent. The researchers concluded that their results "paint a consistent picture of high levels of error in similar research" (p. 378). The implication here concerns Platonist school mathematics potentially applied to a surgical ward. The results are obviously worrisome.

Researchers Hoyles, Noss and Pozzi (2001) decided to spend 80 hours hanging out on the wards with 12 pediatric clinical nurses to observe how they prepared dosages for their patients. During that time, the researchers observed 30 instances of drug administration that involved 26 different types of ratio and proportion calculations.

The results? No errors were ever made. "Most nursing literature on drug calculation…of early research in mathematics education focused on individual performance on '*decontextualized*' written tests" (p. 11, emphasis added). These 12 nurses' calculations were *contextualized* in the reality of a hospital ward.

Obviously these results, 75 versus 100 percent, are evidence that abstract decontextualized mathematics (knowing *that*) must be qualitatively different than concrete contextualized mathematics (knowing *how*). There certainly is a disconnect between the two. What contextualized proportional reasoning *did* the nurses actually carryout? Their professional nurses training at university provided them with *procedural* math for their workplace. Here is their procedural algorithm (Hoyles et al., 2001): "The actions that a nurse would perform in identifying and handling three quantities when preparing a drug: Look at the drug dose prescribed on the patient's chart ('what you want'); next note the mass of the packaged drug on hand ('what you've got') and then the volume of solution ('what it comes in')" (p. 13). Then use your calculator and double check the calculation. There were no algebraic formulae mentioned.

A parallel science education study sheds greater light on workplace mathematics. Aikenhead (2005) conducted 24 *in situ* interviews over a semester with six acute-care nurses working on a hospital surgical ward. The research question was: What scientific curriculum content (decontextualized knowledge) did they use to decide what to do next for their patient, given a set of observations and calculations they had just made? Two short answers are:

1. "Five out the six nurses used specific contextualized science knowledge ('procedural knowledge,' p. 251) without necessarily comprehending its analogous decontextualized science knowledge ('declarative knowledge,' p. 251)." When asked what they would do if they needed to know the theoretical (declarative) knowledge; their response was, "Ask a doctor" (p. 268). Getting the job done effectively and efficiently was the nurses' priority. Taking time to work out and explain to themselves the theory that informed their

procedural knowledge would be a waste of time and energy. That theoretical knowledge had been applied earlier by medical researchers or doctors. The five nurses' interview transcripts "were almost devoid of references to declarative scientific knowledge (except for the use of anatomical terms)" (p. 266).

2. "The transcripts of one nurse, Terry, however, were replete with descriptions and explanations from a scientific worldview perspective [as evidenced in] the following exchange.

   Terry: … And that's also monitoring what's happening with those chest tubes.

   Glen: That's when the evidence comes in.

   Terry: Oh, absolutely. And for that you really have to understand the physics of what's going on with those chest tubes. You have to understand why those chest tubes are there in the first place." (pp. 266-267)

Terry was certainly a STEM person who applied his decontextualized curriculum knowledge effortlessly.

His non-STEM colleague, Gia, attended to a different patient who had an issue identical to Terry's patient, described just above. Aikenhead (2005) learned that:

> Gia, on the other hand, described how she successfully solved a chest-tube problem but her account was formulated on professional knowledge in nursing (i.e., what patients do when they pull their chest-tube equipment with them when they go to the bathroom) rather than a scientific explanation of differential gas pressures in closed or open systems. This is not to say that Gia could not describe how a scientist would explain her patient's situation (she was not asked for that information), but rather, a scientific explanation for her was not relevant to the problem-solving task at hand. (p. 267)

Declarative (abstract decontextualized) knowledge to be remembered and then applied is obviously very different from its analogous procedural knowledge, which is knowing: "what to do, how to do it, and how to communicate it; this produces data to be analyzed by drawing on the quantitative decision-making model 'concepts of evidence' (Gott & Duggan, 2003)" (Aikenhead, 2005, p. 251).

These two workplace investigations provide insights into the qualitative differences between workplace, contextualized or a procedural understanding of mathematics/science and a qualitatively different but analogous pure, decontextualized, declarative mathematics. These insights can be added to those gained from the realistic critique of student diversity. Both insights will be helpful in a discussion on a 21st century renewal of the curriculum for math-disinterested, math-shy and math-phobic students (section 4). But first, an investigation into teachers' experiencing the bridge between in-school and out-of-school mathematics.

3.1.3 Teacher Candidates in a Pre-Service Mathematics Education Course

Knowing that current and future mathematics teachers will most likely be math-oriented, math-curious, or math-interested people, one might hypothesize that they will have as much trouble understanding contextualized, procedural, workplace mathematics as do math-disinterested, math-shy and math-phobic students have understanding abstract decontextualized pure mathematics, to varying degrees.

That was what Nichol's (2002) research investigated: How well do these mathematics education majors *apply* the decontextualized pure mathematics content to the everyday world of local businesses? In the context of a provincial renewed curriculum to coordinate secondary school mathematics with "workplace applications, it was hoped that mathematics could be made more accessible to a larger number of students" (Nicol, 2002, p. 292). A faculty of education curriculum and instruction course was designed to help prepare 22 teacher candidates for this curriculum change. The course included "the integration of field experiences with course work" (p. 293). This involved networking with employees who use mathematics in local businesses. During the teacher candidates' visits to various businesses, they interviewed workers about the mathematics they used. Then, based on this networking experience, the teacher candidates designed a teaching unit for high school students to teach them the workplace mathematics that they had just learned. Most of the teacher candidates were less than successful.

Nicol (2002) pointed out that workplace mathematics is embedded in commercial activities with the purpose of getting the job done. This differs from the isolated separate subject of mathematics in schools. Abstract decontextualized knowledge has no context; a far cry from embedded workplace mathematics. School mathematics' intellectual purpose expects students to bank concepts and processes for "solving problems that may not emphasize the relational and connected aspects of mathematical reasoning, making such topics as proportional reasoning a difficult mathematical concept for students" (p. 294). Because the *purpose* for doing mathematics-in-use differs dramatically from the *purpose* for applying decontextualized Platonist mathematics learned in school, the chasm between the two seems wider.

But it widens even further. From an epistemic perspective, the integrated or *holistic nature* of workplace mathematics contrasts with Platonist school mathematics' *reductionist nature*. This is not a dichotomy, but instead a continuum when observed on site with its complexities. Students' predilections towards *a degree of* a holistic or reductionist epistemology helps explain the spectrum of students' cultural self-identities with respect to mathematics (Table 1). About 70 percent of students tend to be more comfortable as holistic reasoners to varying degrees than are reductionist reasoners. The issue is amplified further for Indigenous students whose core culture is highly holistic (Aikenhead, 2017). The 30 percent are likely reductionist reasoners to varying degrees. Their predilections form a core to their epistemic preferences. My claim is that Platonist, workplace, real-world, or culture-based school mathematics will not alter those predilections to much of an extent, but they will dramatically improve students' engagement in school mathematics when the pedagogy *and* content harmonize with a student's predilections.

For some prospective teachers, "being at a workplace did not necessarily make mathematics more accessible" (Nicol, 2002, p. 297). Perhaps it was a case of epistemic disharmony between the prospective teachers and workplace mathematics. As a result, most prospective teachers focussed on general skills when they visited the businesses:

> Communication, flexibility, collaboration, problem solving, use of technology, and versatility were mentioned by prospective teachers as skills employers were looking for in their potential employees. No prospective teachers reported observing in the workplace or learning from those they interviewed of a need for extensive algebra, or use for ideas of mathematical proof, modelling, and mathematical rigor in workplace tasks. (p. 298)

These results were duplicated in Lunney Borden's (2011) research, in which teacher candidates in mathematics classes remembered "ideas as things to know, rather than processes to use" (p. 12). Perhaps they found it difficult to escape from the reductionist nature of Platonist mathematics. If so, they need to become empathetic with a mirror image: the majority of students' parallel imprisonment by the holistic nature of their self-identities.

When completing their assignment to develop a teaching unit, "some prospective teachers did not notice the mathematical or pedagogical potential of a workplace context. Instead, they re-created its context to design an interesting but not necessarily authentic workplace problem for students to solve in their unit assignment" (Nichol, 2002, p. 300). The declarative (Platonist) mathematics and procedural (contextualized) mathematics were such two very different paradigms that the procedural mathematics was foreign to them; and therefore, it became a blind spot.

> The relationship between knowing and doing is complex and interactive. Yet, prospective teachers spoke about how employees claimed that the purpose of mathematics was often not realized for them in school and only became clear at work. Situated, problem-based curricula, such as that found in workplace contexts *can help to connect the abstract mathematics of school and its uses in the world*. (p. 304, emphasis added)

Problem-based/problem-posing pedagogy is therefore highly recommended to help connect in-school mathematics with out-of-school mathematics (Andersson & Ravn, 2012; Boaler, 1998; Boaler & Selling, 2017), a topic discussed in section 3.1.4.

Nicol (2002) made several specific recommendations:

1. Teacher educators could mentor prospective and practising mathematics teachers by arranging a guided field trip to local businesses and industries.
2. Expand the usual content in "functional mathematics" from just measurements and computations to including "spatial and geometric reasoning, with chance and probability" (p. 305).
3. "[S]upport prospective teachers' need in drawing upon their own experiences outside of school as a context for understanding mathematics in work" (p. 305).
4. Organize work experiences that develop teachers' "understandings of mathematics through 'realistic' functional workplace problems" (p. 306); for instance, architectural or engineering companies.

To which I would add:

5. Assign teacher candidates a course project to write a report on contextualized mathematics in a particular workplace or for an everyday-life event. Then assign them the short teaching unit, as before. But follow-up with a critique of these projects for the benefit of the whole class. Put simply, rather than *teaching* the topic mathematics-in-use, *mentor* students on a concrete topic embraced by mathematics-in-use.
6. Dramatically change the high school mathematics curriculum for the 70 percent math-phobic, math-shy, and math-disinterested students; but leave such courses open as an elective for the 30 percent students to pursue. (See section 5.)

3.1.4 Connecting the Out-of-School with the In-School Mathematics

"Practicing de-contextualized symbolic algebra manipulation routines seldom engages young teenage students and as a captive audience, in a core subject, many who find this thinking difficult become disinterested or even resentful" (Pierce & Stacey, 2006, p. 214). In order to determine an alternative way of teaching symbolic mathematics, these two researchers investigated, in depth, the reactions of Year 9 students expressed during interviews. Pierce and Stacey also interviewed their teachers' who made real-world connections a priority. By noting the teachers' reactions to four specific "real world problems" (p. 216) for students to solve, Pierce and Stacy composed the following list as the teachers' criteria for assessing those problems: (a) "the mathematical solution involves specific [abstract] mathematics" (p. 216); (b) "increased student interest, engagement and improved attitude toward mathematics" (p. 217), and (c) "prompt mathematical learning by showing its relevance" (p. 217).

The four noteworthy problems for students to solve were: "Barbie Bungee (linear functions)" (p. 218), "Dirt Bike Jumps (quadratics)" (p. 218), "Biggest box (quadratics)" (p. 219), and "McDonald's arches (quadratics)" (p. 220). Some internet and commercially available measuring and plotting electronics were involved. No mention was made, however, of connecting any of these real-world problems to the real world of math-in-use or math-in-action. This observation has direct implications for renewing the mathematics curriculum's content (section 5).

Pierce and Stacy (2006) were disappointed to learn that some teachers relied solely on "the halo effect" (p. 214) – spending off-topic time on feel-good activities unrelated to a specific curriculum topic. Their example was eating breakfast at McDonald's when photos of the golden arches were taken for later analysis. Their perspective suggests they may have a blind spot with respect to teaching the math-phobic, math-shy, and math-disinterested students. Many of these students excel when learning with their heart, sole, body and brain.

The pedagogy of student-centred, project-based and problem-posing school mathematics successfully engaged students in Sweden (Andersson & Ravn, 2012; Andersson & Valero, 2016) and in the UK (Boaler, 1998; Boaler & Selling, 2017). In heterogeneous working groups, both innovations connected students with the out-of-school world by their gaining authentic experiences with Western mathematics not found in conventional school mathematics classrooms. Students learned, used, or tweaked what mathematics was known to them in order to get an everyday job done or puzzle solved. Simply put, the Swedish and UK students generally investigated Platonist mathematics on a need-to-know basis in a context determined by the job or puzzle.

Conventional school mathematics tends to be "Detached and taught in isolation, [pure] mathematics loses many of its attributes as an enormously important part of our society, culture, and science and the students lose their ability to handle complex situations where mathematics is in action" (Andersson & Ravn, 2012, p. 322). Andersson collaborated with a teacher colleagues of hers during a one-semester research project "to bridge the gap between students' experiences in society and the mathematics classroom" (Andersson & Valero, 2016, p. 200).

Three project blocks were introduced into an upper secondary social science program's mathematics course established by the Ministry of Education. The three projects were: (a) Making Your Dreams Come True (financial literary), (b) Newspaper Flyer Workshop on Critical Argumentation (how numbers can manipulate readers), and (c) Students' Ecological Footprints on Earth (an integration of statistics into environmental science). Explicit attention was given to the "societal background and critical mathematics content" (Andersson & Valero, 2016, p. 209) for all three project blocks; in other words, for mathematics-in-use and mathematics-in-action. The lessons were typically open-ended activities but structured to give enough guidance to senior secondary students. For example, their directions for the project “Newspaper Posters with Mathematics Argumentation” began this way:

> The task of today is, in small groups, to create a number of newspaper posters that hit people, engage people, arouse curiosity, reflections and/or emotions – with a mathematical content! The goal is for you to acquire insight into how big the penetrating power of numbers can be in advertisements and newspaper articles. There are 54 articles in “Convention on the Rights of the Child.” Choose the one that interests you the most and focus on that specific one. (p. 208)

Some insights into (a) the Swedish project, (b) the spectrum of students’ cultural self-identities with respect to mathematics (Table 1), and (c) the effect of high-stakes standardized tests; all came from assessments composed by students. For example, Henrik chose to replace one of the three suggested projects and do a geometry project instead. He posed the problem, “Why is a milk packet [Tetra Pak] shaped in the way it is and what calculations has Tetra Pak performed to create such a good product?” (p. 220). Prior to taking this course, Henrik was proficient in mathematics like a math-interested student, but his experiences with conventional classes caused him to be a math-disinterested student overall, according to his own self-assessment. Here is his assessment of his experience with the innovative course (Andersson & Valero, 2016):

> I am very happy with the semester and I feel I have achieved the best I could. The most interesting and instructive parts were the projects and theme work. Then, it felt really realistic and meaningful, because we not only worked with facts but actually used it to create something new and creative. I took the national test last week, and that is really not my favourite and I performed quite bad—and lost some of my interest and motivation for mathematics. (p. 220)

The Tetra Pak project was particularly meaningful because Henrik posed the problem himself. To support students wandering into new territory requires teachers to expand their role into being a student of mathematics along with students (Meyer & Aikenhead, 2021b). Accomplishing something authentic with mathematics is a prime example of mathematics-in-use. To assure that students engage with the nature of mathematics, hold a follow-up class discussion concerning the societal context and critical perspectives of the particular mathematics.

Boaler and Confer (2017), along with several other studies, confirm that standardized exams can do damage to some students, such as changing their feelings that they can learn math (a *growth mindset*) into their feelings they are not good at math (a *fixed mindset*) (Boaler, 2016).

And a final observation, Henrik changed his categorization back and forth for his self-identity with respect to mathematics. This shows the power that such exams have on many students. Nevertheless, Henrik did learn something significant about his Swedish culture’s package industry, and in doing so, participated meaningfully in workplace mathematics.

Andersson and Valero’s (2016) conclusion that legitimated alternatives to Platonist school mathematics, is also evidenced in Boaler’s (1998) extensive three-year teaching project, with its follow-up longitudinal research eight years later, reported by Boaler and Selling (2017). They investigated the long-term effects of *active* versus *passive* mathematics instruction.

In two schools in England, Phoenix Park and Amber Hill, a cohort of students were taught mathematics very differently for Years 9 to 11. All school subjects taught at Phoenix Park stressed an active engagement in: “the construction of conceptual understanding” (Boaler and Selling, p.

79), a mathematics “identity formation” (p. 81), taking on the roll of “students and doers of mathematics” (p. 81), “learning for life” (p. 91), “heterogeneous grouping” (p. 80), and “open-ended projects that the [four] teachers had designed, and in which they introduced new content only when students needed the knowledge to move forward in their projects” (p. 80).

On the other hand, all subjects taught at Amber Hill stressed their usual passive learning: the teachers introduced the topic, the students practised the problems in the textbook, the teacher corrected the students’ work. The schools were located in the same neighbourhood that had a fairly low social economic status.

In both schools, Boaler “followed entire cohorts of students (n = 290) through their mathematics classes for three years, from age 13 to 16” (Boaler and Selling, 2017, p. 79). Four major conclusions were evident (Boaler, 1998; Boaler & Selling, 2017):

1. Although the students were introduced to similar mathematics content, which was taught by equally qualified teachers using the same national curriculum, the study showed that the two groups of students had engaged in different practices as they were learning mathematics. Through engaging in different learning practices, each group of students learned to engage with mathematics differently.
2. The students at Phoenix Park scored at significantly higher achievement levels than the Amber Hill students on a range of assessments, including the UK’s national examination in their third year. This, despite the fact that the students had scored at the same levels on standardized tests three years earlier at age 13. The Phoenix Park students also scored at higher levels than the national average, despite being at significantly lower levels when they entered Phoenix Park.
3. The equitable nature of the Phoenix Park approach was apparent: analyses of examination scores showed that there were no achievement differences by gender, ethnicity, or social class; an unusual and important achievement for a school. At Amber Hill, typical patterns of social class difference emerged.
4. One of the main conclusions of the initial study was that the two approaches gave students opportunities to develop different identities as students, with about 80 percent of the Phoenix Park students developing identities as active mathematics users, and the majority of the Amber Hill students developing identities as passive receivers of knowledge.

Of much greater significance, perhaps, were the results from the follow-up study conducted eight years after the students left their schools. “Did the differing forms of identity and expertise that students had developed at school persist into their working lives and impact their use of mathematics in life?” (Boaler & Selling, 2017, p. 82). Young adults who had attended each school (Amber Hill 181) and (Phoenix Park 107) were located and 10 from each group were interviewed; using two balanced samples based on several factors.

Rich and lengthy results arose around many topics, including:

- *learning as identity formation* (e.g., “for adolescents who are learning to see themselves as people who make decisions, have ideas, and act with agency, these passive forms of engagement may present a conflict.” p. 83);
- *employment and socioeconomic opportunities* were significantly greater for the Phoenix Park graduates;

- *using mathematics in life* (e.g., “The Phoenix Park participants appeared to have moved seamlessly from their mathematics classrooms into the mathematical demands of the workplace, whereas the Amber Hill participants did not” (p. 90). They “communicated frustration with their school maths approach and dismissed most of school mathematics as irrelevant to their work and lives” (p. 97); and
- *identity and expertise* (e.g., “At Phoenix Park, the teachers did not focus only upon mastery of content; their goals were much bigger. They wanted to develop inquiring, problem-solving, and responsible young people” (p. 93). This contrasted with Amber Hill where teachers developed “authority dependent students who often could not figure out problems themselves at work).

With respect to *mathematics-in-use* specifically, 10 out of 10 Phoenix Park graduates found “school mathematics helpful …[in a] seamless transition into workplace mathematics; whereas, 10 out of 10 Amber Hill graduates spoke to an irrelevant “distinct disconnect” (p. 90).

3.1.5 Employment and Mathematics-in-Use

The 21st century has substantially changed mathematics out-of-school requirements for both non-STEM and STEM employment (Borovik, 2016), of which the former is of interest here. “New patterns of division of labour have dramatically changed the nature and role of mathematical skills needed for the labour force and correspondingly changed the place of mathematics in …mainstream education” (p. 347). “Nowadays mathematics (including many traditional areas of abstract pure mathematics…is used in our everyday life thousands, maybe millions, of times more intensively than 50 or even 10 years ago” (p. 349).

However, this technology, such as smart phones that can read and solve school mathematics problems, has highly sophisticated mathematics hidden within the devices. What used to be mathematics knowledge and proficiencies in a mathematician’s head, have now been moved into computer systems: out of sight, out of mind. “Studies of the actual demands of everyday adult practices reveal that most occupations involve only a low level of mathematical content and expose the disparate natures of everyday and school mathematics” (Gainsburg, 2005, p. 1). Simply put, the gap between in-school and out-of-school mathematical practices has widened so much it has become a crisis of relevancy for the math-disinterested, math-shy and math-phobic students – the 70 percent graduating from secondary schools. For many students, lower and upper secondary mathematics is a formidable rite of passage into adulthood; nothing more.

Gainsburg (2008) concluded, “Teachers would undoubtedly profit from development activities that engaged them in co-constructing an understanding of what makes a classroom task ‘real’” (p. 216). Section 3.1.4 addressed this issue by showing how difficult a task it was for future mathematics teachers (Nicol, 2002). Research by university mathematics educators’ needs to address that task. Unfortunately, Gainsburg appears to join the PISA proponents who pay attention to only the highly abstract declarative mathematics by focusing on “teacher beliefs about how to help different kinds of students learn mathematics” (p. 199), when the more fundamental problem would seem to be the curriculum teachers are forced to follow, with its misplaced sense of rigor and its lack of procedural mathematics found contextualized in the real world.

3.2 Platonist Mathematics-in-Action[7]

In an appropriate subsection entitled "Mathematics and Power," Greer and Skovsmose (2012) introduce their book edited, *Opening the Cage: Critique and Politics of Mathematics Education*, with the following statement: "It is arguable that one of the greatest shortcomings of mathematics education is that …scant attention is paid to the societal effects of activities in which mathematics is instrumental" (p. 12). They quote D'Ambrosio (2010, p. 51) to define the territory in general and to suggest one reason why mathematics enjoys such a high status: "It is clear that mathematics provides the foundation of the technological, industrial, military, economic and political systems and that in turn mathematics relies on these systems for the material bases of its continuing progress" (p. 51). As midwives to artificial intelligence, mathematicians delivered surveillance capitalism to our world (Blumenthal, 2018). Mathematics is active in society's "predatory lending practices, how companies pursue young people to borrow money" (Andersson & Valero, 2016, p. 209).

Mathematics' ubiquitous presence "across all sectors of society much of its hidden entanglement in human affairs is for the good, *benefiting humankind*. However, …the near invisible and often unnoticed presence of mathematics means there is all the more reason to subject it to an *ethical audit*" (Ernest, 2019, p. 9, emphases added). It can uncover "corporate designed Apps to conceal car pollution, such as the Volkswagen fraud" (p. 11).

Mathematical modelling is a predominant process for creating the mathematical language of algorithms to understand some specific aspect of the real world. Yasukawa, Skovsmose and Ravn (2016) describe four general perspectives from which to interrogate mathematics modelling to help one critically examine modeling's ubiquitous presence in society.

1. A *descriptive* perspective: Models represent something, such as a complex irrigation system on large, tall, vertical rice paddy terraces in the Philippines. Mathematical models are powerful but they can be deceptive: "They present aspects of reality which can be confused with reality itself" (p. 85). An ethical audit would address, "What it includes and excludes …Whose reality counts?" (p. 86). For example, the rice paddy terraces modeling would not work at first because it excluded "the ethic of cooperation among farmers" (Aikenhead, 2017, p. 133).
2. An *inscriptive* perspective inquires into the beliefs of the mathematicians creating the algorithm. This is how White male biases were detected in mathematical modelling.
3. A prescriptive perspective relates to making a decision, which is never value neutral even though airline companies claim it is because a computer, not a person, made the decision to bump you off your flight.
4. A *subscriptive* perspective is about the effect of modelling on the general public (Yasukawa, et al, 2016):

    > As mathematical modeling becomes pervasive, we at the same time subscribe to the general process of making our life world measurable and calculatable. It appears ideal for many politicians to apply the 'regime of justification through numbers' to silence

[7] As defined previously, the phrase "mathematics-in-action" used in this article is a subset of Greer and Skovsmose's (2012) "mathematics in action" (p. 4), in order to clarify different types of content for the mathematics classroom.

> critique or hide the real decision-making process when the mathematical model is put into action. (p. 94)

An excellent teaching resource for examples of modeling that high school students could investigate from all four perspectives is O'Neil's (2017b) *Weapons of Math Destruction*.

3.3 The Culture and Nature of Platonist Mathematics

The internal features and characteristics of Platonist mathematics are often related to the actions or inactions taken in its name by others (e.g., section 3.2). So much has been written on this topic by the critical mathematics education (CME) group, that in the context of this present article, only a short and targeted synopsis is offered here.

The fundamentals of Platonist mathematics can be based on these ontological, epistemological, and axiological axioms (Linnebo, 2018): (a) "Existence: There are mathematical objects;" (b) "Abstractness: mathematical objects are abstract;" and (c) "Independence: Mathematical objects are independent of intelligent agents and their language, thought, and practices" (website quotes).

From this absolutist stance on the philosophy of mathematics, a characterization of Platonist mathematics was deduced, which includes: value-free, non-ideological, acultural, purely objective in its use, universalist in the sense of being universally true, and therefore, the only acceptable way of mathematizing. "However, my argument is that the choice of an absolutist philosophy of mathematics is itself a values-based choice" (Ernest, 2016a, p. 211) and subsequently any derived statement "is a consequence of this initial choice. Thus it cannot be claimed that mathematics is ethics free purely on the grounds of logical necessity" (p. 212). Nor can one claim it is free of ideologies, such as purism, quantification, universalism, objectivism, and rationalism (Ernest, (2016a,b).

The above cluster of presuppositions and others have been analyzed thoroughly and critically by many CME scholars (e.g., Bishop, 2016; Ernest, 1988). They conclude that this characterization of Platonist mathematics is simply a façade or propaganda to legitimize their highly abstract mathematics. And the CME scholars are in agreement with the following more accurate cluster: Platonist mathematics is value infused, ideological, cultural, non-objective[8], and its truthfulness is inversely proportional to its accuracy at describing reality. The famous American mathematician, Reuben Hersh, when asked, "Is mathematics objective?" replied, "It is objective from the point of view of the person who must learn it or apply it. It is however internal to culture and society; so in that global sense of humanity as a whole, it can be called subjective. Its objectivity is in its transpersonal intersubjective societal status" (Sriraman, 2017, p. 9). Einstein (1921) summarized mathematics' accuracy succinctly: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" (website quote).

The existence of abstract mathematical objects seemed to me to be an oxymoron. I posed the ontological question, "What are they?" I found Sriraman (2004) helpful, where he explored the psychology research on mathematicians' creativity when they develop a research question. He

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[8] In the context of objectivity versus subjectivity, the idea of academic objectivity in general was rejected with reasons, in a published piece entitled "Objectivity: The Opiate of the Academic?" (Aikenhead, 2008).

found six different approaches identified. One is called the mystical approach, which means that a mathematician's "creativity is the result of divine inspiration or a spiritual process" (p. 22). Setting aside speculation over Ancient Greek mythology, I changed my question to: "How can we explain their concrete appearance?" Anthropologist Hall (1976) had a rational explanation, in my view: Plato's philosophical concept "purity of mind" (p. 192) and Plato's axiom that the universe is made up of abstract mathematical objects, "amounted to an intellectual mirage: *What has been thought of as the mind is actually internalized culture*" (p. 192, original emphasis). Thus, Plato's source of his abstract mathematical objects is his culture, thus causing Platonist mathematics to be fundamentally cultural rather than mystical. And therefore, as a corollary perhaps, a belief in abstract mathematical objects identifies Platonist mathematics as spiritual. This conclusion arose as a very unexpected dichotomy: Platonist mathematics: either cultural or spiritual; the choice is yours. Phillips (2009) offers a way to choose spiritual.

The discussion above pertained to the *internal* human dimensions of Platonist mathematics' cultural content, its internal cultural identity so to speak. As defined in sections 3, the *external* human dimensions of Platonist mathematics are comprised of two aspects:

1. *mathematics-in-action*[9] is preoccupied mainly with power and politics (section 3.2) found in, for example, contexts in which Platonist mathematics is highly complex, too abstract to teach its highly sophisticated algorithms of artificial intelligence, for instance; but for which public issues are not properly identified and discussed in school mathematics classes. "A further area of values is that of ethics, concerned with the good and right treatment of other humans, as well as all living things and the environment" (Ernest, 2016a, p. 203). Greer and Mukhopadhyay (2012) assessed this situation this way:

   > [P]eople generally are ill-prepared to react critically, and with agency, to these circumstances and are underserved in this regard by their education, and by forms of discourse within society. As a consequence of this lack of critical agency, people are subject to many forms of control, resulting in a combination of powerlessness and uncritical compliance. (p. 233)

   The mathematics curriculum itself is another example of the hegemony of Platonist mathematics into the lives of students and teachers.

   Simply put, mathematics-in-action includes both the social/political/ethical aspects of pure Platonist mathematics and Platonist mathematics in social/political/ethical issues. "[C]ritical mathematics education proposes conditions in which students become critical of the role of mathematics in society" (Sriraman, 2016, p. x).

2. *mathematics-in-use*, defined in section 3.0 and explored further in section 4.0, pertains to the real-world, occupational, workplace, everyday contexts related to mathematizing in which students can participate to understand contextualized concepts, skills, and strategies (procedural knowledge) associated with, derived from, or analogous to, the Platonist (declarative) content in a STEM-oriented curriculum (e.g., arguing over the results of political polls, thereby critiquing how the mathematics was done).

[9] A reminder from a footnote in section 3, that "mathematics-in-action" differs from Greer and Skovsmose's (2012) definition of "mathematics in action" (p. 4) because their definition encompasses *both* "mathematics-in-action" and "mathematics-in-use."

4 Mathematics Curricula as Political Documents

This section addresses Platonist mathematics' power in education by challenging its sacrosanct curriculum, whose high status is buttressed by both government standardized examinations and the folklore of "This is the way we've always done it." Its power includes being the essential gatekeeper for graduation from high school. As a result, in countries where colonization has occurred, a disparity occurs between the high graduation rates of non-Indigenous students and the much lower graduation rates of Indigenous students (Duchscherer, Palmer, Shemrock et al., 2019). This is incontrovertible evidence that high school curricula are, for the most part, major contributors to systemic racism in those countries. This tars the high school mathematics curriculum as a racist document. Similarly, mathematics curricula help maintain a country's social class structure (Jorgensen, 2016).

Andersson and Ravn's (2012) chapter provides evidence for the fact "that the nature of mathematics does not force upon us an abstract and isolated approach to the learning of mathematics" (p. 322). What was forced upon us, however, was a lethal combination of prestigious university mathematicians living in a political era of progress through economic development and expansionism of the early 19th century. "[E]litist Platonists, channeling the ancient Greeks' belief in the superiority of pure abstract thought (Plato's world of ideas)" (Aikenhead, 2017, p. 89), who insisted that the first public school mathematics curriculum be defined as decontextualized content only; that is, a formal, highly abstract mathematics.

Therefore, first and foremost, it is important to understand why public-school mathematics curricula today are *political* documents.

4.1 The Political Origin of Today's Platonist School Curriculum

The first public-school mathematics curriculum was established in most Western countries about 170 years ago, fraught with political subterfuge (Aikenhead, 2017; Ernest, 1991; Nikolakaki, 2016; Willoughby, 1967). The political scene in the U.S.A. was sketched by Nikolakaki (2016).

> At the beginning of the 19th century, numbers were connected with the idea social progress. …One way to measure the value of the democratic institutions was the use of political arithmetic. Numbers were considered to be objective since they have no subjective value elements, and they were more convincing arguments than opinions and rhetoric. The ability of arithmetic not only to produce progress but also to measure it explains why arithmetic held this position in the educational system. (p. 279)

Prestigious university mathematics departments debated against representatives from local business leaders and factory owners over what the curriculum content would entail: abstract Platonist content (declarative knowledge), or content directly useful to the commercial interests of having educated workers to run the new factories being built (both procedural and some declarative knowledge)?

This ongoing politicking ended with the elite university mathematicians winning. They not only decided what would be acceptable content; but of greater importance they decided what would be silenced (until critical mathematics education appeared on the academic scene). This process of exclusion by the university mathematicians was described as "fallacious reasoning, rhetorical tricks, and political deception" (Aikenhead, 2017, p. 89). "[T]he values of the absolutists are smuggled into mathematics, either consciously or unconsciously, *through the definition of the field*" (Ernest, 1991, p. 259, emphasis added). Aikenhead (2017)[10] summarized it this way:

> The Platonists drew on a binary, "logical versus irrational," …in order to construct their own theoretical binary: "formal mathematical discourse" versus "informal mathematical discourse" (Ernest, 1991, p. 53). They arbitrarily assigned their definition of school mathematics to the formal discourse category; for example, deductive proofs, theorems, and the scientific application of Platonist content. (pp. 89-90)

The *informal* mathematical discourse consisted of the culture and nature of Platonist mathematics (Skovsmose, 2016), which was deemed irrational. Therefore, it was excluded from the curriculum.

The Platonists' covert rhetorical deceptions hid from the public any thought of an alternative to a Platonist school mathematics (until critical mathematics education appeared). Pais (2012) explained a social purpose to their deceptive strategy:

> This concealment is essential to maintain the role of school as an ideological state apparatus. Seeing school as a place free of ideology disables bringing ideological struggles to school. All enterprises undertaken by teachers to unmask the "invisible" ideology are immediately accused of being ideological acts. In this way, the dominant ideology ensures that no ideology is present in school except, of course, the dominant one. The dominant one is precisely the one that presents itself as ideologically free, by positing the importance of mathematics as knowledge and competence [for citizens]. (p. 70)

I would suggest there was also the personal purpose of professors ensuring a pool of mathematics students properly prepared to become mathematics majors in Departments of Mathematics. And finally, Pais (2012) also pointed out, "the ideological injunction that you really need mathematics to attain [full] citizenship" (p. 65). This tends to be repeated many times by teachers justifying their subject to their students. Many of them know, however, the claim is an exaggeration; for example, "Really? Think about people you know. Aren't there many who do not have a solid grounding…in mathematics that are living full and productive lives? Isn't it offensive to tell such people that they are dysfunctional?" (Greer and Mukhopadhyay, 2012, pp. 239-240).

As a result, no significant curriculum change has occurred over the last 170 years (Aikenhead, 2017), a testament to how Platonist school mathematics became the "culturally established norm" (Andrews, 2016, p. 13). This means that a significant alteration to the mathematics curriculum requires altering a country's established norm.

How well did the university mathematics professors' 19th century win work out? As reported above, their excessively abstract and crowded curriculum of today was assessed by polling (Ipsos, 2005) young adults (ages 20 to 39) of whom 37 percent hated mathematics while only 23 percent found it their favourite subject. These data indicate that the decision was an acceptable one for a significantly large proportion of students, but the win succeeded well with a small minority. My claim is that the time has come to revisit that 19th century decision, with a clear understanding that life in the 21st century is much different.

The Platonists' call for an Ancient Greek type of rigor serves as their mythical armour to repel those who would reform their curriculum. To be sure, however, it is a curriculum that

[10] A much more detailed summary account can be found in Aikenhead, 2017, in the section "A Hidden Platonist Agenda" (pp. 89-90).

properly serves, by and large, the math-oriented, math-curious, and math-interested; about 30 percent of graduating students.

4.2 Politics of School Mathematics Innovation Today

In their edited book, *Opening the Cage: Critique and Politics of Mathematics Education*, Skovsmose and Greer (2012b) and many other mathematics educators tend to assume that excellent *research* and well-argued *policy* directly influence classroom *practice*. My 50 years as a research science educator and now a mathematics educator have taught me that both research and policy are necessary, but they are woefully insufficient. They lack political action. All three (research, policy, and practice) normally act quite independently (Aikenhead, 2020), but they can be energized in synchrony with successful politics, as depicted in Figure 1. When there is political will, the political gear will move, and so will research, policy, and practice along with it. Remove the political gear and the three revert to having a slight influence from afar.

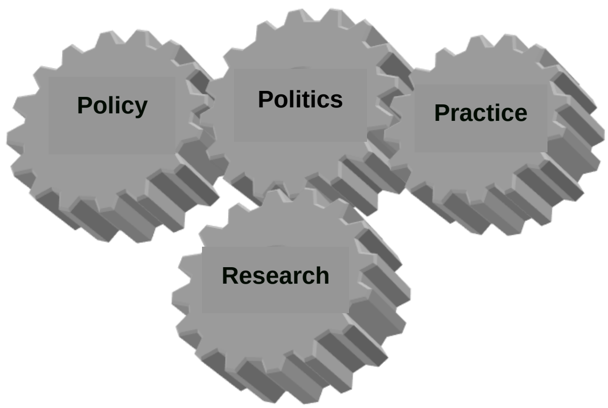

Figure 1

As a way to enact Figure 1, Aikenhead (2020, p. 685):

1. Describes an overarching, down-to-earth *political strategy* for science and mathematics educators, along with a case study to illustrate its effectiveness; and
2. Documents an accompanying *educational strategy* that could be retuned to the characteristics of one's education jurisdiction.

Both strategies are clarified below, following this short description of their context. The case study began in the Province of Saskatchewan about 13 years ago when the Province's curriculum was changed so science classrooms became enhanced with Indigenous perspectives on Mother Earth. Today this innovation continues to help lead a national trend for the next few generations of science and mathematic education teachers (Conference Board of Canada, 2020). The science curriculum had previously been changed in the 1980s to include a science-technology-society-environment emphasis, which today would be recognized as the equivalent to Greer and Skovsmose's (2012) definition of "mathematics in action" (p. 4), a theme discussed above (section 3.1.1).

Aikenhead (2020) also includes a much shorter case study that illustrates the same political and educational strategies in action for mathematics education. This Saskatchewan innovation includes both Indigenous and mainstream culture-based school mathematics. The "mainstream culture-based mathematics" (in press) refers to the workplace, real-world, functional, school mathematics discussed in this present article.

The *political strategy* followed in the two case studies is not exactly a new one. It is taken from the Platonists' playbook and is informed by years of personal experience. Aimed at the powerful detractors of innovative change in education, the political strategy is simply: "Do it to them before they do it to you" (Aikenhead, 2020, in press).

The 19th century representatives from local business leaders and factory owners perhaps failed to implement this strategy against the elite universities' mathematics departments who created the dichotomy formal versus informal mathematical discourses, allocating Platonist mathematics to the former while burying the latter out of site. In other words, the Platonist have already done it to us, and now we know the opposition's playbook.

The four-point political strategy requires: (a) political savvy in terms of knowing one's educational jurisdiction, (b) solid research evidence documenting what is wrong with the status quo, (c) salient reasons to support the innovation proposed, and (d) solid research evidence on what can be anticipated from the innovation.

The solid evidence comes from implementing a four-point *educational strategy* for change: (a) publish an evidence-based innovative *policy* article; (b) conduct and publish a *research* or *research & development project* that implements the policy to establish the innovation's credibility; (c) then begin the *politics* of supporting a scaled-up implementation, which can eventually (d) encourage *practice* within classrooms of the educational jurisdiction.

Knowing that the innovation is the ethically right thing to accomplish is essentially immaterial beyond one's personal motivation. What leads to success is for the innovation to take a political route. And a political route is highly dependent on the specific country, province, or state; and on the individual innovators themselves. Success requires a team of collaborators over time, along with unanticipated positive coincidences. All the descriptions in the Saskatchewan case study constitute public knowledge and are documented in Aikenhead (2020).

4.3 Specific Political Action to Take

To carry out the "Do it to them before they do it to you" strategy in a savvy manner (and they *will* do it to you), the following are offered as suggestions to be modified or ignored, because each country has its unique cultural and political features related to mathematics education (Andrews, 2016).

1. Know your strongest detractors and identify supporters whose qualifications, political status, sports and entertainment status, or public image status would be helpful to your critical mathematics education (CME) innovation.
2. Your detractors will call you and your innovation derogatory names (e.g., radical, undermining democracy, where's the rigor? soft math, the *very* new math, enemies of the economy, etc.). Names position one in the public sphere. Decide ahead of time on what names will best label your detractors. For instance, the most absolutist detractors may earn the moniker "member of the Pythagorean cult." The general public does not require historical knowledge of Pythagoras. They only need to hear the word "cult." Position your detractors in the public sphere *before* they position you.

3. Some of your detractors will include STEM enthusiasts. Therefore, accentuate how your innovation will benefit the 30 percent by promoting the International Baccalaureate in Mathematics for these STEM-oriented students. In other words, “out rigor” your detractors.
4. Cultivate connections with people in the public media. Many are always looking for controversy, debates, and documentaries. Cleverly timed press releases from the CME community are always helpful to one’s cause. In politics, a public presence is more important than published articles, although the latter are absolutely necessary as a paper trail for politicians and bureaucrats to read.
5. Begin personal discussions with key people at key post secondary institutions to negotiate what non-STEM programs they offer would accept a culture-based school mathematics innovation as an acceptable mathematics prerequisite.
6. Every three years, the media delights in reporting on the controversy when the OECD releases its PISA mathematics scores. Make sure that your media contacts are primed ahead of time with critiques of PISA; for example, it is “a political project masquerading as an educational tool” (Sjøberg, 2015, p. 1). For a detailed critique of the science tests and how they are used, see Sjøberg (2016) and Sjøberg and Jenkins (2020). These critiques apply to the PISA mathematics tests, as well. A critical question posed by Sriraman (2016, p. xi) is, “Are the types of competencies being touted by PISA really essential or ‘critical’ to the schooling and societal needs of this country?”

   Ask key politicians why they are only interested in students’ scores, when there are extensive data on: (a) equity (e.g., immigrants compared with non-immigrants, high versus low neighbourhood social economic status schools) and (b) frugality – commonly known as the bang for your buck (e.g., money spent on education compared to the student mathematics scores). Here is what you should know. The PISA 2012 test report (Aikenhead, 2017):

   > ranked Finland, Estonia, and Canada at 11, 12, and 13 (respectively) on the basis of student performance alone. These rankings were behind many East Asian countries. The much more sophisticated three-factor analysis resulted in Finland, Estonia, and Canada being tied for *the top* PISA ranking number 1. (p. 125, original emphasis)

   The politicians’ reason for ignoring the more rational three-variable analysis is simple: when the marks are based on test scores alone and a country’s marks are low, the public blames the teachers. But when three variables are considered, the public tends to blame the politicians for not spending enough money on education or they are not spending it judiciously.

   Make sure your media contacts understand the mathematics *procedural* knowledge of confidence limits, but not necessarily their *declarative* (abstract) form. Then the media people will understand that countries can be tied in rankings even though they have different averages. When your country is tied with four other countries, it looks better and it is rational.

   The most intensive series of investigations into PISA scores has been the concern over why Finland’s scores have been consistently superior. Andrews (2016) summarized the many countries’ attempts and failures to duplicate Finnish pedagogy in mathematics, and he concluded: “In short, the available evidence shows that culture may play a more significant role than pedagogy in determining the educational achievements of [a] country” (p. 19).

Pais (2012) described the politics of PISA to control the mathematics content taught in many countries:

> [W]e are living in an age of measurement in which pressure is put on teachers, schools, and governments to increase educational results measured by mass-scale comparative studies such as … the OECD’s Programme for International Student Assessment (PISA). These international, comparative studies are to an increasing extent brought into the political sphere, placing pressure on national governments to regulate their educational systems according to the standards stipulated by those tests?” (p. 74)

The commercial forces of globalization need to be on the political agenda of the CME community.

7. The popular PISA discussion, “Are my country’s mathematics scores trending up or down from previous years?” is addressed by the OECD (2019) in terms of “statistically significant differences,” an abstract concept to be sure. For your media contacts, develop for them the concept of “significant *educational* differences” – how much would it cost to raise your country’s average by 5 points on the test’s 600-point scale? Is it worth the millions of dollars/Euros/pounds? And do you know how closely your mathematics curriculum corresponds to the PISA test’s content regime. Different countries stress different proficiencies and a country’s culture, with respect to education, matters (Andrews, 2016).
8. Locate important non-STEM people who have succeeded in the eyes of the public. Interview them on the topic of how or what mathematics, if any, helped them succeed. This is powerful information to counter the hypothetical claims of the Platonist enthusiasts about rigorous mathematics leads to adult success.
9. In section 3.1.1, you were challenged to do a personal survey. An excellent political project is to conduct an academic survey on a large scale. Include items for agreement or disagreement by respondents, for instance: (a) “Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement” (National Council for Teachers of Mathematics, 2000, p. 4), (b) All citizens must have a solid grounding in mathematics to function effectively in today’s world; or (c) There are many people who do not have a solid grounding in mathematics, but who are living full and productive lives.
10. The “pièce de résistance” of research projects would be testing the mathematical proficiencies of specific public groups composed of anonymous participants, such as a group of politicians, university professors, and public figures listed in item 1, just above. Test at the Year 6, 8, and 10 levels. It is of prime importance to get trustworthy evidence to back up the myth-slaying arguments the CME can develop against what Platonists often claim are the benefits of highly abstract and rigorous mathematics. Imagine the news headlines: “Centennial School’s Year 8 Students Score Higher on a Math Test than Members of Parliament.”

11. Obtain funding to organize, and publish the findings of, public interdisciplinary meetings that discuss key issues for which the CME has recently released a research report (e.g., items 8-10). Make sure the media attend. Invite high schools to send some articulate students to participate in the meetings. For example, Ernest's (2019) article, "Privilege, Power and Performativity: The Ethics of Mathematics in Society and Education" would make an excellent basis for a public forum of key public figures, the press, as well as academics. Hold it in a venue with easy access to the general public. Make a public issue out of it. His paper will be of interest to a wide range of people, from household mothers and fathers to mathematics department heads in schools and universities. Include media-based evidence in the presentation, such as Cathy O'Neil's (2017a) TED Talk, "The era of blind faith in big data must end."

And the list goes on.

5 Renewed Curriculum Content for the 70 Percent of Students Ages 12 to 18

Skovsmose and Greer (2012a) concluded, "For too many people, their experience of school mathematics is personally, emotionally, and intellectually dehumanizing. It does not have to be like that" (p. 383). They were suggesting alternatives to pure Platonist school mathematics. It is important to keep in mind, however, that pure mathematics' Platonist philosophy works well for about 26 percent to 30 percent of high school graduates.[11]

What are the most promising relationships between, on the one hand, out-of-school mathematical practices largely contextualized by both an everyday employment procedural knowledge issue and a social-political-ethical issue; and on the other hand, how mathematics might be contextualized in a school setting?

The question has generated a great interest for some time now in professional journals; for example, from Gainsburg's 2008 research article to the theme of the teachers' articles published in issue 10 of the 2020 volume of the NCTM's *Mathematics Teacher: Learning & Teaching PK-12* (e.g., Stephan et al., 2020). Unfortunately, the conversations have been highly restricted to an implicitly imposed "*formal mathematics discourse*" (see section 3.2.1) that places on pedestals a 19th century mathematics curriculum for largely STEM-oriented students, all in the name of rigor. For example, noticeable in articles published in issue 10 was the vector-like subtext that tended to direct the reader to move *from* the classroom *to* the real world, and not so much in the opposite direction. In contrast, this article emphasizes the opposite direction, in the following section based on the wisdom, "*Mathematics becomes best understood by how it is used*" (Barta, Eglash & Barkley, 2014, p. 3, emphasis added).

By learning procedural mathematics in several real-world contexts, students can then generalize by *transferring* ideas from several different authentic contexts towards a more abstract analogue of the procedural contextualized mathematics they have learned. A form of abstract mathematics is possible, but not mandatory, for a renewed curriculum pathway for the 70 percent majority of students 12 to 16 years of age.

[11] In the spirit of full disclosure, I must report that I belong to this group and that Platonist school mathematics served me very well.

5.1 Learn Your Culture by How Mathematics Is Used in It

A broad brushstroke picture of a renewed curriculum for Years 7 to 12 (in North America, 12 to 18-year-olds) could be described here from two different emphases: culture-based and humanistic school mathematics – two sides of the same coin. For reasons of brevity, this article addresses the *culture-based emphasis*. This section unfolds in two stages. First, each of the two major projects described in section 3.1.4 is compared to some defining features of culture-based school mathematics already established in sections 3.1, 3.2 and 3.3. Secondly, a module and one lesson illustrate culture-based school mathematics further.

In Boaler's (1998) research with Phoenix Park School, students proposed or took on mathematics-related problems by learning *abstract declarative knowledge* on a need-to-know basis working in heterogeneous groups. The out-of-school venue was generally the everyday world. "[S]tudents had been 'apprenticed' into a system of thinking and using mathematics that helped them in both school and non-school settings" (p. 41). It barely qualifies as culture-based school mathematics, however. More details would be helpful to determine the degree to which students explicitly connected the mathematics to their culture; drawing on content selected from sections 3.1, 3.2, and 3.3. Comparing the two schools, the researchers noted that both schools addressed the same Platonist content, but the pedagogy differed remarkably. This led them to conclude that the Phoenix Park School's success was less about *what* you teach, and more about *how* you teach. Teaching was the focus of the research project, not CME curriculum content.

Many more details were made available in Andersson and Valero's (2016) research collaborating with a teacher in one school. Their semester-long project exemplifies culture-based school mathematics by embracing both mathematics-in-use and mathematics-in-action. Drawing for a moment from section 3.3 (The Culture and Nature of Platonist Mathematics), Andersson and Valero's (2016) research could have included follow-up questions that introduced the cultural features within Platonist mathematics itself; for instance, questions that direct students' attention to relevant presuppositions, values, or ideologies inherent in Platonist mathematics. They are hidden just below the surface of Andersson and Valero's "societal background and critical mathematical content," which is certainly related to section 3.2 and perhaps section 3.1.

Moreover, the connection between out-of-school and in-school mathematics could have been tighter had some projects included an occupational role playing. For example, in Making Your Dream Come True, students could have played the role of a bank employee giving advice to a recent high school graduate, rather than casting the students in the more personal role of the graduate. These are choices made depending on one's agenda. The Newspaper Flyer project was implicitly an occupational role play in journalism, but was not cast explicitly that way. If so, teachers may have invited a local journalist into class to react to the students' posters and to be interviewed by the students about the use of mathematics in journalism.

The CME research community could serve school mathematics well by developing teacher resources. Lessons or modules could be produced by collaborating to document the procedural knowledge of community businesses and other institutions (mathematics-in-use); and to produce vignettes related to societal issues of interest to students in Years 7 to 12 (mathematics-in-action), and shedding light on the culture and the nature of Western mathematics. Include teachers on your design team and have them teach the first version to develop them further in the crucible of classroom practice. The best feedback will like come from the students.

For example, I was motivated by Skovsmose's (2016) discussion of the airline industry maximizing profits by using sophisticated mathematics "digital information and communication technology" (p. 9). My unpublished module, Airline Bumping, for 13-year-olds draws mainly on slightly edited newspaper clippings gathered over a year, and ending with the Canadian government's enacting passenger rights legislation. The contents are listed in Table 2, which require about seven daily sessions at 20 to 30 minutes per session.

Table 2. Contents of an unpublished module "Airline Bumping"

Module	Conventional math content	Math-in-use	Math-in-action	Culture of math	Indigenous perspectives
Airline Bumping Year 8	Ratio, rate, proportional reasoning. Introduction to linear relations and equations. Modelling and problem-solving. Probability based on historical data.	Airline industry contextualized mathematics. Critical analysis of media articles containing authentic data and their visual displays.	Math algorithms decide how many seats are overbooked, and who gets bumped. Government regulations. Passenger rights.	Associate emotionally with math. Mathematical algorithms are hidden from sight.	Sustainability is fundamental to Indigenous worldviews.

This is one of six modules for Years 7-12, some co-authored by teachers and some by a university mathematician. The modules are waiting to be the focus of a research project with a local school division; postponed due to COVID-19. In "Airline Bumping," the airline industry's procedural knowledge includes "load factors," which becomes "bums to seats" (journalism language), and finally "ratios" (declarative knowledge); learned in that order. The "Introduction to linear relations and equation" found in Table 2 refers to translating airline English statements into algebraic equations without manipulating them. It is an exercise to become familiar with the language of algebraic mathematics.

In one of the sections to the Year 7 module, "Math in the Everyday World," students become a beginner employee at a custom picture framing business. After their business "internship" to designing picture framing with matting is finished, their at-home project is to pick a fairly flat object that has personal significance, and design a picture framing for it. No assembly is required. Invite the business owner to send an employee the help the teacher assess the designs. He or she could be a welcomed guest to discuss the mathematics and give hints on making better designs. By the end of their project, students have mastered the addition and subtraction of fractions. All six modules will exemplify a wide range of culture-based and humanistic school mathematics.

6 An Envisioned Cultural Framework for Culture-Based School Mathematics

Bishop (1988) perceived mathematics education as providing all students with an enculturation experience – "a creative, interactive process engaging those living the culture *with those born into it*" (pp. 88-89, emphasis added). Bishop appears to foster a one-size-fits-all approach, which is at odds with the diversity of students' mathematics self-identities that range from math-phobic to math-oriented (Table 1); that is, encompassing the approximately 70 percent and 30 percent groups of students, respectively (section 2).

"Students' understanding of the world can be viewed as a cultural phenomenon (Spindler, 1987), and learning at school viewed as culture acquisition (Wolcott, 1991)" (Aikenhead, 1996, p. 8). The view of learning mathematics as culture acquisition affords an intuitive, holistic, and rich appreciation of students' experiences in a mathematics classroom (Hawkins and Pea, 1987; Wolcott, 1991). By extension, teaching can be viewed as cultural transmission. In other words, Wolcott would say that mathematics students experience cultural transmission-acquisition episodes when learning mathematics; one type of which is enculturation.

However, if enculturation were attempted with the 70 percent group, educational anthropologist Spindler (1987) might classify the episode as assimilation, which we know leads to hegemony and social injustices (Aikenhead, 2017; Ernest, 2019; Jorgensen, 2016).

Between the two extremes of enculturation and assimilation lies the cultural transmission-acquisition experience of *acculturation* (Spindler, 1987), understood as the *self-selected* modification of one's currently held ideas or customs under the influence of another culture (Aikenhead, 2015; Eisenhart, 2001). This means that a student changes a concept, belief or habit, or adds new ones to their self-identities or to their ways of knowing the world, without changing their core self-identity. A key phrase in the definition of acculturation is "self-selected modification." But self-selection can either be made freely because the change is attractive to a student, or can be made under pressure by their teacher to conform to the change. Still, it is a self-selected change, but made neither happily nor permanently. Further fine-tuning of the description for acculturation is required: (a) *autonomous* acculturation respects a student's intellectual independence, and (b) *coercive* acculturation pressures students explicitly or implicitly.

From the point of view of a student, however, the distinction, between "coercive acculturation" and "assimilation," relies vaguely on the degree to which: (a) their teacher is honestly unaware of being coercive (i.e., a case of coercive acculturation); and (b) the teacher intended to be coercively acculturating (i.e., a case of assimilation). These distinctions have direct ramifications to the ethics of mathematics education (Boylan, 2016; Ernest, 2016a, 2018, 2019; Harris, 2017; Ravn and Skovsmose, 2019; Yasukawa et al., 2016).

The triad enculturation, acculturation and assimilation, is one way to understand the cultural transmission-acquisition process that all students experience in their mathematics classes. Another way, of course, is to consider the curriculum's content: on a continuum from one extreme of being highly contextualized by local culture-based procedural mathematics; to the other extreme of being highly decontextualized, abstract, declarative mathematics. The former is mathematics content to be generalized idiosyncratically by students towards a more abstract understanding of a mathematics topic (section 5).

The following is a sketch of a cultural framework for a culture-based school mathematics curriculum:

Years K-6: Culture-based mathematical enculturation into local mathematizing for all students. Cull the current amount of content in order to give teachers time to innovate. This is what creative conscientious elementary teachers have always done. Develop rational innovative assessments to avoid the predictable negative effects that standardized testing has on students (Boaler, 2016; Ernest, 2019; Simeonov, 2016).

Years 7-9: Enculturation into culture-based adult numeracy, spatiality and logicalness (Ginsburg, Manly & Schmitt, 2006) for all students; plus: mathematics-in-use, mathematics-in-action, the culture and nature of mathematics, mathematics as a human endeavour; and Platonist enrichment for math-oriented, math-curious, and math-interested students.

Years 10-12: (a) For the large majority (70 percent): acculturation into mathematics-in-use, mathematics-in-action, the culture and nature of mathematics, mathematics as a human endeavour, culture-based and evidenced-based adult numeracy, spatiality, and logicalness; including an optional two-year enrichment apprenticeship/internship for trade enthusiasts.

(b) For the small minority (30 percent): enculturation into the culture of Western mathematics, contextualized in architectural design, engineering problem solving, and scientific explorations; enriched with mathematics-in-use, mathematics-in-action, and the culture and nature of mathematics. For example, Alrø and Johnsen-Høines (2016) in Norway offer an exemplar joint collaboration among a school, industry and a university teacher education department. The authors stress the dynamic: teach Platonist mathematics only on a need-to-know basis. For instance, the student teachers' introduction to statistics prior to meeting with their industry mentors was a disappointing failure because the need-to-know would arise until the school students began their work at the company.

Years K-6 are about students coming to know fundamental ideas about doing Western mathematics personally, at home, and in the community. In countries with Indigenous citizens, introduce aspects of their Indigenous mathematizing (Mayer & Aikenhead, 2021a,b); ideas and insights that will evolve with each engaging cross-cultural experience.

Years 7-9 continues this trend and attends to discovering latent mathematical potential that students bring to the mathematics classroom, especially for those who have been systemically marginalized in school mathematics. Students who appear they may have potential for a deep understanding of Western mathematics will receive specific enrichment in that direction.

Years 10-12 specializes in a focussed preparation for responsible, diverse, real-life adulthood; first by culling the current obsolete curriculum content; leaving time and energy for innovating with new curriculum content. A highly porous flexible interface between parts (a) and (b) should be maintained. Moving easily between the two needs to be an established protocol in all schools.

7 Conclusion

The literature cited in this article suggested there is an on-going, dynamic, and inseparable tension between: (a) what content to teach, and (b) the pedagogy for teaching it. The article sought to inform mathematics educators what is possible in these two dimensions, in order to help teachers reach a balance between the two, appropriate for a specific lesson. Equally important, however, is a third-dimension that ties content and pedagogy together in a holistic three-dimensional space: *the context*. For a large majority of students in lower and upper secondary mathematics, the current Platonist curriculum causes a perturbation through this three-dimensional space that forces teachers into a two-dimensional survival mode (content and pedagogy), leaving little or no time for teachers to innovate with the third dimension. The triad, content-pedagogy-context, is exemplified by variations within culture-based school mathematics described in this article.

What do mathematics educators need to know in order to help the 70 percent of graduating students become savvy non-STEM adults, long into the future so they can:

1. Draw judiciously on their acquired understandings of mathematics-in-use, mathematics-in-action, the culture and nature of mathematics, and abstract mathematics; to better serve and survive in their 21st century cultures?
2. Accomplish *their* personal goals commensurate with their own mathematical potential?

The critical mathematics education (CME) community has opened the conversations in support of mathematics educators including the "*informal mathematics discourses*" banished during the politics of the public school's establishment in the 19th century. My call is for the CME group to engage in the *politics* of the 21st century and revisit that 19th century curriculum decision for the benefit of the 70 percent. This article has attempted to outline how it might be done, modified according to each country's culture and politics (Andrews, 2016).

As I understood him, Borovik (2017) connected the dots plotted from his mathematician future employment data. The images that the connections produced clearly conveyed a bifurcation: one side exponentially descending into the infinity of the small; the other side had already ascended into the infinity of the large. What do mathematicians do with two contrary infinities? They recognize a crisis of global proportions: school-level mathematics has not adequately prepared the growing number of "end users" (p. 306) to survive what the "mathematical makers" (p. 306) have made disappear from the end users' consciousness. That is my interpretation.

For the record, here is what Borovik (2017) actually wrote:

> [T]he current crisis in school-level mathematics education is a sign that it reaches a bifurcation point and is likely to split into two streams:
>
> - education for a selected minority of children / young people who, in their adult lives, will be filling an increasingly small number of jobs, which really require mathematical competence (I call them *mathematical makers*); and
> - basic numeracy and awareness for the rest of the population, *end users* of technology saturated by mathematics – which however, will remain invisible to them. (p. 309)

This article strived to deconstruct this crisis sufficiently enough that critical mathematics educators could see pathways to advance out of the crisis, mainly by liberating lower and upper secondary school mathematics from its 19th century shackles so it can deal with what had disappeared from school mathematics when the Platonists took charge.

The final word on crises and the CME community rests with Skovsmose (2019):

> Crises might include an overwhelming dynamic that we seem unable to cope with, and we have to recognise the *brutality of crises*. Simultaneously, we have to recognise the *fragility of critique*, as we cannot expect that through a critical activity we will be able to adequately interpret and manage a crisis. To me the brutality of crises combined with the fragility of critique set the parameters for our *human condition*. This is also the condition for critical mathematics education. (p. 14, original emphases)

Compliance with ethical standards: Not applicable.

Conflict of interest: The author declares that there is no conflict of interest.

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