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Enhancing School Mathematics Culturally: A Path of Reconciliation

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ABSTRACT

Culturally responsive or place-based school mathematics that focuses on Indigenous students has an established presence in the research literature. This culture-based innovation represents a historical shift from conventional approaches to mathematics education. Moreover, it has demonstratively advanced the academic achievement for both Indigenous and non-Indigenous students.

Its success has exposed deep fault lines in conventional school mathematics. Many mathematics educators unknowingly embrace problematic, taken-for-granted notions about their school subject that inhibit student engagement and contribute to Indigenous students' low graduation rates. However, innovative researchers and teachers have adapted or developed culture-based teaching materials and strategies that significantly reduce the problems inherent in conventional school mathematics. As a result, these innovators' actions challenge standard curricula and instruction.

These changes coincide with another profound transformation taking place in countries with Indigenous citizens. In response to having kidnapped Indigenous children and held them in residential schools in an attempt to rid them of their Indigenous self-identities, Canada's federal government apologized in 2008 and established a process of reconciliation (Truth and Reconciliation Commission, 2016) for all Canadians.

Accordingly, this article has two main goals: to (a) *illustrate* how critical analysis can help educators decide which taken-for-granted notions about school mathematics should continue to be embraced and which ones should be updated because they interfere with the engagement and achievement of most Indigenous students and a majority of non-Indigenous students and (b) identify concrete ways in which mathematics educators, researchers, and curriculum writers can help enhance school mathematics by drawing upon how mathematics is used in both mainstream and Indigenous cultures and in a way that simultaneously promotes both academic achievement and reconciliation.

These goals lead to the following questions answered in this article:

1. What conventional taken-for-granted notions impede student achievement?
2. Which of these conventional notions continue to be held by many innovators who have enhanced school mathematics culturally?
3. Which innovative taken-for-granted notions improve student academic achievement?
4. Exactly how do researchers or teachers "see" school mathematics content "embedded" in an Indigenous artisan handwork or activity?
5. Which notions found in conventional school mathematics continue to serve students' interests?

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6. How can mathematics curricula mitigate systemic racism and support reconciliation?
7. What specific actions can mathematics teachers, researchers, teacher educators, and curriculum writers take to regenerate what is essentially a Victorian-era, 19th-century elitist curriculum (for Grades 5 and higher) to a 21st-century curriculum in harmony with today's realities?

The article advances a pluralist mathematics perspective that makes explicit the cultural nature of school mathematics within an Indigenous cross-cultural framework of respect and collaboration. Mathematics' cultural nature becomes both a context of instruction and content expressed in a curriculum.

RÉSUMÉ

L'enseignement culturellement sensible des mathématiques à l'école, centré sur les étudiants autochtones, a maintenant une place établie dans la littérature de recherche. Cette innovation fondée sur la culture représente un changement historique par rapport aux approches conventionnelles en enseignement des mathématiques. De plus, il est démontré qu'elle a favorisé le succès scolaire des étudiants autochtones et non autochtones.

Son succès a mis en lumière de graves lacunes dans l'enseignement traditionnel des mathématiques à l'école. En effet, de nombreux enseignants de mathématiques adoptent sans le vouloir des notions problématiques, tenues pour acquises, de leur matière scolaire, qui gênent la participation des étudiants et contribuent aux résultats médiocres des élèves autochtones. Cependant, des chercheurs et des enseignants innovateurs se sont adaptés et ont mis au point des ressources pédagogiques et des stratégies d'enseignement qui tiennent compte de la culture des étudiants et réduisent la portée des problèmes inhérents à l'enseignement conventionnel des mathématiques à l'école. Ce faisant, ces innovateurs remettent en question les curriculums et les programmes d'enseignement standards.

De tels changements coïncident avec une autre transformation qui a eu lieu dans d'autres pays qui ont une population autochtone. Après avoir kidnappé des enfants autochtones et les avoir internés dans des pensionnats afin de les priver de leur identité autochtone, le gouvernement fédéral du Canada a présenté des excuses officielles en 2008 et a entrepris un processus de réconciliation (Commission de vérité et réconciliation, 2016) pour tous les Canadiens. Cet article a donc deux objectifs principaux : (a) *illustrer* comment l'analyse critique peut aider les enseignants à décider quelles sont, parmi les notions tenues pour acquises en enseignement des mathématiques à l'école, celles qu'il faut garder et quelles sont celles qu'il convient de réviser parce qu'elles entravent la participation et le succès scolaire de la plupart des élèves autochtones ainsi que ceux d'une bonne partie des élèves non autochtones ; et (b) identifier des moyens concrets pour que les enseignants de mathématiques, les chercheurs et les auteurs des curriculums puissent contribuer à améliorer les mathématiques à l'école en tirant profit des façons dont on se sert des mathématiques aussi bien dans la culture majoritaire que dans les cultures autochtones, et ce de façon à promouvoir à la fois le succès scolaire et la réconciliation. Ces objectifs m'amènent aux questions suivantes, auxquelles je réponds dans l'article :

1. Quelles sont, parmi les notions conventionnelles tenues pour acquises, celles qui entravent le succès scolaire des étudiants?
2. Lesquelles de ces notions conventionnelles continuent d'être soutenues par de nombreux innovateurs qui ont contribué à l'avancement culturel des mathématiques à l'école?
3. Quelles sont, parmi les notions conventionnelles tenues pour acquises, celles qui favorisent le succès scolaire des étudiants?
4. Comment les enseignants et les chercheurs voient-ils les contenus mathématiques à l'école comme « parties intégrantes » du travail ou des activités d'un artisan autochtone?

5. Quelles sont, parmi les notions conventionnelles tenues pour acquises en enseignement des mathématiques à l'école, celles qui continuent de servir les intérêts des étudiants?
6. Comment les curriculums de mathématiques peuvent-ils atténuer le racisme systémique et favoriser la réconciliation?
7. Quelles actions les enseignants, les chercheurs, les didacticiens et les auteurs des curriculums de mathématiques peuvent-ils entreprendre pour faire évoluer ce qui est essentiellement un curriculum élitiste de l'ère victorienne (à partir de la 5^e année scolaire) vers un curriculum du 21^e siècle qui soit en harmonie avec les réalités d'aujourd'hui?

L'article met de l'avant une perspective pluraliste qui rend explicite la nature culturelle des mathématiques à l'école dans un cadre interculturel autochtone de respect et de collaboration. Ainsi, la nature culturelle des mathématiques devient aussi bien un contexte d'éducation qu'un contenu exprimé dans le cadre d'un curriculum.

Introduction

Respect is more than tolerance and inclusion—it requires dialogue and collaboration.—8Ways (2012, p. 4)

In many countries with Indigenous citizens, an increasing number of mathematics educators and ministries of education understand the merits of enhancing school mathematics with Indigenous ways of knowing, doing, living, and being—Indigenous perspectives (Alberta Education, 2006; Fyhn, Sara Eira, & Sriraman, 2011; Lipka, Wong, & Andrew-Ihrke, 2013; Meaney, Trinick, & Fairhall, 2012). These merits have motivated research and development (R&D) projects that draw upon Indigenous perspectives to promote greater academic achievement by Indigenous students.

The strategy has “increased student mastery of science and math concepts, deeper levels of student engagement in science and math and increased student achievement in math and science” (U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education, 2008, p. 13; see also Lipka, Webster, & Yanez, 2005; Meaney et al., 2012; Richards, Hove, & Afolabi, 2008; Sakiestewa-Gilbert, 2011). Greater achievement is also found for non-Indigenous students in those same classrooms (Adams, Shehenaz Adam, & Opbroek, 2005; Aikenhead & Michell, 2011; Lipka et al., 2013; Richards et al., 2008; Rickard, 2005).

To improve Indigenous students' well-being in school mathematics, researchers have developed a number of major R&D agendas: to explore culturally responsive teaching (e.g., Enyedy, Danish, & Fields, 2011) or place-based teaching (e.g., Sterenberg, 2013b); to interrogate school curricula (e.g., Jannok Nutti, 2013; Lipka et al., 2005); to investigate teacher professional development (e.g., Donald, Glanfield, & Sterenberg, 2011; Furuto, 2014); and to produce culturally specific teaching materials required by all of the above major R&D agendas. This last item defines the focus and scope of this article.

My purpose is to illustrate how a critical analysis¹ of researchers' and teachers' words, plans, and actions can be helpful to mathematics educators, especially to those of non-Indigenous ancestry, who aim to nurture Indigenous students in their academic achievement while strengthening their cultural self-identities. The school mathematics literature reports on the processes and products of R&D projects that aim to do this. Drawing on this evidence and on personal experience in Indigenous cross-cultural science education, I offer suggestions about what to consider and what to avoid in school mathematics and future R&D projects. My purpose is not a comprehensive review of the literature.

Notably, cultural perspectives and identities within any identifiable Indigenous group are far from homogeneous. By referring to a group's singular name, I do not suggest that the group is homogeneous, even though the group may embrace some fundamental ideas unanimously.

Educators need to be vigilantly aware of not inadvertently recolonizing the people they intend to serve. Māori researcher and educator McKinley (2001) warned that effective teaching of Indigenous students requires educators to deeply understand past political–social events that have caused current

political–social power imbalances between, for example, school curricula and Indigenous communities. This colonial-initiated imbalance inevitably favors non-Indigenous researchers or teachers. Accordingly, this political–social issue pervades this article’s text and subtext. Mathematics educators who are unaware of, or ignore, this political–social power imbalance will tend to be blind to their privileged position in society and, thus, blind to some important interpersonal dynamics with Indigenous adults and students. McKinley’s (2001) article, “Masking Power With Innocence,” could be called *privilege-blindness*. It can disrupt teaching or a project aimed at creating teaching materials for Indigenous students. It will also disrupt teachers engaged in culturally responsive or place-based mathematics instruction (Belczewski, 2009; Russell & Chernoff, 2015).

For example, if a researcher or teacher subscribes to the popular notion that many Indigenous students have “lost their culture” (St. Denis, 2004, p. 43), the researcher or teacher implicitly blames the Indigenous community for the loss—being “reckless caretakers of their culture” (p. 43). This attitude will certainly raise barriers and impede progress in an R&D project or during instruction. However, if a privilege-savvy educator (i.e., a person politically and socially informed in terms of privilege-blindness) subscribes to the notion that European colonizers *almost obliterated* the students’ Indigenous culture, then blame rests on the colonizers’ shoulders for Indigenous students’ losing facets of their own culture. This important shift of emphasis, from blaming an Indigenous group to blaming the colonizer, pivoted around a critical analysis of the word *lost*.

My intent is to promote *privilege-savvy* by discussing positive exemplars in the literature and by critiquing instances similar to “lost their culture” but in the context of school mathematics. In response to various types of privilege blindness found in the mathematics education research literature, I seek to expose instances of masking power with innocence, critically examine their consequences, explore their political racialized roots, and propose an evidence-based alternative for the benefit of all students.

I invite the reader on a journey from *tolerance* and *inclusion* of Indigenous perspectives in school mathematics to an in-depth *dialogue* and *collaboration* with Indigenous communities.

Our journey takes a rather cyclical route through topics and issues that prepare us to appreciate exemplary innovations (see section Examples of Enhancing School Mathematics Culturally), to recognize directions for their improvement (Correcting Lingering Impediments to Student Success), and to appreciate the political action (Mathematics Curricula Revisited) required to effect changes for enhancing school mathematics culturally.

This journey entails the following:

- Political–Social Contexts and Education Contexts, in which many mathematics teachers and researchers find themselves when involved with Indigenous students. Issues include contemporary consequences of colonization, privilege blindness, the mathematics curriculum, the high status of school mathematics, and culture-based alienation of students.
- Mathematics Clarified, which provides an in-depth analysis of the nature of mathematics. Topics deal with generally held presuppositions (i.e., taken-for-granted notions) concerning what school mathematics content is and what it means. It also addresses the historical manipulations that have rendered school mathematics unnecessarily challenging for so many students.
- Early Innovations, which recognizes early innovators who pioneered the historical shift to culture-based school mathematics.
- Avoiding Present-Day Appropriation and Marginalization, which explores how appropriation and marginalization of Indigenous cultures can occur despite the best of intentions. It offers ways to avoid these problems by paying attention to what gets lost in translation between Indigenous and Euro-American cultures.
- Making Connections, which identifies connections between features of R&D projects and their degree of opportunities for reconciliation; between the context of engaging students and the degree to which they see themselves in that context; along with a critical analysis of ethnomathematics.
- Examples of Enhancing School Mathematics Culturally, which describes 9 notable projects illustrating innovative presuppositions about teaching mathematics that improve student achievement. The section also describes conventional presuppositions that continue to serve students.

- Correcting Lingering Impediments to Student Success, which deals with some impediments to student achievement found in conventional school mathematics and in some innovative projects claiming to enhance school mathematics culturally. Topics include the hegemony of mathematics, myth blindness, misinterpretations of quantified indicators of student success, and, finally, researchers' and teachers' beliefs in being able to "see" school mathematics content "embedded" in an Indigenous handcraft object or everyday activity.
- Mathematics Curricula Revisited, which revisits the mathematics curriculum by summarizing ideas and implications that have accumulated throughout the monograph. The section suggests concrete action for innovative mathematics educators to take in their own political–social context; broadly interprets opposition to culture-based mathematics as professing a search for certainty in political–social and educational contexts that, in reality, comprise degrees of *uncertainty*; and identifies how mathematics curricula are unavoidably related to Canadian reconciliation with Indigenous people.
- The Conclusion, which synthesizes major implications for all stakeholders of school mathematics.

Political–social contexts

If Indigenous knowledge and pedagogy are to be integrated effectively into the national and provincial curricula, educators must be made aware of the existing interpretative monopoly of Eurocentric education and learn how the fundamental political processes of Canada have been laced with racism.—Marie Battiste (2002, pp. 9–10)

Consequences of colonial genocide

Many Indigenous peoples worldwide have faced colonial *genocide* (Woolford, Benvenuto, & Hinton, 2014), "the destruction of those structures and practices that allow the group to continue as a group" (Truth and Reconciliation Commission, 2016, p. 3). It takes the form of violence, starvation, cultural erosion, racism, oppression, and marginalization (Daschuk, 2013). As a direct result, with a few notable exceptions, Indigenous people currently suffer degrees of deprivation in social assistance, housing, health care, education, employment, and criminal justice.

In Canada, a country of three founding nations (Indigenous, French, and English; in chronological order), Indigenous peoples include three very different groups: First Nations, Métis, and Inuit (in order of decreasing population), together known as Indigenous peoples.² About one half of the First Nations population currently lives on federal government–designated reserves, originally established to force them onto mostly nonproductive pieces of land to make room for European settlers to build a nation. At the same time, First Nations people were prohibited by law from participating in that economic–political nation-building development. Since the late 17th century, First Nations have signed treaties with the British Crown, but Canada's federal government has only honored them partially, as it pleases.

The First Nations population in western Canada plummeted in the late 19th century due to engineered epidemics and orchestrated starvation (Daschuk, 2013). The federal government stole children from their families for long periods of time and forced the children into residential schools. The policy behind the government-funded, church-run schools attempted to kill the Indian in the child. The mortality rate for children in western Canadian residential schools was known to be 30 to 50% in 1910, yet residential schools were continued mostly unchanged for decades. The deputy superintendent of Ottawa's Department of Indian Affairs dismissed these data thusly: "This alone does not justify a change in the policy of this Department, which is geared towards *the final solution of our Indian Problem*" (quoted in King, 2012, p. 114, emphasis added). Atrocious conditions prevented children from learning how to be loving parents. Later, when they had children, their cruel parenting skills learned at residential schools tore apart traditionally strong family units on reserves. Intergenerational dysfunction followed, often unabated.

Since then, government discriminatory policies toward Indigenous people continued even more covertly, thus provoking an enormous accumulation of negative consequences. For example, reserve schools are currently funded at about 60% of what non-Indigenous schools are given, per pupil (Cut-hand, 2012). "We have normalized racial discrimination against [Indigenous] children as a legitimate

fiscal restraint measure” (Adams, 2016; quoting Blackstock, president of First Nations Child and Family Caring Society, which successfully sued the federal government recently in a human rights court over the government’s discriminatory low funding of child social services). Other pertinent political–social facts are described in Aikenhead (2017).³

These childhood casualties are sources of malaise among many Indigenous students entering mathematics classes today. The casualties imply that all teachers and researchers need to become privilege-savvy and consistently express it through their words, plans, and actions (McKinley, 2001).

Although the consequences of colonial genocide are brutal, times are now beginning to change. The Prime Minister of Canada apologized in 2008 for the cruel and oppressive treatment many Indigenous people suffered in residential schools. As a result, Canada has now been challenged by its Truth and Reconciliation Commission’s (2016) final report, which concluded with a directive to all Canadians to reinvent the relationships between Indigenous and non-Indigenous citizens. The Commission’s 10th Call to Action includes “improving education attainment levels and success rates” and “developing culturally appropriate curricula” (Truth and Reconciliation Commission, 2016, p. 165), giving impetus to enhancing school mathematics culturally.

In this context, educators in Canada and elsewhere are beginning to do their part toward such reconciliation by committing themselves to a professional journey in collaborating with Indigenous communities to enhance school subjects, including mathematics, with Indigenous perspectives. Examples appear throughout this article.

A postcolonial response

What are the implications for developing Indigenous-relevant teaching materials for mathematics in any country with a history of colonialism? A large majority of non-Indigenous citizens tend to be blind to the many privileges and power they enjoy; blind to the historical facts that account for those privileges and power; and unconcerned with Indigenous people’s current oppressive circumstances (Truth and Reconciliation Commission, 2016). This blindness includes mathematics teachers who may very honestly be perplexed by the idea that they benefit from many social and cultural privileges unavailable to many, but not all, Indigenous families. This is understandable. When a person lives within only one cultural milieu, it can define for them what normal is, and normal soon becomes invisible, thereby creating privilege-blindness.

Escape from privilege-blindness into a pluralistic inclusive world is made possible if people engage in a cultural immersion or some cultural enlightening experience (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b; Fyhn et al., 2011; Lunney Borden, Wagner, & Johnson, 2017; Michell, Vizina, Augustus, & Sawyer, 2008), where people have postcolonial conversations with colleagues and perhaps even a postcolonial relationship with Indigenous neighbors. The term *postcolonial* does not mean that colonialism has ended. Instead, it means that colonialism is explicitly recognized and efforts are made to diminish and extinguish its power, a process called *decolonization* (Battiste, 2013). This is the current political–social context in which school mathematics educators and researchers find themselves. Belczewski (2009, p. 197) summarized her “White teacher” learning moments acquired when she developed her own lesson plans and teaching materials for First Nations students: “I have come to realize that for my contribution to go above tokenistic culturally relevant content, it is not only the content that must be decolonized but also myself.”

Decolonizing present-day school mathematics happens at many levels. There are major changes such as explicitly including Indigenous perspectives in curricula beyond window-dressing (Mathematics Curricula subsection and Mathematics Curricula Revisited).

There are also very minor changes, such as writers showing respect and extending privileges to Indigenous cultures. One example of this is the capitalization of the words *Indigenous* and *Aboriginal*. (In Canada, the terms are interchangeable, but *Indigenous* is preferred by most Indigenous Canadians.) The mass media and publishing industry do not normally extend this sign of respect, but most Indigenous scholarly authors do. The ethical principle here is: Do what most Indigenous scholars do and mitigate the historical, colonial power imbalance and thus challenge a systemic power structure masked by innocence,

convention, ignorance, or racism. This is but one small, yet highly poignant, incident of decolonization. Others appear throughout this article.

As mentioned above but worth repeating: to decolonize school mathematics, successful educators adhere to a fundamental two-part policy. Indigenous students should be able to (a) master and critique mathematical ways of knowing without, in the process, devaluing or setting aside their culture's ways of knowing, doing, living, and being and (b) at the same time, strengthen their own cultural self-identities. The two goals are not a binary, as some educators seem to believe. They occur simultaneously.

Mathematics curricula

R&D project researchers and classroom teachers strive to meet provincial, state, and federal standards, but they also address an urgent need to contextualize this content in their communities' local cultures (Furuto, 2014, in press; Lunney Borden et al., 2017). Standards are powerfully expressed in the form of mathematics curricula, which are currently out of synchrony with enhancing school mathematics culturally.

A curriculum's voluminous content may alone prevent most teachers from exercising the flexibility needed to provide a culturally responsive or place-based mathematics program (Lipka et al., 2005). This explains the lingering tension between the legal authority of the mathematics curriculum and the evidential authority of what works well in culturally responsive or place-based Indigenous education (Examples of Enhancing School Mathematics Culturally). For the vast majority of competent teachers, curricular constraints will stifle innovation toward enhancing school mathematics culturally.

Small R&D projects are usually successful, but the process of scaling-up is a major roadblock erected by the conventional curriculum and its major stakeholders (The Opposition subsection).

But innovative mathematics curricula, crafted to advocate teaching mathematics culturally, can still be inadequate unless they provide reasonably informed directives to teachers, appropriate local teaching materials, and directions for adequate professional development opportunities required to implement a decolonizing curriculum. Teachers cannot be guided by a curriculum *hypothetically envisioned* by its writers who lack the experience of introducing Indigenous perspectives into their own mathematics classrooms.

One example will illustrate how critical analyses of our words, plans, and actions apply to such a curriculum. The Saskatchewan Grade 6 mathematics curriculum (typical of other grade levels) is open to including Indigenous perspectives in its listing of four goals: logical thinking, number sense, spatial sense, and *mathematics as a human endeavor* (Saskatchewan Curriculum, 2007, emphasis added; all quotes are from the website, which has no page numbering). This last goal creates promising opportunities to include Indigenous perspectives. For example, teachers are directed to create experiences for students to develop "an understanding of mathematics as a way of knowing the world." This is expected to be achieved, first by empowering "teachers to understand that mathematics is not acultural. As a result, teachers then realize that the conventional ways of teaching mathematics are also culturally biased." This type of transformative empowerment usually requires a substantial profession development program (Aikenhead et al., 2014; Chinn, 2007; Furuto, 2014, in press; Lunney Borden et al., 2017).

The Saskatchewan curriculum lists outcomes and indicators to define and suggest (respectively) what students should learn or do. The Grade 6 mathematics curriculum has only one Indigenous-related outcome out of a total of 19 outcomes (similar to other grade levels). That one outcome addresses Indigenous perspectives in the context of learning about numbers and is accompanied by three indicators (Saskatchewan Curriculum, 2007):

1. Gather and document information regarding the significance and use of quantity for at least one First Nations or Métis peoples from a variety of sources such as Elders and traditional knowledge holders.
2. Compare the significance, representation, and use of quantity for different First Nations, Métis peoples, and other cultures.
3. Communicate to others concretely, pictorially, orally, visually, physically, and/or in writing what has been learned about the envisioning, representing, and use of quantity by First Nations and

Métis peoples and how these understandings parallel, differ from, and enhance one's own mathematical understandings about numbers.

Out of a total of 145 indicators, 5 relate to Indigenous perspectives, which include the 3 above. Their expectations of students seem quite unrealistic for all but a few exceptional students, unless substantial scaffolding is present to guide and support them. What is being masked with innocence in the Saskatchewan curriculum?

Unlike the Saskatchewan school science program (Aikenhead & Elliott, 2010), the mathematics program has made no progress in producing supportive mathematics teaching materials such as textbooks, monographs, modules, or websites. As a result, the mathematics curriculum exemplifies tokenism by its superficial and inadequate inclusion of Indigenous perspectives, despite its promising rhetoric to the contrary.

The intransigent power of the curriculum works against updating it with contemporary presuppositions about the cultural nature of school mathematics content, a topic discussed throughout this article (e.g., *Mathematics Clarified* and *Mathematics Curricula Revisited*).

Who is the author?

As far as readers are concerned, a major political–social context is the author's personal background and relationships with the Indigenous people with whom an R&D project is undertaken or about whom they are writing (Bang & Medin, 2010; Lunney Borden, 2013). It is an issue of credibility revealed through self-disclosure.

Tân̄si. Glen Aikenhead *nit'siyikâsan*. I am of British ancestry and a Treaty 6 Canadian. My ancestors immigrated to the Ottawa Valley, Ontario, in 1820, to establish a farmstead on a piece of Algonquin hunting land. I acknowledge and honor the Algonquian Nation as inhabitants of that sacred land, which shadows my life today.

I grew up in rural Alberta and urban Calgary. I have been a science and mathematics teacher in Calgary and in two international schools. I have always embraced a humanistic perspective on science, which was honed during my graduate studies at Harvard University. In the early 1970s, this humanistic approach launched my career at the University of Saskatchewan with a research program making school science realistic and relevant for all students. My university teaching included science methods courses for First Nations and Métis teacher education programs. My students inspired me to learn more about Indigenous ways of knowing.

My Caucasoid, male, middle-class, science–mathematics-related identity has always given me a privileged status. Since the early 1990s, I have invested this cultural capital in being an ally to my Indigenous colleagues and friends. Their visions of science and mathematics education have always defined my R&D agendas.

By being taught by patient and wise Elders, I matured to the point of offering a policy statement for Indigenous cross-cultural science education (Aikenhead, 1997). This led to a community-based, collaborative project, "Rekindling Traditions" (Aikenhead, 2002a). Its purpose was to demonstrate how place-based science teaching materials could be developed from an Indigenous perspective.

Since my retirement in 2006, the Saskatchewan Ministry of Education added Indigenous knowledge to its science curriculum in collaboration with Elders and then published a Grades 3–9 science textbook series with Indigenous perspectives developed in full collaboration with Elders (Aikenhead & Elliott, 2010), with whom I had the honor to share editing responsibilities. Books were published: *Bridging Cultures* (Aikenhead & Michell, 2011) and a professional development resource book for implementing culturally responsive science teaching (Aikenhead et al., 2014).

Conclusion

Every country has unique and complex political–social contexts that determine educational policies. Canada's policy now rests mainly on the social justice issue of reconciliation. Norway's policy is formulated on government legislation protecting Sámi culture as an important heritage to maintain in Norway

(Fyhn et al., 2011). Other reasons that encourage educational jurisdictions to enhance their school mathematics culturally include the following: to meet student assessment standards defined by governmental authority; to counter the negative consequences of globalization experienced by developing nations; and to change a government's economic base from (a) an Indigenous population forced by colonial genocide and racism to draw heavily on social welfare to (b) a government reliant on employed Indigenous citizens contributing to the well-being of the country. Appropriate education contexts motivate that transition.

Education contexts

If we are to understand how Aboriginal and Eurocentric world views clash, we need to understand how the philosophy, values and customs of Aboriginal culture differs from those of Eurocentric cultures.—Kainai Elder Leroy Little Bear (2000, p. 77)

The intergenerational dysfunctions identified in the Consequences of Colonial Genocide subsection have a direct bearing on most Indigenous students' interest and achievement in schooling, particularly in mathematics. This influences postsecondary attendance specifically and employment in general. Statistics are both overwhelming and well known worldwide (Anderson & Richards, 2016). The vicious cycle of poverty will continue unless a host of interventions are implemented simultaneously. Education has one small but crucial part to play. Its role in developing Indigenous-related curricula and teaching materials for mathematics can certainly help.

In this section, I address three pervasive education topics that inform an important context for understanding the cultural enhancement of school mathematics: improvements in developing teaching materials, the high status of school mathematics, and culture clashes that alienate many Indigenous students to varying degrees.

Improvements in developing teaching materials

Although exemplary R&D projects are contributing substantially to decolonizing mathematics education (Examples of Enhancing School Mathematics Culturally), a critical examination reveals some subtle, vestigial, hegemonic presuppositions that cause privilege blindness and thereby undermine the projects' full potential for decolonization (Correcting Lingering Impediments to Student Success). Based on the past few decades, Jorgensen and Wagner (2013) drew attention to mathematics researchers' and educators' move away from

1. deficit assumptions about Indigenous students and toward focusing on the assets students bring to the classroom;
2. stereotyping and romanticizing Indigenous cultures and toward knowing the realistic diversity within any Indigenous group; and
3. grasping onto outdated ontologies (people's presuppositions about reality), epistemologies (how people come to know what they know, and the kind of knowledge it is), axiologies (people's values that guide their thinking and behavior), and ideologies (doctrines that determine how people or institutions treat others), all four of which are responsible for the low achievement, especially for many Indigenous students (e.g., Lunney Borden, 2013; Russell & Chernoff, 2013).

Jorgensen and Wagner's (2013) observations highlight some fundamental values and observations that can serve as critical analysis criteria.

Doolittle (2006) and Sterenberg (2013a) warned about contextualized school mathematics in Indigenous cultures that have ended up as tokenism, stereotyping, or other neo-colonial devices, with the negative consequence of marginalizing Indigenous students. For example, Sterenberg (2013a) repeated an insulting word problem: "Imagine a band of 250 Aboriginal People. Each tipi can hold approximately eight people. Calculate how many tipis would be needed to house the entire band" (p. 21). This is insulting because Indigenous people would not divide themselves in the hypothetical way stated in the word problem. Relational and spiritual factors would dominate. And the required hypothetical state of mind itself, captured in the word *imagine*, is a value embraced strongly in the culture of school mathematics

but would generally be foreign to an Indigenous culture in the context of people choosing a tipi to enter, because it conflicts with how to live in a good way.

Using written recipes for Indigenous food is quite a popular activity for teaching measurement and proportionality. But as Jannok Nutti (2013) pointed out, *authentic* Swedish Sámi cooking follows oral nonquantitative directions. Thus, a quantitative word problem is an inauthentic representation of Indigenous cultures (Aikenhead, 2017).

Teaching school mathematics culturally can generally avoid tokenism, stereotyping, and other neo-colonial devices by *collaborating* with Indigenous people who have the relevant gifts or expertise to participate. Many examples are found in the Appropriation subsection and Examples of Enhancing School Mathematics Culturally.

Unfortunately, a number of promising curricula and R&D projects never reach fruition because they became mired in the process of *consultation*, which does not sufficiently share power with, or transfer power to, Indigenous Elders or communities to make the final decisions concerning education outcomes and teaching materials. There are practical advantages to reasonably reversing the status quo power imbalance so it favors an Indigenous community's interests (e.g., Aikenhead & Elliott, 2010).

High status of school mathematics

Historically, Eurocentric cultures manifest superiority and aggressive authority that have the capacity to colonize (Historical Appropriation subsection), a propensity taken on by 19th-century mathematics educators who ensured an elite status for their mathematics curriculum (subsection on Norway). Today it continues as the school's highest status subject (Venville, Wallace, Rennie, & Malone, 2002). Hence, mathematics teachers today enjoy a paucity of pressure to defend, rationalize, or modify their school mathematics curriculum content in junior and senior secondary grades (D'Ambrosio, 2007). This elite status contributes to school mathematics' ideology and philosophy of education, which in turn energizes the resistance to enhancing school mathematics culturally (A Hidden Platonist Agenda subsection).

For example, students' assessment in this high-status subject becomes an unquestioned screening device for graduation and entry into specific postsecondary educational programs (Boylan, 2016). Students who are most likely to fail this screening tend to belong to marginalized and low-social-economic-status groups (Nasir, Hand, & Taylor, 2008). The screening process provides social power to the more privileged students who make it through (Russell, 2016). This reality faces all students participating in most mathematics learning environments but especially Indigenous students, a situation that Russell and Chernoff (2013) argued, with considerable force, is ethically indefensible.

Perhaps this high status stems from society's and students' perception of mathematics "as the antithesis of human activity—mechanical, detached, emotionless, value-free, and morally neutral" (Fyhn et al., 2011, p. 186), as well as being decontextualized, abstract, philosophical, and rigid and therefore being more difficult to learn (Nasir et al., 2008). This perception (a) creates a culture clash for such students (Aikenhead, 1997; Russell & Chernoff, 2013; Culture Clashes That Alienate Many Indigenous Students subsection); (b) convinces them that mathematics is not relevant to who they are and where they are going—their self-identities (Ishimaru, Barajas-López, & Bang, 2015); and (c) spurs them to resist this assimilationist instruction, usually by dropping out of class or "playing Fatima's rules" (Aikenhead, 2006, p. 28) to make it appear as if meaningful learning has occurred, when it has not (i.e., playing the system).

In addition, there is the issue of the validity of its high status. Is what school mathematics claims to accomplish a misrepresentation? For instance, the promise that learning mathematics trains the mind to think logically was empirically tested during the 1970s and rejected as an old wives tale. The promise was then changed to one of developing a labour market to benefit a nation's economic competitive edge (The Altar of Platonist Content subsection). Accumulated research in economics fails to support this promise (Aikenhead, 2006, 2017; Lunney Borden & Wiseman, 2016). The validity underscoring school mathematics' high status seems specious. The screening is "a hoop to jump through just to prove you could ... a false boundary in one's life" (Russell, 2016, p. 44).

The political-social high status of conventional school mathematics is a major barrier to reconciliation and to a renewed curriculum and instruction commensurate with 21st-century realities.

Culture clashes that alienate many Indigenous students

A specific fundamental culture clash between mathematical and Indigenous ideas of reality (i.e., their ontologies) illustrates two very different cultural ideologies. The confidence that scientists hold in their mathematical representations of the physical universe can cause many people to believe that Indigenous ideas of reality are based on superstition. But at the same time, confidence in Indigenous rationality causes some Indigenous people to think that mathematical representations of reality are based on superstition. This was illustrated by Lakota Elder Vine Deloria (1992), who stated:

The present posture of most Western scientists is to deny any sense of purpose and direction to the world around us, believing that to do so would be to introduce mysticism and superstition. Yet *what could be more superstitious* than to believe that the world in which we live and where we have our most intimate personal experiences is not really trustworthy, and that another mathematical world exists that represents a true reality? (p. 40, emphasis added)

Thinking that someone is rational or superstitious would seem to depend upon which culture-based worldview a person adheres to (Bang & Medin, 2010; Medin & Bang, 2014).

The culture of school mathematics often clashes with a student's worldview, cultural identity, or life in general (D'Ambrosio, 1991; Davison, 2002). Quantification itself seems to control students' lives in arbitrary ways, such as representing their achievement with numbers assumed to validly define an assessment of what they have learned (Doolittle & Glanfield, 2007). “[S]o much of the power of [Western] Mathematics comes from the security and *control* it offers” (Bishop, 1988b, p. 151, emphasis in original). Quantification encourages people to believe that they can objectify things, events, or people by stripping them of their qualitative, subjective, and spiritual attributes. “The mathematical valuing of ‘right’ answers informs society, which also looks (in vain of course) for right answers to its problems” (p. 152).

Boylan (2016) illustrates a much different way in which mathematics can affect people's lives:

A significant capitalist response to the environmental crisis has been to enlist mathematics in the search for market solutions. Under the banner of green capitalism, mathematics is being used as a means to extend the commodification of natural resources in new ways. ... The value and worth of the natural world and our relationship to it are transmuted into valorisation; everything—water, trees, clean air, biodiversity and ecosystems—can be given a price. (pp. 402–403)

State, national, and international testing results exert enormous power over citizen opinion, over education policy, and over what happens in school mathematics classrooms; all leveraged by the ideology of quantification (Misinterpretations of Quantified Assessment subsection). This ideology, experienced by all students, is located in the content of school mathematics, albeit tacit, and hence seldom made explicit to students. Instead, students are expected to accept the ideology—an assimilation or indoctrination into the culture of school mathematics and into Eurocentric-based cultures. Some students figure out what their school mathematics is up to, and they resist in varying degrees and ways, especially Indigenous students (Enyedy et al., 2011). Some students will aim for only a pass mark, and others drop out.

Another source of alienation arises from the fact that, for instance, axioms, sets, proofs, algebra, and geometry are all abstract inventions of the intellect (Bishop, 1988b; Boylan, 2016; Ernest, 1991; François & Van Kerkhove, 2010). These abstract representations or metaphors can be *superimposed* on the physical world in order to make sense out of it through a mathematical lens. Ernest (2013) noted a “distinction between *concept definition* (formal, explicit, publicly justifiable description) and *concept image* (visual and other representations and associations). Concept images represent a deep level of meaning” (p. 5, emphasis added).

Interestingly, these metaphoric images are originally created by the human mind according to Einstein (1930), who wrote cited in Director, 2006):

It seems that the human mind has first to construct forms independently, before we can find them in things. Kepler's marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone, but only from the comparison of the inventions of the intellect with observed fact. (p. 113)

School mathematics seems alien to students who do not have a Kepler-like worldview or have not yet decided to play within mathematics' ideology or have not mastered using its abstract mathematizing in their everyday concrete world (François & Van Kerkhove, 2010).

Alienation also arises for many Indigenous students when mathematics is incorporated into school science; the ideology of quantification tags along implicitly (Hogue, 2011). In the context of explaining the ways of thinking in high-status scientific paradigms, Aikenhead and Michell (2011) pointed out, “The presupposition of quantification assumes that the material world is governed by objective mathematical relationships. Theoretical physicists are prone to say ‘the language of nature is differential equations’” (p. 52).

I would emphasize that some students (Indigenous or non-Indigenous) will feel very comfortable with the ideology of quantification found in most school mathematics and physical science classes, if and only if their worldview harmonizes with a quantitative perspective that makes sense to their lived experiences.

In contrast to the ideology of quantification, Indigenous Elders generally live according to *qualitative* responsibilities to act in a good way, which trumps any decision to be controlled by quantitative symbols (e.g., numerals). Elders have not been colonized into allowing the ideology of quantification to dictate their behavior. They are wise in knowing that the rectilinear concept of time is a European cultural invention (Bolter, 1984). Elders understand time as cyclical, sometimes in a spiral type of way (Aikenhead & Michell, 2011).

In Euro-American cultures, it does not make sense to ask, “What is an *average* high school student?” because the mathematics concept of average is irrelevant in this context (Hough, 2015, p. 24). In other words, calculating a mathematical average takes an *intellectual* understanding of school mathematics content, whereas knowing when and where to use that concept requires a *wisdom* understanding of the cultural context of school mathematics content. *Both the intellectual and wisdom components should be involved in understanding school mathematics*, for a number of reasons discussed in this article, especially Mathematics Clarified.

A distinction between intellectual and wisdom understanding represents another fundamental source of cultural clash for many Indigenous students. Indigenous cultures give much higher priority to the *wisdom tradition* of thinking, reflecting, doing, and being. When that view is normal for Indigenous students, imagine how disappointed they must feel when studying mathematics and discover that the subject is *only* about an *intellectual tradition* of thinking, a much narrower view of education. Mathematics may seem inferior and largely irrelevant compared to an Indigenous student’s taken-for-granted wisdom tradition of understanding.

From an Indigenous perspective, enriched by subjective relationships and responsibilities with everything in Mother Earth, objectification through quantification can show a lack of respect at the very least and can border on oppression at worst, depending on the circumstance. That explains why, before contact with Europeans, most Indigenous groups had no need for an elaborate mathematics system (Elder Leroy Little Bear, personal communication, February 27, 1999), with Mayan and Inka civilizations, for example, being exceptions.

On the one hand, analytical objectification, a component of the quantification ideology, can cause serious concerns for both Indigenous and non-Indigenous students whose wholistic⁴ qualitative worldviews do not resonate with the mathematical value of objectification. On the other hand, mathematical representations of events and mathematical problem solving are valuable cultural capital in many everyday circumstances and particularly in certain high-paying occupations, such as engineering. Thus, culture clashes must be resolved or at least mitigated for Indigenous students in order to help them escape the socioeconomic cycle of poverty described above. Culture-based school mathematics has a proven record for resolving or diminishing students’ culture clashes (Examples of Culture-Based School Mathematics).

This resolution can occur with an agenda to walk in both worlds (Stereberg & Hogue, 2011)—the analytical objectified world of mathematics and an Indigenous world rich in relational subjectivities (subsection on Mathematical Pluralism). A similar agenda is called *two-eyed seeing* (Hatcher, Bartlett, Marshall, & Marshall, 2009), which means learning the strengths of each culture’s way of knowing, illustrated in the Algonquins of Pikwàkanagàn First Nation Project subsection. Either agenda speaks to the wish of Indigenous parents for a locally determined educational balance between the two worlds (subsection on Sweden).

Learning school mathematics with academic rigor should not mean that students need to devalue their cultural heritage. Indigenous perspectives have motivating value in school mathematics for both

Indigenous and non-Indigenous students (Davison, 2002; Donald et al., 2011; Furuto, 2013a; Fyhn et al., 2011; Lipka et al., 2005).

Cross-cultural teaching materials that reveal the ideology of quantification to students should distinguish between what students *believe* (in the sense of making it part of their own worldview) and what they merely *understand* (in the sense of describing or using it accurately; Aikenhead & Michell, 2011). For example, students should not be expected to believe the ideology of quantification, because that would be indoctrination. But they should be expected to understand it as pragmatic content that prepares them for living well in the dominant society. By expecting an understanding, we tend to reduce students' resistance to school mathematics by diminishing their feelings of culture clash.

This strategy appeals to many high school students (both Indigenous and non-Indigenous) who see mathematics as simply foreign to their worldview (Aikenhead & Michell, 2011). Perhaps if school mathematics were taught as if it were an economically potent foreign culture, then some Indigenous students might see themselves as being able to appropriate rich knowledge from the dominant culture.

Intellectual clashes also stem from “ways of thinking” and “styles of communication” (Lunney Borden, 2013, p. 14). Divergent epistemologies have been detailed by Aikenhead and Michell (2011, pp. 99–120) and studied in depth by Bang and Medin (2010). Teachers and researchers who create teaching materials for Indigenous students need to be conversant with sources of cultural conflict described in this article, in order to develop scaffolding strategies to help students bridge what sometimes feels like an epistemic hiatus (Stoet, Bailey, Moore, & Geary, 2016). And researchers and teachers need to be mindful that “it is incorrect to believe that [Indigenous] students who speak English also think English” (Lunney Borden, 2013, p. 14).

Not only do students' divergent ways of thinking influence their experiences with school mathematics but mathematics itself has divergent identities that historically have been withheld from teachers and students, which are clarified in the following section.

Mathematics clarified

Mathematics is as much an aspect of culture as it is a collection of algorithms.—Carl Benjamin Boyer (1906–1976)
American historian of mathematics. Quoted from an unknown 1949 calculus textbook

Which taken-for-granted assumptions about mathematics continue to impede Indigenous students' achievement by creating culture clashes? What role do values and ideologies play in mathematics? What deceptive assumption has become arbitrary dogma? What are the alternatives? How are mathematics curricula related to reconciliation? These questions set the agenda for this section.

Some fundamentals of mathematics

As discussed in the Culture Clashes that Alienate Many Indigenous Students subsection, various culture clashes account for many students' difficulty learning school mathematics. These clashes do not reflect any deficit in students, but they do stem from school mathematics' worldview-based ontological, epistemological, and axiological presuppositions. These beliefs are embraced unconsciously by most mathematics educators, school administrators, and ministries of education and, therefore, the beliefs are subliminally inculcated into the general public who have lived in mathematics classes during their youth.

Ernest (1988, website quotes) distinguished among three different sets of beliefs about the nature of mathematics:

1. An *instrumentalist belief* about mathematics as a utilitarian “accumulation of facts, rules and skills to be used in the pursuance of some external end.”
2. A *Platonist belief* about mathematics “as a static but unified body of certain knowledge. Mathematics is discovered, not invented.” In other words, a “doctrine that mathematical entities have real existence and the mathematical truth is independent of human thought” (*Collins English Dictionary*, 1994).

3. A *cultural belief* about mathematics “as a dynamically organized structure, located in a social and cultural context” for the benefit of problem solving; and a “continually expanding field of human creation and invention.”

The instrumentalist belief is judged as being naïvely simplistic (Ernest, 1988, 1991). Moreover, in Anyon’s (1980) ground-breaking research into how mathematics teaching varies according to the social-economic status of a school’s neighborhood, an instrumentalist approach was shown to be the pedagogy of choice in high-poverty working-class neighborhoods. Being simplistic, classist, and racist (Jorgensen, 2016; Martin, 2006), an instrumental belief is ignored in this article.

The Platonist belief

The ever popular Platonist belief is usually referred to in the educational literature as academic or conventional mathematics. It characterizes mathematics as being value free, decontextualized, acultural, non-ideological, purely objective, always consistent, generalizable, universalist in the sense of being universally true, and thus *the only acceptable way of mathematizing* (a term specifically defined as counting, locating, measuring, designing, playing, and explaining; see subsection Mathematical Pluralism). This cluster of presuppositions has been analyzed critically by scholars other than Ernest (1988, 2016b); for instance, Donald et al. (2011, p. 75) wrote: “Mathematics appears to be universal because of the prevalence of absolutist philosophies that view mathematics as timeless because it builds on logics of deduction.” Mukhopadhyay and Greer (2012) go a step further:

The development of a multicultural and humanist view of mathematics challenges the supremacist position maintained by many mathematician educators who regard abstract mathematics as the crowning achievement of the human intellect, and school mathematics are the transmission of its products. (p. 860)

As indicated by Ernest’s (1988) characterization of a Platonist belief (just above), a centuries-old philosophical debate underlies these critiques: Is mathematics *discovered* or *invented* by humans? Platonists predominantly champion the “discovered” position due to their belief that the universe itself is innately composed of mathematical abstract objects, there to be discovered. Hence, mathematics must be universal, absolute, certain knowledge, and beyond the influence of humans. This tangential philosophical debate is summarized by Aikenhead (2017).

Bishop (1990) revealed value-laden reasoning expressed by many Platonist mathematics teachers: “[T]o decontextualize in order to be able to generalize, [that] is at the heart of Western mathematics” (p. 57). Platonist mathematics teachers will deny that school mathematics content is value laden and, at the same time, they obviously cherish two of its *epistemic values*: decontextualization and generalizability. More examples of values are found in the Euro-American Mathematics Cultural Content Made Explicit subsection.

A Platonist belief creates an image detrimental to many students’ participation and achievement in school mathematics (Fyhn, 2013). Indigenous cultures, for instance, generally share presuppositions characterized as value-laden, contextualized, cultural, ideological, mostly subjective, and embracing multiple truths. Thus, a curriculum defined by a Platonist belief alone exacerbates the culture clash faced by most Indigenous students. For example, its universalist value logically dismisses their culture’s mathematizing, thereby maintaining the colonial-based power imbalance between the mathematics curriculum and Indigenous communities. The Platonist belief ossifies privilege-blindness, thereby masking power with Platonic innocence. (A detailed explanation that draws on language-laden cognition is found in the Historical Appropriation subsection).

The Platonist belief in schools and curricula is very subtle. Its popularity in schools tends to rest on its claim to objectivity—the opiate of the academic (Aikenhead, 2008). In various contexts of this article, I draw attention to pervasive Platonist presuppositions that impose a neo-colonial stance on Indigenous students and a doctrinaire influence on a large majority of students (Doolittle, 2006; Doolittle & Glanfield, 2007).

Toward a cultural belief

Some mathematics educators and researchers have chosen to replace the Platonist belief with a *cultural* or *culture-based* understanding of mathematics, with which to enhance school mathematics for the

specific benefit of Indigenous students (Early Innovations and Examples of Enhancing School Mathematics Culturally). Even though these educators' plans and actions have been implemented, many of these educators have yet to modify all of their Platonist concepts and expressions to accurately match their intentions. Therefore, their language has not completely shifted to expressing a cultural belief.

For example, when teachers tell students that they “see” mathematics all around them, teachers assume that all students “see” what they “see.” The quotation marks signify that in this context the word does not really mean to observe; instead, it means to conceptualize and then project that conceptualization onto the world around them (Historical Appropriation and Projectionism subsections). A disbelieving high school student may logically ask, “Take me to the mall and show me a quadratic equation.”

A cultural understanding of school mathematics aims to position students so that they experience mathematics as a human endeavor (François & Van Kerkhove, 2010). As such, mathematics is rooted in the culture of those who created the knowledge system (Ernest 1991), and it can be related meaningfully to students' cultural self-identities (Ishimaru et al., 2015). Bishop (1988b, p. 155) recognized this double function when he stated that mathematics is a product of its developer's culture and “as a cultural product, [it] is now strongly shaping Western culture as a whole.”

To experience mathematics as a human endeavor is to engage in a repertoire of its sense-making cultural practices (Boylan, 2016). This view of culture as everyday practice lends itself to teaching and producing projects in which students must negotiate, supported by their teachers, among multiple ways of understanding; that is, multiple cultures, such as an Indigenous culture and the culture of school mathematics.

Mathematical pluralism

In a scholarly milieu of Cambridge University (White, 1959; Wilder, 1981) informed by contemporary ideas such as mathematics as a cultural system, Bishop (1988b) helped identify the cultural nature of mathematics as a pan-human activity:

Recently, research evidence from anthropological and cross-cultural studies has emerged which demonstrates convincingly that the mathematics which we know is a culture-bound phenomenon, and that other cultures have created ideas which are clearly “other mathematics.” (p. 145)

Worldwide, the cultural product of mathematics develops in tandem with people's everyday cultural activities. When the origin of various mathematics is linked to activities—*mathematizing* processes—then mathematics linguistically connects with Indigenous people's verb-based ways of expressing themselves (Ascher, 1991; Battiste, 2002; Fyhn, 2013; Lunney Borden, 2013; English/French Language-Laden Confusion/Misconceptions subsection). Indigenous and non-Indigenous students benefit from a mathematics curriculum and pedagogy that convey this cultural activity perspective (Introduction).

Bishop (1988b, p. 146) argued that language can act like conceptual *tools*, including mathematical *symbolism*: “Mathematics, as an example of a cultural phenomenon, has an important ‘technological’ component.” He then concluded that all mathematical cultural activities “relate to the physical and social environment in some way and ... the functions of this symbolic technology called mathematics are concerned with relating [humankind] to [the] environment in a particular way” (p. 147). Bishop's general characterization of mathematics can be distilled into the following definition:

In any culture (including Euro-American cultures), their mathematics is a symbolic technology for building a relationship between humans and their social and physical environments.

Mathematics taught in schools “cannot be separated from ‘Western’ cultural and social history” (Bishop, 1988b, p. 151) without misrepresenting a fundamental feature of the subject, a feature that significantly reduces culture clashes. Therefore, this cultural feature that connects school mathematics content to its cultural environment improves students' success.

Bishop's (1988b) research conceptualized six foundational or universal *mathematizing* processes (cultural practices) present in mathematical systems created by various major cultures throughout history: counting, locating, measuring, designing, playing, and explaining. In a sense, these processes offer a simple definition of the superordinate *mathematics*. Lunney Borden's “Show Me Your Math” website (SMYM,

2017) begins with a video in which a Mi'kmaw Elder and several students energetically define mathematics by these six processes. The foundational processes suggest that a wide range of mathematizing can be found in a local Indigenous community (Aikenhead, 2017). By becoming familiar with these six cultural practices, teachers and researchers will be cued into an early step in producing successful teaching materials and lessons.

Throughout history, the intellectual invention called *mathematics* has produced radically different culture-based mathematical systems that have built symbolic relationships between humans and their environment. Consider, for example, Indigenous Australian mathematics in use tens of thousands of years ago (Watson & Chambers, 1989), Yup'ik mathematics (unknown date; Lipka et al., 2013), Mayan mathematics (about 2,000 years ago; NOVA, 2016), Polynesian mathematics (about 1,000 years ago; Ball, 2013), and the latest Japanese mathematics (about 400 years ago; Wikipedia, 2016).

Every major culture has had a mathematics knowledge system; therefore, many different mathematics knowledge systems exist; ergo, mathematics is culturally *pluralist* but not relativist. Platonists value binaries very highly (e.g., universalist or relativist and nothing in between). But pluralism rests comfortably in between. Pluralism is simply a logical implication of Bishop's (1988b) characterization of mathematics.

From a grammar perspective, this means that the term mathematics functions as a *superordinate* concept, representing a set of multiple mathematics knowledge systems that grammatically function as *subordinate* concepts. By analogy, the term *furniture* is a superordinate concept with respect to the set of subordinate concepts: chair, table, bed, and bookcase.

Ogawa (1995) established the same distinction for a superordinate *science*, elaborated upon by Aikenhead and Ogawa (2007), who identified subordinate sciences: Eurocentric science, Indigenous ways of living in nature, Islamic science, and Japanese ways of knowing *seigyo-shizen*.

For clarity and accuracy, the mathematical system conventionally taught in schools and universities needs to be *identified by its cultural association*. And what is that cultural association?

Yousan

An answer is found in the following short historical account. Japanese mathematics that Japanese people call *Wasan* effectively served Japan's Edo society period, 1603–1868. When Japan was suddenly forced by American navy gunboats to join the globalization movement in 1868, a mathematical system Japanese people called *Yousan* was introduced officially into Japan. *Yousan* is taught in schools worldwide today.

Historically, *Yousan* had been developed within Euro-American cultures through invention (Ernest, 2016b) and much appropriation from other cultures over time (e.g., Babylonian, Egyptian, Greek, Hindu-Arabic and Chinese cultures; Historical Appropriation subsection).

A Japanese point of view is revealed by a literal translation of *Yousan* into English: *foreign over the ocean calculation* (Uegaki, 1990). Japanese scholars realized that *Yousan* was a cultural creation that harbored ideologies, values, and perspectives foreign to those of Japan's (Kawasaki, 2002). For example, the *Yousan* counting system uses the same names for numbers no matter what is being counted. "Mathematics is the art of giving the same name to different things" (Henri Poincaré, French mathematician, early 20th century, cited in Verhulst, 2012, p. 154). On the other hand, the *Wasan* counting system, which continues today to some degree, has different number names depending on the objects being counted. For instance, number names differ somewhat when counting people, long skinny things, thin flat things, clothes, etc., all of which subtly relate to an endemic Japanese worldview.

According to Japanese common sense, therefore, *Yousan* was consistent with Ernest's (1988) *cultural belief* about mathematics, and its cultural association was indeed Western or, more accurately, Euro-American.⁵ But Platonist content was also present in *Yousan*, of course. As a result, *Yousan* combines Ernest's two categories, a *Platonist belief* and a *cultural belief* about mathematics, in a yin-yang paradoxical way. This means that both beliefs function simultaneously. They are an amalgam. One belief is not subsumed within the other belief.

Accordingly, I propose to replace the literal translation of *Yousan* (foreign over the ocean calculation) with a more precise translation: *Euro-American mathematics*. The phrase expresses a cultural association,

a pluralist stance, and an amalgam of the *cultural* content of mathematics along with its conventional *Platonist* content.

Grammatically speaking, Euro-American mathematics (EAM) is a subordinate concept to the superordinate mathematics. Moreover, *EAM articulates a meaningful concept of what can be taught in school mathematics; one that leads naturally to enhancing school mathematics culturally, for the benefit of most students.*

If what is taught in schools is just called “mathematics” or “Mathematics,” then its cultural association is suppressed in accordance with the Platonist belief (subsection on a Hidden Platonist Agenda) and consequently the name continues a hegemonic agenda that claims that school mathematics is acultural, value free, nonideological, and universalist and that no other mathematical system is worth considering.

According to Bishop (1990, p. 1), Western (i.e., Euro-American) mathematics is “one of the most powerful weapons in the imposition of Western culture.” This agenda of power, enacted through the ideology of quantification among other ideologies (Euro-American Mathematics Cultural Content Made Explicit subsection), is one aspect of the culture of conventional school mathematics that tends to alienate Indigenous students (Culture Clashes that Alienate Many Indigenous Students subsection), thereby ultimately contributing to high dropout rates in schools and the cycle of socioeconomic poverty (Culture Clashes that Alienate Many Indigenous Students subsection) and its concomitant devastating consequences for the well-being of students and their communities. Breaking this cycle is a high-priority agenda item for 21st-century school mathematics in an era of reconciliation.

A hidden Platonist agenda

Why has the Platonist belief about the nature of school mathematics prevailed to this day? Who is served by this belief? Can school mathematics evolve in the 21st century to become in tune with today’s political–social and educational realities (Political–Social Contexts and Education Contexts)?

During the 19th century in the era of Queen Victoria (1819–1901), the Industrial Revolution caused the development of the public education system in Europe and then, soon after, worldwide. Mathematics would be adopted as a core subject, but which version: the elite academic mathematics or a practical relevant mathematics that held contemporary promise? A public debate ensued.

The elitist Platonists, channeling the ancient Greeks’ belief in the superiority of pure abstract thought (Plato’s “world of ideas”; Kawasaki, 2002, p. 25), fought for *decontextualized* content. This view is consistent with the idea that mathematics content is *discovered* as abstract objects that constitute the universe, not invented by humans (The Platonist Belief subsection). Platonists eschewed worldly practical knowledge (Plato’s “phenomenal world;” Kawasaki, 2002, p. 25) connected with human activity, a position that would produce a curriculum of *contextualized* content. These two views mirrored the ancient Greek class-based society: the elite aristocracy versus the artisans and slaves. The Platonists argued for public schools to mimic the elite 19th-century British Latin Grammar Schools. As it turned out, the Platonists won the debate.

Their rationale for defining the school subject their way contained a deceptive strategy that somehow rendered itself invisible to future generations—a hidden agenda perhaps?

Just above, Bishop (1990, p. 1) connected “Western mathematics” directly to the act of colonization. D’Ambrosio (1991, p. 12) concurred: Euro-American “[m]athematics is also the imprint of Western culture.” Its cultural identity expresses itself, for instance, when it is used to control people. That is certainly not decontextualized knowledge. How could mathematics be acultural, as the Platonist claimed?

Ernest (1991, p. 259) objected to their belief “in the absolute objectivity and neutrality of mathematics.” The Platonists vehemently claimed that mathematics was value free, despite a public repository of mathematics values and ideologies (a few already mentioned above but expanded upon in the Euro-American Mathematics Cultural Content Made Explicit subsection). Ernest (1991) answered, “[T]he values of the absolutists are smuggled into mathematics, either consciously or unconsciously, through *the definition of the field*” (p. 259, emphasis added). A hidden agenda does exist, perhaps.

Ernest (1991) was referring to the following rhetorical sleight of hand. When establishing the first public school mathematics curriculum in the 19th century, the Platonists drew on a binary, “logical versus irrational,” invented by “Western culture dating back to Socrates, Plato, and Aristotle” (Hall, 1976,

p. 213), in order to construct their own theoretical binary: “*formal mathematical discourse*” versus “*informal mathematical discourse*” (Ernest, 1991, p. 53). They arbitrarily assigned their definition of school mathematics to the formal discourse category; for example, deductive proofs, theorems, and the scientific application of EAM Platonist content. This assignment is consistent with their philosophical position that mathematics is *discovered* as abstract concepts constituting the universe, not *invented* by humans.

The *informal* discourse category, on the other hand, contained all of the features that made mathematics a human endeavor; for example, its political–societal contexts (Skovsmose, 2016); its ideologies and values by which it operates (Euro-American Mathematics Cultural Content Made Explicit subsection); and its ontological, epistemological, and axiological presuppositions; in short, EAM cultural content.⁶

Einstein (1921) provided a parallel explanation by noting a “new departure in mathematics which is known by the name of mathematical logic.” The truth and certainty of mathematical logic is found in deductive proofs of theorems. A theorem is based on definitions and a set of axioms, from which a conclusion is derived by the logic of deduction. Consequently, mathematical logic’s axiomatic system is a self-consistent closed system by design. Truth of theorems, therefore, is *relative to* the set of axioms from which the theorem was deduced. Although an axiomatic system conveys an aura of truth, certainty, and the power to control, among other attributes, formal mathematical discourse seems somewhat relativistic in its truth claims—truth is relative to a particular set of assumed axioms—more EAM cultural content to teach.

Einstein (1921) wrote that “logical-formal” content (i.e., ideology-free within a closed, self-consistent, deductive system) was separated from “intuitive” content. “[T]he logical–formal alone forms the subject-matter of mathematics, which is not concerned with ... intuitive or other content *associated with* the logical-formal” (emphasis added). This *associated* content had been included in European Renaissance mathematics (Wikipedia, 2017).

The explanations from Ernest and Einstein converge. In essence, Platonists’ arguments rest on an arbitrary, Age-of-Reason type of social license to *define* the school subject of mathematics by inventing a theoretical binary that disassociates formal from informal discourses. Ernest (1991) hints at a magic trick: “[A]t the heart of the absolutist neutral view of mathematics is a set of values and a cultural perspective, as well as an ideology which renders them invisible” (p. 260)—a deceptive hidden agenda, to be sure. This ideology is the Greek-based European impulse to reify, essentialize, and vigorously promote arbitrary binaries that only exist in the mind.

Is this a case of masking power with a rhetorical sleight of hand? *In the context of educational policy and curriculum development* for schools, has the philosophic absolutists’ social license not expired well before the 21st century? In today’s political–social context of reconciliation dedicated to rid society of the same type of Age-of-Reason social license that enacted residential schools (Consequences of Colonial Genocide subsection), is it not time to revoke the Platonists’ social license that needlessly depresses high school graduation rates of Indigenous and non-Indigenous students?

Because the culture of school mathematics is tied up with some people’s self-identities and worldview-based beliefs—people who currently have the *power* to control the school mathematics curriculum and graduation requirements (High Status of School Mathematics subsection)—the issue can stir emotional reactions (The Opposition subsection).

The above critical analysis of Platonists’ hidden agenda exposes a deceptive rhetorical trick. By doing so, the analysis helps set mathematics education on a path toward a negotiated sharing of power between mathematics curricula and Indigenous communities. This coincides with Recommendation 10 from Canada’s Truth and Reconciliation Commission (2016): “improving education attainment levels and success rates” and “developing culturally appropriate curricula” (p. 165). Examples of restoring a more equitable balance of power are highlighted in Examples of Enhancing School Mathematics Culturally.

The critical analysis also challenges mathematics educators, education administrators, curriculum developers, and ministries of education: “It is now time to reconceptualize school mathematics’ curriculum content in terms of 21st century cultural practice, by replacing its 19th century Platonist belief with a Euro-American cultural belief contextualized by reconciliation” (Aikenhead, 2017, p. 40).

Definitions clarified

Now that this article has explicitly taken on a contemporary cultural understanding of mathematics, I pause to acknowledge that some words and expressions have taken on a more precise meaning. A summary of definitions is required.

Mathematics, without a qualifier, *will have a superordinate meaning only*, the content of which is all of the mathematical knowledge systems that have developed historically in specific cultures. In operational terms, a mathematics knowledge system is a culture's composite of Bishop's (1988b) six universal mathematical processes (subsection on Mathematical Pluralism). In grammatical terms, these knowledge systems, such as Japanese mathematics (Wasan), carry a *subordinate* meaning of "mathematics," which always requires a qualifier (e.g., Japanese) or a totally unambiguous context.

Another existing subordinate knowledge systems could accurately be named *Euro-American mathematics* (EAM). Its content encompasses the following:

1. academic mathematics conventionally taught in schools and universities (Platonist content or Platonist mathematics) wholistically amalgamated with
2. content associated with EAM's cultural identity (illustrated in the Euro-American Mathematics Cultural Content Made Explicit subsection); its influence on political–social–economic–military contexts, known as mathematics-in-action⁷ (Irvine, 2017; Skovsmose, 2016); its everyday use, another type of mathematics-in-action (Fisher, 2017; Irvine 2017); its history (Ernest, 2016b; Nikolakaki, 2016); and its ontological, epistemological, and axiological presuppositions (Bishop, 1988b, 1990; Corrigan, Gunstone, Bishop, & Clarke, 2004; Ernest, 1988, 1991).

Mathematics' cultural content has been suppressed by the Platonist belief about the nature of mathematics (Yousan subsection). Thus, these cultural features, implicit in the 19th century, will today become explicit in school mathematics according to the intellectual and social maturity of students.

For the sake of clear precise communication, EAM content will continue to have two categories (Platonist content and cultural content), although in practice they are wholistically interwoven, just like Indigenous epistemologies would naturally conceptualize them.

A second point of clear communication needs to be mentioned: What is the consequence of *not* distinguishing precisely between the superordinate and subordinate meanings of mathematics? The answer is fuzzy or confused reasoning (Aikenhead, 2017). If someone unconsciously switches between the two, then any conclusion he or she reaches is likely illogical, a situation Aristotle labeled the *fallacy of equivocation* (Philosophy Department, 2017).

To summarize, from this point forward I shall use the following expressions as defined here:

- EAM or EAM content—both Platonist content and EAM's cultural identity content combined as an amalgam.
- EAM Platonist content—the conventional content currently taught in most schools and universities (representing an intellectual understanding).
- EAM cultural content—the cultural identity content of EAM (representing a wisdom understanding).
- EAM educators—educators who deal with EAM content explicitly.
- School mathematics—teaches EAM content principally, in keeping with Ernest's (1988) culture belief category and Bishop's (1988b) pluralism.
- Platonist school mathematics—conveys the pretence of being an acultural school subject; synonymous with conventional or academic mathematics.
- Enhancing or teaching school mathematics culturally—signifies that EAM cultural content is taught along with instances of Indigenous mathematizing⁸ beyond tokenism. Synonymous phrases will be *cross-cultural* or *culture-based* school mathematics. For Indigenous groups, such as the Mi'kmaw First Nation, I use the action form—mathematizing—to convey their verb-based language-laden cognition (Marginalization subsection).

Figure 1 depicts the relationships among key categories related to enhancing school mathematics culturally.

The regions A to D in Figure 1 are outlined here:

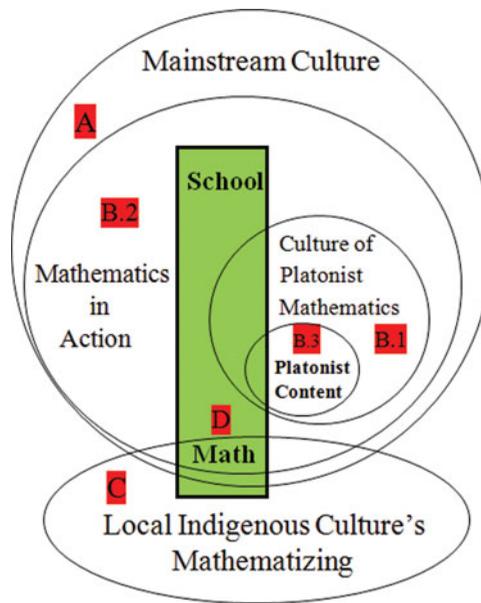


Figure 1. A schematic showing relationships among the components for enhancing Euro-American school mathematics culturally. (Proportion of overlapping has no significance.)

- A. The outer circle represents the content of a country's Euro-American mainstream culture.
- B. The inner three circles represent content in EAM, composed of the following:
 1. The culture of Platonist mathematics: its history, ideologies, values, and presuppositions.
 2. Mathematics-in-action in a country:
 - i. Mainstream culture's everyday activities or ideas that explicitly or implicitly involve either Platonist mathematics or analogues of it; including commerce, the trades, etc.
 - ii. Explicit or implicit societal influences that Platonist mathematics has on mainstream and Indigenous cultures for which people or corporations make social, legal, or ethical decisions.
 3. Platonist content.
- C. The ellipse represents the mathematizing found in an Indigenous culture.
- D. The shaded rectangle represents culture-based school mathematics that teaches the four content areas B.1, B.2, B.3, and C, thereby enhancing school mathematics culturally.

One important relationship revealed in [Figure 1](#) is that Platonist mathematics is a subculture within the culture of EAM.

Knowing that mathematics knowledge systems in some ancient cultures have contributed to today's EAM Platonist content, we could ask: How have these cultures all but disappeared in school mathematics, making it appear as if it were acultural? An answer lies in the Eurocentric habit of appropriating from other cultures, a topic addressed in the following subsection.

Of course, contemporary appropriation and marginalization are processes to be avoided in teaching and in R&D projects that enhance school mathematics culturally. These issues are addressed in the Appropriation and Marginalization subsections, which draw heavily from the following discussion explaining a mechanistic process of historical appropriation.

Historical appropriation

About 5,000 years ago, "mathematics had ritual functions" (Ernest, 2016b, p. 382) as well as record-keeping, accounting, and land surveying. "[T]he reliability of calculation, measures and numerical records was also understood as part of the idea of justice, taking on ethical as well as utilitarian and ultimately epistemological value" (p. 382). Ernest added:

By the time of the Pythagoreans, elements of number mysticism had developed. ... Three centuries later, Plato conceptualized numbers as self-subsistent entities existing in an ontological category apart from things bodily and mundane. Plato's ontology included numbers as ideal abstract forms, unchanging entities that could be known with certainty. ... A thousand years later, the emergence of modern science in the work of Galileo, Descartes, Newton and others prioritized numbers as the real existents behind our sensory experiences. Numbers were understood as invariant objects that characterized the most reliable and permanent knowledge of the world, part of the structure of the universe. (pp. 384–385)

The Eurocentric impulse to appropriate from other cultures can account for how European mathematicians throughout the centuries seem to have imported ways of mathematizing from earlier cultures but then reconstructed those ideas to fit the European mathematical philosophy or ideology of the time. How does this appropriation process occur exactly?

A mechanism is constructed out of Kawasaki's (2002) linguistic domain of *language-laden cognition*. It reveals that a word's meaning in one culture's language (e.g., pre-renaissance European mathematics) is attached to a cluster of associated concepts related to that culture's collective worldview. For instance, to understand an English/French concept beyond superficiality is to be aware of some of its associated concepts. A non-English/French language (e.g., Arabic) will reflect a very different collective worldview and likely a different cluster of peripheral concepts. This perspective resonates with how Ojalehto and Medin (2015) characterize the relation between culture and concepts:

Emerging trends in semantics, agency, and causal concepts converge on the idea that individual minds are grounded in systems of social relations, from the languages we speak to the (cultural) practices we engage in, and concepts must be understood as elements of those systems. (p. 9)

As a contemporary illustration of peripheral concepts, consider the following Grade 9 indicator in the Saskatchewan Mathematics Curriculum (2007): "Describe examples of where First Nations and Métis, past and present, lifestyles and worldviews demonstrate one or more of the *circle properties* (e.g., tipi and medicine wheel)." Note that the phrase *circle properties* refers to a *decontextualized* meaning of the term *circle*, with its cluster of peripheral Platonist concepts such as point and plain, as well as associated values such as decontextualization, intellectual purity, consistency, and objectivity—all examples of the Platonist content of EAM. The phrase *circle properties* does not refer to the Indigenous concept of something circular (in Plains Cree, wāweyiyāw). Instead, wāweyiyāw has peripheral concepts of a Plains Cree community's subjective, wholistic, and spiritual meanings of something circular.

The curriculum's indicator merely encourages students to *superimpose an EAM concept* (e.g., circle) *onto Indigenous objects*. This neo-colonial act of projectionism is revisited with other examples in the Projectionism subsection. Doolittle (2006, p. 20) criticized projectionism because it ignores substantial, Indigenous, peripheral concepts:

"The tipi is a cone," I have heard countless times. But that is surely wrong; the tipi is not a cone. ... It bulges here, sinks in there, has holes for people and smoke and bugs to pass, a floor made of dirt and grass, various smells and sounds and textures. There is a body of tradition and ceremony attached to the tipi which is completely different from and rivals that of the cone.

Without identifying "a body of tradition and ceremony" as peripheral concepts, mathematics professor Doolittle of Mohawk ancestry illustrated the importance of peripheral concepts when people move between the culture of EAM and the culture of an Indigenous community.

Peripheral concepts may subtly address three different types of cultural presuppositions, stated here with Indigenous examples: (1) *ontological*: dependent relational existence and sacredness; (2) *epistemological*: wholistic and paradoxical reasoning (Maryboy, Begay, & Nichol, 2006); and/or (3) *axiological*: "community solidarity, respect for the earth, and respect for elders" (Lunney Borden, 2013, p. 9); and Native Hawaiian values of mākana (*to care for*) and lokomaika'i (*to share with each other*; Furuto, 2013a).

A worldview encompasses ontological ideas of "what ought to exist" (Kawasaki, 2002, p. 26). For European mathematical thinkers who speak Standard Average European (SAE) languages (Whorf, 1959), their ideas fit Plato's "world of ideas," characterized by the purity of universalist objectivity that celebrates abstract nouns, as Kawasaki (p. 25) explains. For non-SAE mathematicians whose first language is Egyptian, Hindu, Arabic, or Chinese, for example, their "what ought to exist" tends to be more like Plato's

“phenomenal [material, tangible] world” that celebrates subjective placed-based processes or action. For each group, their peripheral concepts will reinforce their cultural ontological stance.

The historical European appropriation from ancient cultures is a translation issue. An SAE mathematician (a) may not even understand the peripheral concepts of a non-SAE mathematician or (b) may find those peripheral concepts irrelevant to the SAE mathematician’s world of ideas. In either case, the peripheral concepts are lost in translation. To be more specific, they have been stripped away.

Simply put, the historical appropriation process is one of deconstructing an idea from one culture (i.e., removing or stripping away its peripheral concepts) and then reconstructing it to fit another culture (i.e., adding peripheral concepts that make sense in that culture or subculture). This mimics what engineers or the general public do when they “apply scientific ideas” to an everyday situation or issue of interest to them (Aikenhead, 2006, p. 30).

In a rather parallel way, Ernest (2016b), explained historical appropriation by European mathematicians:

Eurocentric ideology ... has dominated historical and philosophical thought for the past 200 years. This ideology elevates rationality based on deductive reason as the highest intellectual good. ... The “afro-asiatic roots of classical civilization” have been neglected, discarded and denied. Thus, [their] vital developments in number and calculation ... are unrecognized as the essential foundation for all of mathematics including proof. (p. 385)

What might seem as a challenge to the grammatically subordinate identity of Euro-American mathematics, Donald and colleagues (2011, p. 75) mentioned “the myth of mathematics as a European discipline.” They understood school mathematics as a pluralist collection of mathematical knowledge systems from several ancient civilizations. This idea of a collection is indeed correct, in keeping with Ernest’s (2016b) explanation. But Euro-American *mathematics* has a subordinate meaning of “mathematics,” with a cultural content component that includes a more complex explanation involving language-laden cognition and peripheral concepts.

Thus, I respond to their challenge by noting we are both correct, I believe. This paradox can be easily resolved. We have two different stories to tell about ancient cultures’ contributions to EAM. Each story is relevant for different audiences: either (a) school mathematics is mainly a collection of multicultural knowledge systems with historical roots that Platonist mathematicians neglected, discarded, or denied or (b) school mathematics is mainly appropriated knowledge with a Eurocentric imprint (i.e., the original culture’s peripheral concepts were deconstructed and then reconstructed with Eurocentric peripheral concepts) plus its own mathematical inventions described in Ernest (2016b). This more complex explanation happens to describe a cultural feature of EAM (i.e., EAM cultural content related to the history of EAM). EAM certainly owes a debt of gratitude to the cultures that originally inspired earlier European mathematicians to appropriate ideas from them.

The imprint of appropriation will tend to alienate many Indigenous students because it manifests Eurocentric cultural features associated with superiority and aggressive authority that have the capacity to colonize. At the same time, this imprint of superiority and aggressive authority helps explain the high status enjoyed by Platonist school mathematics in a Euro-American culture (High Status of School Mathematics subsection).

Showing agreement with Bishop’s (1990) comment that Western mathematics is “one of the most powerful weapons in the imposition of Western culture” (p. 1; Yousan subsection), D’Ambrosio (1991) identified Platonist school mathematics’ identity and ideology this way:

The history of Mathematics is identified with its progress through colonialism, industrialization and the emergence of the great European Empires of the XIXth century. Mathematics is also the imprint of Western culture. This is easily identified with the decline and subordination, conquest and colonization and destruction of old empires of what is now called the Third World and with populations labelled as “minorities.” (pp. 12–13)

The Eurocentric impulse to appropriate from other cultures drew upon its own what-ought-to-exist, objectivity-conscious, and language-laden cognition, thereby ignoring or not being able to “see” (i.e., conceptualize; Projectionism subsection) the subjectivity-conscious content of other cultures’ language-laden cognition.

This describes, for instance, the British colonizers who could not understand the complex genealogy-based mathematics system in use by Australian Indigenous families (Watson & Chambers, 1989). The colonizers' inability to understand Indigenous mathematizing illustrates how the universalist stance of Platonist school mathematics can ignore and therefore demean Indigenous mathematizing. Is this history repeating itself today when mathematics educators adhere to the Platonist belief about school mathematics?

To prevent school mathematics from reenacting what happened in Australia, mathematics educators can participate in today's renewal of school mathematics (Examples of Enhancing School Mathematics Culturally). Boylan (2016) wrote:

Mathematics is a cultural product of our ancestors and positions humans as “participants in the great, age-old human conversation that sustains and extends our common knowledge and cultural heritage”; such a recognition entails “acknowledging that the conversation is greater than yourself” ... This suggests a responsibility to mathematics itself. (p. 402)

Accordingly, this article emphasizes mathematics educators' responsibility to critically scrutinize conventional school mathematics and then determine appropriate 21st-century Euro-American and Indigenous cultural practices that could serve as contexts for teaching school mathematics in an era of reconciliation. Which heritage of school mathematics should be passed on to future generations—a Platonist belief or a cultural belief within which Platonist content resides?

Euro-American mathematics cultural content made explicit

Because the suppressed EAM cultural content may not be familiar to many readers, it is described here in more detail. Masking power with innocence thrives if this content is kept invisible for future generations. As Bishop (1988a, p. 82) pointed out, “Mathematics, as a cultural phenomenon, only makes sense if [its] values are also made explicit.” His idea transfers well to the experience of most Indigenous students (Furuto, 2013a).

This is excellent advice for mathematics curriculum developers and teachers. A value-free knowledge system does not make common sense to students whose worldview is replete with, and relies on, values. Making EAM cultural content explicit, therefore, repositions school mathematics as a value-laden knowledge system that can make common sense to more students, Indigenous or non-Indigenous. Whether or not students agree with EAM's values can be sorted out by requiring students to least *understand* those values, even if students do not *believe* them (Culture Clashes that Alienate Many Indigenous Students subsection). Assessment in cross-cultural school mathematics is restricted to what students understand and can do. Assessing what they believe is indoctrination.

Ernest's (1988) cultural belief about mathematics led him to identify cultural values inherent in Platonist content. The following are expressed in the form of nonbinary dyads with respect to the value's importance: “Abstract is valued over concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, general over particular, theory over practice, the work of the brain over the work of the hand, and so on” (Ernest, 1991, p. 259).

In a more recent axiological study of “Western mathematical knowledge,” Corrigan and colleagues (2004, p. 10) identified six ideologies, each associated with a cluster of values undergirding Platonist content. The ideologies are rationalism, objectivism, control, progress, openness, and mystery (Aikenhead, 2017). This helps to discourage mathematics teachers from privileging EAM as being superior, rather than just different and coexisting with Indigenous mathematizing.

However, neither Ernest nor Corrigan and colleagues (2004) mentioned a cluster of aesthetic values dear to the hearts of mathematicians: elegance, simplicity, and beauty, any of which would apply to my personal favorite Platonist equation, Euler's unity ($e^{\pi i} = -1$) reworked as $e^{-2\pi i} = 1$.

In reality, of course, EAM *cultural* content interpenetrates EAM *Platonist* content. But the two are occasionally treated as separate categories in this article only for the convenience of discussing their respective content.

Now that cultural identities of EAM school mathematics have been exposed, we turn next to the pioneer educators who began to do something about it.

Early innovations

The real voyage of discovery consists not in seeking new landscapes but in having new eyes.—A famous paraphrase from French novelist Marcel Proust (1923)

Noted here are early innovations: in Brazil by D'Ambrosio, in countries where critical mathematics education scholars lived, in Aotearoa New Zealand by Māori educators, and in Alaska with Yup'ik communities. Each innovation addressed a different priority.

D'Ambrosio (1991) is credited as being the pioneer educator for teaching school mathematics enmeshed in local culture. He drew upon the cultures of Brazilian communities when teaching school and university mathematics and science in the 1950s and 1960s (Ascher, 1991). His inspiration was the Brazilian politics of social justice in the context of globalization. “Ethnomathematics was forged in the experiences, reflections, and hopes for a better quality of life” (Furuto, 2013a, p. 39), one of many mathematics-in-action topics.

D'Ambrosio has many followers, some of whom embrace agendas modified from his original agenda. Today, meanings of *ethnomathematics* include making school mathematics more relevant to different cultural or ethnic groups, describing how cultural values influence school mathematics, and studying the mathematizing in different cultures and subcultures (Ascher, 1991; Doolittle, 2006; Jannok Nutti, 2013; NOVA, 2016). This diversity has turned the term ethnomathematics into a slogan or vague metaphor, which serves as an influential rallying cry to improve students' experience in school mathematics (Ethnomathematics subsection).

During the period 1973–1988, the academic field of critical mathematics education was initiated. Greer and Skovsmose (2012) outlined its chronology. Early participants included M. Frankenstein, O. Skovsmose, M. Fasheh, A. Bishop, and P. Ernest. The field encompasses social justice, the curriculum, antiracist education, and the philosophy, politics, and ethics of mathematics education. Their influence endures today (e.g., Ernest, 2016a; Skovsmose, 2016).

Another early innovation occurred in Aotearoa New Zealand in the 1970s and 1980s. A large minority of Māori people, fearing the extinction of their language (te reo Māori), began establishing independent total-immersion schools that offered a “bicultural” approach to school mathematics (McMurchy-Pilkington & Trinick, 2002). In 1989, along with te reo Māori becoming an official language of the country, the Ministry of Education took responsibility for the Māori education system. Next came the 1992 decision to *translate* national curriculum documents into Māori, which led to a Māori version of the 1996 mathematics curriculum (Pāngarau) that was used in bilingual and total immersion Māori schools.

As discussed above (Historical Appropriation subsection), much can get lost or distorted in translation. Pāngarau included Māori vocabulary (including newly invented terms) and syntax but only to the extent that the vocabulary and syntax conformed to an English version of EAM Platonist content (McMurchy-Pilkington & Trinick, 2002, p. 468): “Some of the contexts and the exemplars are from a Māori perspective, but the concepts and ideas are underpinned by western thinking.”

McMurchy-Pilkington and Trinick (2002) warned that Pāngarau acted like a Trojan horse by surreptitiously assimilating Māori students into thinking in a Euro-American way (i.e., “cognitive imperialism”; Battiste, 1986, p. 23), thereby continuing the country's ongoing agenda of neo-colonization. Their concern has materialized, according to Russell and Chernoff (2013, p. 114): “Unintentionally, these endeavours to bring school mathematics into the Indigenous languages have actually resulted in further losses to the languages and culture.”

The pioneering group in Alaska (University of Alaska Project subsection) recognized the potential for positive outcomes when reducing culture conflicts between Indigenous students and EAM Platonist content (Lipka, 1994). They set about to reverse the alienation felt by many (but certainly not all) Indigenous students in conventional mathematics classes. Fundamentally, the Mathematics in a

Cultural Context (MCC) program is about interrogating the culture of school mathematics conventionally taught in Indigenous communities, in order to renegotiate with those communities an improved culture of schooling (Lipka, 1994).

These early innovations have helped the field mature to the point of having a noticeable presence in the research literature. Teachers and researchers have been inspired to join in. Out of fear of appearing disrespectful, they have wanted to learn how to avoid appropriating Indigenous knowledge or marginalizing Indigenous students.

Avoiding present-day appropriation and marginalization

Without mutual respect and mutual responsibility, the truth is we can achieve very little.—Prime Minister Kevin Rudd, February 13, 2008, from his apology to Australia's Indigenous peoples for the stolen generations (Rudd, 2009)

Many high-quality R&D projects discussed in this article begin with dialogues of respect, humility, and patience with Elders and knowledge holders in order to discuss language-laden cognition informally. They inform each other about, for example, their what-ought-to-exist presuppositions that might cause misunderstandings. Most researchers and teachers quickly lose their fear of accidentally insulting an Indigenous student or knowledge holder after talking about such fears with an Elder. This type of dialogue requires people to maintain an open mind in order to avoid a Eurocentric impulse of authority and to develop humility in appreciating the local Indigenous subjectivity-conscious activities from which mathematics educators will develop culture-based teaching materials. This helps avoid appropriation and marginalization.

Appropriation

Keene (2016, 6.33–6.44 min), of Cherokee ancestry, defined *cultural appropriation* as “taking content from another culture, erasing the original meaning, and using it as you see fit;” that is, using it for a purpose not intended by the source. In a contrasting view, Beatty and Blair (2015, p. 5) described it as “taking of Indigenous knowledge to use within a different cultural context, without truly understanding the cultural significance of the knowledge.” They raise an important point that we should understand what meaning (what peripheral concepts) is being erased from the original piece of knowledge.

Both views of appropriation express important essences, but they both ignore a tenet of Indigenous epistemology: knowledge belongs to (is related to) the person holding that knowledge. This implies that if I want to use that knowledge, permission must be given by the knowledge holder according to local protocol. Notably, permission depends on how I intend to use the knowledge. My request for permission is not a yes or no proposition; there are conditions and responsibilities I must follow. For instance, do I intend to incorporate it into my personal life only? Or can I share it: orally with my immediate family only, orally with students, in writing in teaching materials for other teachers to use, or digitally on a website available to the world? And when I do use it, my obligation is to show respect by explicitly acknowledging the permission granted by the Elder or knowledge holder, so that others know to whom the knowledge is related. In short, to avoid appropriation we must ensure that the knowledge holder has the power to make a fully informed decision: “Yes, go ahead”; “No, that wouldn't be appropriate”; or “OK, provided that. ...”

It is informative that Keene's (2016) condition “erasing the original meaning” points to the deconstruction process of stripping away Indigenous peripheral concepts (Historical Appropriation subsection). An Elder or knowledge holder can give me permission to do that but only if I am aware of the key peripheral concepts. To become aware usually entails experiential learning, plus engaging in a type of in-depth exchange of taken-for-granted ideas about people's worldviews, language-laden cognition, and values. Such a conversation many call “third-space dialogue” (Lipka, Sharp, Adams, & Sharp, 2007). Vickers (2007, p. 592), of Tsm'syen ancestry, called them “camping spots where we can dialogue” between cultures (e.g., Indigenous and non-Indigenous). The self-explanatory metaphor *camping spots of dialogue* captures a deep coming to know each other (Cajete, 2000).

If educators are not cognizant of the cultural significance of peripheral concepts being stripped away, they run the high risk of marginalizing Indigenous students by not being transparent about the transformation process that turns an Indigenous artisan object, process, or idea into a school mathematics lesson.

Marginalization

Educators also want to avoid unconsciously marginalizing Indigenous perspectives. Therefore, the following analysis of the process by which marginalization can occur will be helpful. The issue has fairly complex nuances to explore.

A linguistic explanation of marginalization parallels the language-laden cognition explanation of historical appropriation found in the Historical Appropriation subsection. But in this subsection, the explanation is specific to producing teaching materials.

When mainstream researchers and teachers visit, for instance, Nehiyaw (Plains Cree) communities, each culture group brings its own language-laden cognition to their conversations. Of course, Indigenous language-laden cognition will vary slightly from community to community. Kawasaki (2002, p. 24) clarifies who the actors are: on the one hand, the well-intended “objectivity-conscious” educators who tend to occupy mainly a world of ideas expressed in noun-based, abstract, decontextualized, symbolic language and, on the other hand, the “subjectivity-conscious” Nehiyaw community mainly in their phenomenal world expressed in verb-based Nehiyawewin (Plains Cree language). Of course, Indigenous languages contain highly abstract concepts as well, which harmonize with a group’s collective worldview.

An SAE noun-based language is often deficient in capturing the complex peripheral concepts attached to an Indigenous verb-based construct. (Examples were given in the Historical Appropriation subsection.) Moreover, subjectivity-conscious ideas are not usually relevant to the objectivity-conscious mind and will tend to be ignored. This subtle disappearance or dilution of an Indigenous perspective in the teaching materials—an instance of marginalization, to be sure—can make an Indigenous student or community feel violated. According to Doolittle (2006), students will likely

feel that their culture has been [marginalized] by a powerful force for the purpose of leading them away from their culture. ... Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures. (p. 20)

What can mathematics educators do to prevent it from happening?

Participating in a camping spot of dialogue, one example of a cultural immersion, is a *foundational prerequisite* for (a) anyone initiating a project that develops teaching materials; (b) teachers expected to implement them (Aikenhead et al., 2014; Belczewski, 2009; Chinn, 2007; Furuto, 2013b, in press; Fyhn et al., 2011; Lunney Borden et al., 2017; Michell et al., 2008); and (c) curriculum writers who compose outcomes and indicators (Mathematics Curricula and “The Opposition”). Cultural immersions are filled with superb learning moments, including those that guide educators around linguistic pitfalls such as confusion or misconceptions.

English/French language-laden confusion/misconceptions

The English and French languages seem to have a built-in colonizing vocabulary and syntax that hamper mathematics educators in *expressing* respect for the integrity of Indigenous cultures (Garrouette, 1999), despite their heartfelt respect for those cultures. A critical analysis of two expressions reveals detrimental neo-colonial messages.

1. “Tlingits manipulated their natural environment with great skill, requiring understandings of mathematical concepts and physics” (Bradley & Taylor, 2002, p. 53).

On the contrary, the Tlingit people of eastern Alaska survived for millennia without having contact with EAM Platonist concepts or Newtonian physics. Although meant as a compliment, the statement privileges Euro-American thinking by using it as the universal standard against which

to compare Indigenous thinking. Bradley and Taylor unintentionally projected their colonizing epistemology onto an Indigenous group—“cognitive imperialism” (Battiste, 1986, p. 23).

When is a compliment not a compliment? Consider the case of a school principal at a Grade 12 graduation ceremony. An Indigenous student receives a diploma with a top-of-the-class distinction. Instead of complimenting the student by saying, “We’re proud of you; we knew you could do it,” the principal exclaims with enthusiasm, “You’ve done exceptionally well for an Aboriginal student.” The student does not feel the praise but an unconscious putdown of the student’s ethnicity. This example highlights a neo-colonial masking power with a compliment (see Aikenhead [2017] for other examples).

2. “These [Yup’ik] practices form a coherent and generative set of concepts which *incorporate* geometry, fractions, ratios, and proportional reasoning” (Lipka et al., 2013, p. 132, emphasis added). An English vocabulary (e.g., geometry, fractions, etc.) is used here to describe Indigenous concepts, rather than either using an Indigenous vocabulary or restating the English to read, “These [Yup’ik] practices form a coherent and generative set of practices, which Euro-American mathematicians may recognize as analogous to geometry, fractions. ...” Without being edited, Lipka and colleagues’ (2013) statement, at best, implicitly depreciates the integrity of Yup’ik concepts. At worst, it conveys epistemological privilege and cognitive imperialism.

Just because the practices used by Yup’ik people can be described in English or French and with Platonist concepts does not mean that the people were using those concepts. And if they did not use those concepts, the concepts do not exist in Yup’ik culture. Garrouette (1999), Princeton doctoral graduate of Cherokee ancestry, pointed out the need to respect the integrity of Indigenous cultures in this way.

Avoiding these types of misunderstandings is a matter of describing Indigenous content by using a vocabulary that *clearly conveys an Indigenous perspective*, not a Euro-American one. For instance, rather than theoretical physicists announcing that “the language of nature is differential equations,” a reasonable revision might read: “Some scientists understand the physical world in terms of differential equations” or, perhaps, “Paradigmatic preferences among some scientists are metaphors borrowed from the field of differential equations.” Or as anthropologist Nesper (1994, p. 20) expressed it, “Mathematics equations in physics are representational technologies.”

A powerful technique for acquiring an Indigenous understanding is back-translation. For instance, ask a Nehiyaw-speaking person who is well acquainted with English/French to translate into Nehiyawewin (the Cree language) the word *knowledge* in the context of learning. In Nehiyawewin, as in many Indigenous languages, there is no such word. Thus, the person will utter a Nehiyaw expression normally used by Nehiyawak (Cree people) in the context of learning. Then get another independent Nehiyaw speaker to translate whatever the first speaker said back into English/French in a literal way. This is a back-translation. A Nehiyaw back-translation of *knowledge* usually comes out as “ways of knowing, living, and being” (Aikenhead & Ogawa, 2007, p. 553). This back-translation helps a person appreciate that the English expression *Indigenous knowledge*, when spoken by Nehiyawak, very subtly imposes a Eurocentric epistemology on what they say but do not really mean. The word *knowledge* acts as an assimilating Trojan horse.

How do we describe the complex process that appears to begin with an Indigenous group’s everyday object, activity, or understanding and ends with a culturally contextualized EAM lesson or unit? The literature is replete with verbs such as *to interpret*, *to transfer*, *to connect*, and *to relate*, all of which give no hint as to how it is done exactly. (More context-specific examples are discussed in the Projectionism subsection.)

As described in the Marginalization subsection, an Indigenous everyday object, activity, or understanding must be deconstructed and then reconstructed to fit into the culture of school mathematics. But how does a researcher select an Indigenous piece of artisan handwork, activity, or understanding to be deconstructed? A four-step process can be a general guide:

1. Forge a relationship with an Indigenous knowledge holder or Elder, adhering to local protocols (Lipka et al., 2013; Sterenberg, 2013a). This usually takes time—at least two or three meetings before the relationship is strong enough to embark on a collaborative project.

2. Become familiar with some of the group's ways of counting, locating, measuring, designing, playing, and/or explaining (Bishop, 1988b; Some Fundamentals of Mathematics subsection).
3. Guided by the insight that “it seems that the human mind has first to construct forms independently, before we can find them in things” (Einstein, 1930; cited in Director, 2006, p. 113), mathematics educators will draw upon their professional constructed forms or images (mathematical abstractions, concepts, and processes) to superimpose their mental forms or images on Indigenous groups' mathematizing, *within the educator's understanding of* that mathematizing. This is often accomplished, in part, by non-Indigenous researchers ignoring what they find irrelevant, such as Indigenous peripheral concepts—the same “stripping” process described in the Historical Appropriation subsection. But more important, with help from an Elder, researchers can become cognizant of some of the peripheral concepts otherwise lost during the superimposition and deconstruction phases.
4. Keep track of the content stripped away to use later as content to teach all students in cross-cultural mathematics courses, in order to make the stripping process transparent to everyone. For example, Fyhn (2013) showed how Indigenous mathematizing is integrated with cultural values: When making a square embroidering pattern on a fur parka, “The construction process is explained by the need for balance and harmony between the wearer and the spirit world, and between the user and the task” (p. 355). This will be some of the content stripped away, which should be made explicit to students at some appropriate time. Furuto's (2013a, p. 53) Native Hawaiian R&D project emphasizes “how universal values bind us [Indigenous and non-Indigenous] together,” as practiced by the international Hōkūle'a voyage around the world Hawai'i (University of Hawai'i Project subsection).

This mechanism can be summarized as *superimposing* a Platonist content image on an Indigenous object, process, or idea by using a best-fit trial-and-error method; *deconstructing* it in an Indigenous culture; and then *reconstructing* it in the culture of EAM with EAM peripheral concepts. This three-phase process will be represented by the metaphor of *transformation* (Aikenhead & Ogawa, 2007; Jannok Nutti, 2013). In Figure 1, it could be represented by a bridge from C to B.3, because the two areas do not overlap in terms of mathematizing (Garrouette, 1999).

Centrality of Indigenous languages

An English or French educator's sensitivity toward the issue of a mainstream language monopoly (i.e., language privilege) is sharpened by asking oneself: “Whose language is being spoken? Did I acknowledge that I am thereby privileged by the use of my language in this project?” When non-Indigenous educators learn some local key Indigenous phrases, this conveys sincerity.

Analyzing Indigenous language-laden cognition is central to the success of cross-cultural school mathematics. For example, when Elders observe a natural phenomenon, they might ask, “*Who* did that?” and “What might I do to show my responsibility to my relations in return?” whereas schools only ask students to learn, “*How* does the phenomenon happen?”

Actually, Nehiyaw Elders do not observe in the English objectivity-conscious meaning of the word. Instead, they engage in *kanawapamew* (nehiyawewin for *observing*; Beaudet, 1995), a meaning with peripheral concepts associated with *contemplating* the interrelationships that contribute to a *wholistic* understanding of the phenomenon in terms of everything in creation. In other words: Elders do not *explain* the universe, they *inhabit* it; and Elders do not ask *how* the universe works, they ask what the universe *is* (Battiste & Henderson, 2000).

The chance of “intercultural misunderstandings” is great (Hall, 1976, p. 165). Hall (1976) explains these misunderstandings in terms of the limitations of language itself: “The paradox of culture is that language, the system most frequently used to describe culture, is by nature poorly adapted to this difficult task” (p. 57). Aikenhead (2017) discusses details of Hall's contributions to this conversation, as well as the roles Indigenous languages should play when enhancing school mathematics culturally. Those discussions speak to the issue of communicating cross-culturally. According to Hall (1976, p. 166), language represents explicit features of a culture but not a culture's “out-of-awareness” features. People cannot

articulate their own culture's rules explicitly, but they can certainly tell you when you have contravened an out-of-awareness social rule or not.

This out-of-awareness knowledge informs teachers and researchers about the peripheral concepts associated with cultural processes, such as beading moccasins, from which mathematics educators wish to transform into an explicit language-laden cognition expressed in EAM teaching materials. "Out-of-awareness cultural systems have yet to be made explicit" (Hall, 1976, p. 166) and therefore they are accessible only through "observation of real events in normal settings and contexts" (p. 166). In Aikenhead (2017), mechanisms for doing so are explained and their implications for mathematics classrooms are illustrated; for example, transforming a moccasin beading activity into a school mathematics lesson.

Consider, for instance, a culturally contextualized EAM lesson on probabilities that draw upon playing an authentic Mohawk peach pit bowl game of chance called Lahal (Doolittle, 2006). Suppose the teacher asked students to graph the results in order to discuss an EAM Platonist concept of probability. *But what happens to that Indigenous, peripheral, out-of-awareness understanding when the graphs are constructed?*

If ignored, then Mohawk culture has been marginalized. To get around this consequence, the EAM lesson must have a way to draw students' attention to the process of stripping the game of its subjectivity-consciousness—its peripheral meanings. The lesson has now entered into an EAM cultural content discussion, along with Mohawk mathematizing. In other words, the superimposing–deconstructing–reconstructing process is being made transparent to students in order for them to learn some peripheral concepts of Lahal and some EAM peripheral concepts that replaced them—EAM cultural content. Non-transparency contributes to the marginalization of Indigenous students.

Could students, Indigenous and non-Indigenous, be challenged to articulate the marginalization problem as they draw their graphs? Perhaps students might be able to create a method of combining qualitative ideas on a quantitative graph; that is, a form of hybridized knowledge (Aikenhead & Michell, 2011; Enyedy et al., 2011; Jannok Nutti, 2013; Lipka et al., 2007). Perhaps an Elder might have a solution. Perhaps students' unsuccessful attempts would be sufficient to teach them a rational limitation of EAM Platonist content—it was not created to communicate a subjective consciousness. This in itself is a very powerful lesson about a cultural aspect of EAM. Perhaps students will learn an epistemic presupposition of conventional school mathematics' hidden curriculum: Ignore the context even though one is presented to you.

At the same time, students learn the Lahal game in an experiential way that affords an insight into Mohawk ceremonies and spirituality. Alternatively, the game's ceremonial and spiritual content could be explained by a Mohawk knowledge holder. In other words, students can acquire an *intellectual* understanding of the EAM Platonist concept of probabilities, and in the same lesson, students can appreciate a *wisdom* understanding embraced by Mohawk traditions. This lesson illustrates two-way or cross-cultural learning.

In the eastern lands of Canada, a Mi'kmaw speaker may say *mawikinutimatimk* (translated as *coming together*). But as Lunney Borden (2013, p. 9) pointed out, the translation loses the peripheral concepts of "equity and mutual respect." Perhaps the English translation of *mawikinutimatimk* should be "camping spots of dialogue" (Vickers, 2007, p. 592; Appropriation subsection). Learnable moments, such as discovering peripheral concepts, frequently arise from cross-cultural misunderstandings (Aikenhead, 2017). But learning moments can be predictable: Lunney Borden (2013, p. 14) wisely counsels teachers and researchers to ask, "What's the Mi'kmaw word for ... ?" and be open to learning more about an Indigenous person's vocabulary, syntax, and worldview.

Another excellent source of learnable moments is the process of back-translation (described previously), in which the peripheral concepts associated with an Indigenous central concept are, to some degree, exposed. More examples are found in Aikenhead (2017).

We must realize that an SAE language is an encyclopedic repository of a Euro-American culture. If the teaching materials for enhancing school mathematics culturally are going to be highly successful, then *it is absolutely essential to edit the SAE language* so that it conveys the fact that EAM is a human, social, and cultural enterprise with its own ideologies and its own set of cultural practices. In other words, *a writer must critically analyze his or her SAE language to detect and rewrite expressions that suggest that EAM content is acultural and objective when applied and that no other mathematical system is worth considering.*

Although a thoughtful use of language goes a long way to avoid appropriation and marginalization, teachers and researchers achieve even greater gains by using language specifically to make intellectual and human connections—a high cultural priority with Indigenous students and their community, and the central issue in the following section.

Making connections

The difficulty [in explaining the subject of mathematics] arises not only from the abstract character of the subject but also from the generality and lack of content of its propositions. It is hard to know what you are talking about in mathematics, yet no one questions the validity of what you say. There is no other realm of discourse half so queer.—James R. Newman (1907–1966) American mathematician and mathematics historian, *The World of Mathematics* (1956, p. 1614)

In the literature, two very general types of teaching materials—contextualized and cross-cultural—identify a continuum. At one extreme, R&D projects are contextualized in an Indigenous setting but are conducted with a sole emphasis on EAM Platonist content outcomes. At the other extreme, cross-cultural materials highlight detailed features of Indigenous mathematizing, along with EAM content, both of which students are expected to understand. Teaching materials can be identified by the degree to which Indigenous perspectives contribute a significant presence. To that end, many teachers and researchers claim an ethnomathematics identity.

Of course, the true test of a significant presence only comes with evidence of *how classroom teachers* use those materials (Jannok Nutti, 2013; Lipka et al., 2005; Russell & Chernoff, 2015), a topic rarely found in the literature. Two exceptions are worth noting. First, the Alaskan project Math in a Cultural Context published four case studies: Adams, Shehenaz Adam, and Opbroek (2005); Lipka, Sharp, and colleagues (2005); Parker Webster, Wiles, Civil, and Clark (2005); and Rickard (2005). The results are summarized in the University of Alaska project subsection. Secondly, Russell and Chernoff's (2015) research in classrooms identified critical incidents in which teachers' worldviews and lingering Platonist beliefs disrupted teachers' connections to Indigenous students in contextualized teaching lessons. Students became less engaged as a result.

Both contextualized and cross-cultural approaches aim to resolve an oft-repeated complaint about school mathematics; for example, Hogue's (2013) interview with

a former Blackfoot student, now a medical doctor, who struggled greatly with the sciences and mathematics, said, "It helps if you show it first with something that makes everyday sense to me so that I can see the relationship for myself then let me do it so I can see how it works first." Students need context to make connections and the more familiar that context is, the easier the connections are. (p. 1)

However, this sound advice comes with two crucial qualifications. First, if an SAE word for an EAM concept does not have a simple Indigenous translation, then in accordance with language-laden cognition, the concept will most likely be very foreign to an Indigenous speaker. For instance, the English word *middle*, as in "point to the object in the middle," has no equivalent in the Mi'kmaw language in eastern Canada (Lunney Borden, 2013, p. 19). A back-translation is "go half way" (*aqatayik*). This is a reminder that not all noun-based concepts can be directly translated into verb-based SAE concepts and vice versa.

Secondly, Enyedy and colleagues (2011) discovered that students differ according to how strongly they connect to the peripheral concepts attached to their culture's mathematizing in an everyday situation. At one end of this spectrum, some students with a strong allegiance cannot let go of a construct's peripheral concepts, even momentarily. Consequently, there is little movement *from* their local cultural thinking *toward* the school's Platonist mathematics thinking. At the other end of the spectrum, some students have already figured out an ideological feature of Platonist school mathematics: "[T]he real game [is] to ignore the context" (Enyedy et al., 2011, p. 276), or some students can easily take on a "let's pretend" position in the context of doing mathematics.

Aikenhead (2017) discusses the difficulties in moving back and forth between a culture that celebrates useful subjectivity-conscious features of mathematizing and a culture that celebrates objectivity-conscious decontextualized EAM Platonist content. In other words, many Indigenous students negotiate when they go between, on the one hand, a knowledge-in-action value-laden Indigenous culture and, on

the other, an apparent “let’s pretend,” value-free, EAM Platonist content culture. There would be less culture clash if they moved into a cross-cultural EAM that dealt with both the “let’s pretend” (EAM Platonist content) and the “mind the hidden values and ideologies” (EAM cultural content), a situation familiar to Indigenous students raised on Diné (Navajo) paradoxical reasoning (Maryboy et al., 2006) or to any student experienced in “two-eyed seeing” (Hatcher, et al., 2009).

If teachers and researchers treat Indigenous artisan handwork, practices, and ideas *merely* as cultural objects to be encountered with an EAM lens, teachers and researchers risk enacting a neo-colonial agenda by ignoring the foundational policy that Indigenous students need to strengthen their cultural self-identities while engaging in school mathematics (Russell & Chernoff, 2013).

Some contextualized lessons may nurture stronger Indigenous self-identities for a few students to some degree, but negative reactions can result for other students. Partially quoted earlier but a crucial idea well worth repeating (Doolittle, 2006):

My feeling is that Indigenous students who are presented with such oversimplification feel that their culture has been appropriated by a powerful force for the purpose of leading them away from the culture. The [contextualized teaching materials] may be reasonable but the direction is away from the culture and toward some strange and uncomfortable place. Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures. (p. 20)

More appropriate teaching materials will meet Davison’s (2002, p. 24) challenge “that they make sense both in the context of the [Indigenous] culture and in the context of the school mathematics curriculum.” The standard of appropriateness increases even higher with Enyedy and colleagues’ (2011) position on the connection between researchers and Indigenous community members:

Both parties must legitimately be engaged in the negotiation process in order to avoid creating artificial activities under the guise of authenticity; engaging both parties in *co-construction* of a local meaning of relevance helps to ensure that the end product is, in fact, relevant and meaningful to all of the participants. (pp. 288–289, emphasis added)

The potential quality of contextualized or cross-cultural teaching and R&D projects generally increases when they are characterized by descriptors such as co-constructive, collaborative, cooperative, and cogovernance. Engagement in reconciliation occurs for teachers and all of their students.

Doolittle (2006) shared two word problems that meet the minimum criterion of appropriate contextualized school mathematics: (a) Imagine that you and three friends are sitting on the ground with 72 pennies piled in front of you. What would you do so that each of you got the same number of pennies? and (b) If your big brother took his truck to Calgary, how much would he have to spend on gas? Unexpectedly, some Indigenous students with strong connections to their culture’s epistemology will provide a teacher with a learnable moment about these students’ worldviews (Aikenhead, 2017).

Stenberg (2013a) noted that because of a question’s wording, some Indigenous students *could not see themselves reflected* in the situation described and could not relate to it. Their honest response to questions will hint at what they do relate to. On a much larger scale, when students do not see themselves reflected in the mathematics curriculum, their reactions to it are much the same (Lunney Borden, 2013).

Ethnomathematics

The social justice politics of D’Ambrosio’s (2003) ethnomathematics counters some neo-colonizing ideologies of conventionally taught mathematics courses. He introduced his ethnomathematics to make the school’s Platonist content more inviting and accessible to marginalized minority students (D’Ambrosio, 1991). At the same time, he successfully challenged the universality of Platonist content by initiating a research program that legitimized different mathematizing found in various groups within Brazilian society, including Indigenous groups. In other words, his research and teaching was about mathematics-in-action. Ethnomathematics is expected to be “far more reflective and respectful to Indigenous traditions of thought” (Doolittle, 2006, p. 20). D’Ambrosio’s work has helped mitigate the power imbalance between the mathematics curriculum and marginalized communities, a major legacy, to be sure.

Today D'Ambrosio's pedagogical strategies are recognized as one approach to implementing culturally responsive or place-based school mathematics (Furuto, in press; Sterenberg, 2013b). In his pursuit of peace education, D'Ambrosio (2007, p. 34) wrote: "The main goal of Ethnomathematics is building up a civilization free of truculence, arrogance, intolerance, discrimination, inequity, bigotry and hatred." He gives special attention to the political–social contexts of use for Platonist content in "the technological, industrial, military, economic and political complexes [that are] responsible for the growing crises threatening humanity. Survival with dignity is the *most universal problem facing mankind*" (D'Ambrosio, 2007, p. 25, emphasis in original). He points disparagingly to mathematicians who work in the "weapons industry" (D'Ambrosio, 2007, p. 27).

D'Ambrosio pedagogy can be characterized as contextualized EAM Platonist content that highlights *human interactions and responsibilities* found in students' local community (Greer, Mukhopadhyay, Powell, & Nelson-Barber, 2009). Therefore, his ethnomathematics celebrates the policy of strengthening Indigenous students' self-identities.

EAM cultural content includes this political–social context of Platonist content-in-action (Definitions Clarified subsection). But it also includes ontological, epistemological, and axiological presuppositions that frame Platonist content, and it includes its history. The presuppositions and history portion of EAM cultural content, however, do not seem to exist for D'Ambrosio (1991, 2003, 2006, 2007). By accepting this 19th-century myth that Platonist mathematics is value free, decontextualized, acultural, nonideological, and purely objective, his term *mathematics* in the expression *school mathematics content* consistently conveys this myth, given the fact that he does not discuss any cultural values or ideologies of Platonist mathematics (Euro-American Mathematics Cultural Content Made Explicit subsection) but only its association with the imprint of Euro-American cultures. Every other culture's mathematizing is recognized by him as being cultural, but not Euro-American mathematics.

He seems to embrace an amputated version of mathematics' pluralism. This amputated piece—the presuppositions and history of Platonist content—happens to diminish the culture clash between most Indigenous students and their Platonist school mathematics classroom (Culture Clashes that Alienate Many Indigenous Students subsection). Further examples of D'Ambrosio's blindness to the cultural nature of Platonist mathematics arises from some projects described in Examples of Enhancing School Mathematics Culturally.

As a result of ethnomathematics restricting itself to a truncated pluralism, the hegemonic power imbalance favoring mathematics curricula over Indigenous communities is not *sufficiently* challenged by ethnomathematics. Indigenous students' self-identities could certainly be strengthened much further than they are.

Treating school mathematics content as culture laden is foundational to teaching mathematics culturally. The expression *Euro-American mathematics* means the amalgam of EAM cultural content and EAM Platonist content (Definitions Clarified subsection). As detailed in Correcting Lingering Impediments to Student Success, however, some researchers who subscribe to culturally responsive or place-based school mathematics, which includes EAM cultural content, also insist that their project is based on ethnomathematics, which excludes an important portion of EAM cultural content. Hence, researchers find themselves in a logical fix, a problem they usually just ignore. This is a conundrum that many teachers or researchers face when identifying with ethnomathematics.

Examples of enhancing school mathematics culturally

They could have come up with an education plan that would have complemented Native cultures and, perhaps, even enriched White culture at the same time.—Thomas King (2012, p. 119).

As mentioned at the outset, this article is neither a comprehensive review of the literature nor a clearinghouse for R&D projects. Instead, it critically analyzes school mathematics and suggests what mathematics researchers and teachers should know when enhancing school mathematics culturally. A critical analysis entails identifying positive features that demonstrate best practices (Examples of Enhancing School Mathematics Culturally), as well as negative features that could be improved or avoided in the

future (Examples of Enhancing School Mathematics Culturally and Correcting Lingering Impediments to Student Success). Which innovative taken-for-granted notions (presuppositions) of school mathematics improve student achievement? and Which notions found in conventional school mathematics continue to serve students' interests? Examples of Enhancing School Mathematics Culturally illustrates in concrete terms some of the general issues introduced in earlier sections.

In the context of this article, achievement includes four principal dimensions: (a) students' test scores on relevant EAM Platonist content; (b) students' coming-to-know—a back-translation of “to learn in a deep way” (Cajete, 2000, p. 110)—cross-cultural EAM cultural content and Indigenous perspectives; (c) show-what-you-know assessment activities; and (d) Indigenous students strengthening their cultural self-identities, while joining non-Indigenous students in reconciliation.

A cross-cultural approach often works within place-based or culturally responsive mathematics teaching/learning (citations in Introduction). Both approaches to teaching/learning wholistically embody the emotional, spiritual, intellectual, and physical dimensions of a location where Indigenous people reside. Both combine a wholistic array of students' languages spoken, knowledge developed, and wisdom accumulated.

Because most innovative contextualized and cross-cultural projects are highly localized in keeping with the place-based nature of Indigenous perspectives (Michell et al., 2008; Sterenberg, 2013b), projects are usually small scale and known only within their local rural or reserve educational jurisdiction. Often, financial and human resources are all spent on making a project successful, leaving no resources for wide dissemination. Those projects that involve academic scholars, however, tend to be found in the academic literature. The source of material for this section was delimited to published material in the Northern Hemisphere for greater transfer to North American contexts.

Urban sites have not been the focus in the literature, although University of Alaska Project, University of Hawai'i Project, and Mi'kmaw First Nation Projects subsections include a few urban classrooms. Innovations in most urban schools require a scaling-up process (Elmore, 2003) and political action (Innovative Educators and Researchers subsection). Individual teachers and future R&D projects will be well served by the guidance in this section for enhancing school mathematics culturally.

British Columbia Project

In British Columbia,⁹ where many different First Nations groups live without treaties, the First Nations Education Steering Committee (FNESC) produced the teaching resource *Teaching Mathematics in a First Peoples Context: Grades 8 and 9* (FNESC, 2011). It is a very pragmatic document to help mathematics teachers “extend their existing practice to incorporate new approaches that make the BC school system more reflective of the realities of First Peoples in this province” (p. 7). Reading between the lines, I recognize that a great deal of appropriate effort went into the monograph's development, which represents a contextualized approach (see section Making Connections). However, individual teachers can take the initiative to augment their chosen unit into a cross-cultural approach, as the resource suggests they do.

The project produced 10 unit plans (e.g., Cooking with Fractions and The Water Keepers). Each unit explicitly draws upon artisan handwork, activities, or ideas found in British Columbia's Indigenous cultures. Some units simply show how to use EAM Platonist content (e.g., statistics) in an Indigenous context without much explicit instruction on Indigenous values and relationships attached to the meaning of an activity such as salmon fishing. Teachers' self-initiative could fill in this gap. Other teaching units provide this specific information.

One of many strengths of this resource is its chapter entitled “Making Connections.” “Building strong community links—engaging in *consultation* with local First Peoples and seeking their support for what is being taught—will allow you to provide active, participatory, experiential learning and to localize course content” (FNESC, 2011, p. 15, emphasis added). The resource, however, gives weight to the weaker process of consultation, rather than the stronger *collaboration* (Improvements in Developing Teaching Materials subsection).

At one point in the document, the deficit model of teaching is emphasized (Improvements in Developing Teaching Materials subsection): “The students who come to our school have serious gaps in their

education” (FNESC, 2011, p. 29). When mathematics teachers complain that Indigenous students lack the background for doing well in their class, these teachers are masking power with innocence. By framing the issue as a lack of background knowledge, they implicitly fault the students. A critical analysis exposes a much different perspective: the country’s dominant political–social agenda of colonization forced an “educational debt” on Indigenous students (Bang & Medin, 2010, p. 1023). It is this educational debt that teachers are actually complaining about and perhaps about the fact that mathematics teachers are expected to help “pay off” the debt through teaching mathematics culturally.

The FNEC (2011) resource includes two very thoughtful lists, “First Peoples Principles of Learning” (p. 9) and “First Peoples Principles of Mathematical Teaching” (p. 10). Teachers and researchers will find these ideas helpful in discussions with a knowledge holder or Elder.

The monograph deals with making connections between (a) school mathematics and Indigenous perspectives, (b) teachers and students, and (c) teachers and Indigenous themes and topics. In other words, teachers are offered detailed support to enact one of several foundational aspects of Indigenous worldviews: reality is composed of a web of dependent interrelationships and sacred responsibilities to those relationships.

Blackfoot First Nations confederacy study

Rarely can readers watch the dynamics unfold as a mathematics teacher evolves from a contextualized instructor into a cross-cultural instructor. Sterenberg (2013a) gives a clear account of a teacher’s professional transformation, fundamental to anyone who creates materials for teaching mathematics culturally. In this instance, the material was simply a lesson plan contextualized in the real world of teaching. But the teacher’s journey is replete with advice for any teacher or researcher reticent about making “mistakes.”

Sterenberg, whose ancestry is non-Indigenous, collaboratively coinvestigated with Bryony, a Blackfoot teacher in an Alberta Blackfoot reserve school. Bryony was beginning to integrate Indigenous perspectives with the province’s Grade 9 mathematics curriculum. Sterenberg’s two-part study immersed itself in the local culture by adhering to an Indigenous paradigm of social science research (Donald et al, 2011; Wilson, 2008). The first part focused on how Bryony modified a published unit that taught geometry in the context of house construction. The second part followed Bryony’s journey as she moved along a continuum from emphasizing EAM Platonist content contextualized in a housing module to a more equitably balanced emphasis on both cultures’ mathematizing; that is, a cross-cultural approach (Making Connections).

For Bryony (Sterenberg, 2013a):

Teaching from an Aboriginal perspective is simply finding what is meaningful and relevant to the students that honours the ancestors of the host territory in which teachers live and teach. It means teaching the curriculum and addressing silent identity issues simultaneously by revering the land and people from which the students came. (p. 23)

When examining other teaching resources at hand, Bryony was offended by three features: tokenism, the “pan-Indian homogenization of Indigenous culture” (Sterenberg, 2013a, p. 25), and the exclusion of contemporary Indigenous culture.

Following the house construction module’s instructions, she conscientiously engaged her students in meeting with the reserve’s housing committee, drawing scaled maps of the local land and of blueprints of reserve houses, adding furniture icons in scaled proportions to the blueprints, creating a furniture budget, calculating how long it would take to pay for the furniture, and discussing tax structure and band policies. All of this involved ratios, proportional reasoning, and much arithmetic. There was no mention, however, of relational, spiritual, or emotional aspects of the community’s Indigenous perspectives. Peripheral concepts connected to these aspects give epistemological, ontological, or axiological meaning to Indigenous artisan handwork, processes, or ideas. Thus, one might ask: How authentically Indigenous did the housing unit and furniture activities appear to the students?

Bryony had worked diligently to make her adaptation relevant to her students, but the “house construction unit did not resonate with their experiences” (Sterenberg, 2013a, p. 26). Next “she wondered

what a mathematics project that started from Indigenous mathematizing might look like” (Sterenberg, 2013a, p. 27).

Bryony then made the crucial decision to talk with an Elder about Blackfoot mathematizing and school mathematics. From those conversations and from professional reading, she learned more about an Indigenous notion of teaching from place (Sterenberg, 2013b). In other words, Bryony continued her journey into cross-cultural school mathematics.

Part two of Sterenberg’s (2013a) study was based on the fact that “an emphasis on quantifying procedures rather than *a focus on relationships* often becomes the sole priority in school mathematics” (p. 24, emphasis added). A Blackfoot priority is a placed-based, wholistic, and relational “coming to know” (Cajete, 2000, p. 110). Cross-cultural school mathematics embraces *both* EAM and Indigenous perspectives. And now, so does Bryony.

Following proper protocol, Bryony arranged a field trip to a sacred medicine wheel site. Collaborating with the Blackfoot Elder, she designed four learning centers, a sacred number for many First Nations peoples. Two centers involved EAM Platonist content, where students would measure features of the medicine wheel and investigate quantitative relationships among the measurements. The other two centers dealt with information about the sacred place. Other survival-related activities were planned. Importantly, Bryony’s idea of “authentically relating Indigenous knowledges and mathematics curricula” (Sterenberg, 2013a, p. 27) had become linked to an Indigenous sense of place.

On the field trip, the class was accompanied by four resource experts: the Elder, an archaeologist acquainted with the site, a reflective writing instructor, and a Blackfoot cultural teacher who knew details about Blackfoot mathematizing and the cultural significance of the site. The site visit began in a good way with the Elder saying a prayer, in one of the three Blackfoot confederacy’s languages, and leading everyone in making sacred offerings of tobacco to what some Indigenous people call Mother Earth. The Elder also described the preparatory ceremony conducted the day before. Then the cultural teacher told “the story of the Iniskim, which are sacred calling stones; the Iniskim call out to people to find them” (Sterenberg, 2013a, p. 28). It is an ancient story that relates the site’s small stones to the buffalo. The students became fascinated. At the story’s end they were carried away and wanted to act out part of the story. This would alter the field trip lesson plan. Bryony’s intuition advised her to go with the flow. The day turned out to be an exciting and educational success for the students and a highly worthwhile learnable moment for Bryony.

When she reflected on the day, she realized that rather than begin a lesson with EAM Platonist content (i.e., two of the teaching centres), “she had provided students with the opportunity to respond to the teachings of the land. ... She was flexible in recognizing the significance of learning from place” (Sterenberg, 2013a, p. 28). Bryony stated, “They’d understood why this place was important and were now able to incorporate the math” (Sterenberg, 2013a, p. 28). Sterenberg wrote, “Students seemed to have a positive attitude about their own perceptions that engaging in mathematics was a human and social endeavor” (2013a, p. 29).

This vignette points out the power that stories hold, the power of the land as teacher, and the power of a three-part teaching sequence: first give priority to the local Indigenous culture, follow up with EAM directly associated with students coming to know the land, and, finally, buoyed by students’ motivation, teach the EAM content unrelated to the land but important in the curriculum. The other way around treats Indigenous perspectives as add-ons, Bryony claimed. Sterenberg (2013a) concluded that EAM Platonist content and Blackfoot mathematizing can only be successfully integrated if a teacher begins “from Blackfoot knowledge of the land” (p. 29). Two meanings of integration are explored in Aikenhead (2017).

For me, five important points arise from Sterenberg’s inquiry into Bryony’s project to create teaching materials that will enhance her mathematics pedagogy in a cultural way. First, there are many other mathematics teachers who have parallel stories to tell but are not published. Second, a very worthwhile future research program would be to discover, write up, and publish a compendium of these powerful professional development stories (e.g., Aikenhead et al., 2014; Lipka et al., 2005). Third, Bryony’s lesson plan (the product of her project) is naturally place based and, therefore, it is not easily repeatable except for those few communities on Turtle Island (North America) that are located near a medicine wheel or some

other sacred site. But it certainly has inspirational value for other teachers. Fourth, Sterenberg's research illustrates the richness of results that qualitative research produces. To transpose her research into a numbers-generating instrument, such as a theory-based questionnaire, would strip away her study's core significance, and it would lose its power to influence teachers and policy makers. However, questionnaires have been very useful in identifying people to include in a qualitative study. And fifth, it is all about relationships, relationships, relationships: Bryony's relationship with an Elder moved her planning forward; her students' relationship with the sacred site propelled their engagement forward; and her relationship with her students ensured that she could go with the flow with excellent results.

A key feature, significant by its absence from Sterenberg's (2013a) article, however, is Blackfoot language and back-translations of important terms or expressions. Facets of an Indigenous language add a crucial dimension to forging relationships with students and their community and helping students connect with EAM school mathematics.

Sterenberg and McDonnell (2010) completed another study in the same school with a different teacher (Aikenhead, 2017).

University of Alaska project

Due to poverty, racism, and marginalization similar to Canada's, student alienation was high and achievement was particularly low in school mathematics. Alaskan schools were continuing to colonize their Indigenous students, especially in rural areas.

This was the main issue that motivated the creation of a small Yup'ik teachers' research group associated with the University Alaska Fairbanks in the late 1980s. The group called themselves *Ciulistet*, a Yup'ik word meaning leaders. They collaborated extensively with people in their villages so that their culturally based school mathematics would have strong local roots and become a powerful movement for change throughout southwestern Alaska (Lipka, Mohatt, & The Ciulistet Group, 1998). Outside of Alaska, their cross-cultural innovation gave hope and direction to others.

"Two-way learning occurred, and both Western and Yup'ik systems were valued" (Lipka, 1994, pp. 15–16). The group's action research and ethnographic school-based results led them to realize "the potential of using their culture and language as a means to change the culture of schooling" (Lipka, 1994, p. 14). Ciulistet worked closely with the Alaska Native Knowledge Network (2016).

Ciulistet's and the Alaska Native Knowledge Network's policy was to negotiate what school mathematics will be, one school jurisdiction at a time. They altered the colonial established, power imbalance between each Yup'ik community and its school mathematics (Lipka et al., 1998; Lipka et al., 2005) and created opportunities for two-way learning—cross-cultural school mathematics (Lipka, Yanez, Andrew-Irhke, & Adam, 2009). Ciulistet's capacity development within Yup'ik communities put participating schools on a pathway to decolonization. It became MCC (Mathematics in a Cultural Context) in 2005.

Currently, MCC (2016) has produced 10 modules (e.g., Kayak Design and Salmon Fishing) that supplement the Alaskan mathematics curriculum for Grades 1 to 7. They are intended for Indigenous and non-Indigenous students in rural and urban schools. The modules include "DVD clips of teachers' implementing exemplary lessons, case studies, a *Guide to Implementing MCC*, literacy activities and stories that develop cultural, mathematical, and contextual connections for students" (MCC, 2016).

The Yup'ik language appears in the materials enough to warrant having a Yup'ik glossary at the end of each module. But the proportional appearance of the Yup'ik language must be negotiated in each community: "... [S]ome want schooling to represent their culture while other groups want schooling to teach the dominant society's 'secret language'—the language of power" (Lipka, 1994, p. 22).

A very high standard of project evaluation was established by the pre-MCC group (Ciulistet). Their evaluation's mixed methods approach included both quantitative and ethnographic studies. The former provided a highly appropriate and reliable big picture of what happened, of special interest to administrators and funding agencies.

The extensive quantitative studies covered 5 years (2001 to 2005); collected pre- and posttest scores on state standardized tests that yielded student gain scores from about 3,000 students in 15 schools (Lipka & Adams, 2004; Lipka et al., 2005). Treatment classrooms (where MCC modules were taught)

were paired with control classrooms (conventional school mathematics) in a way that established a rigorous quasi-experimental research design. The studies produced the following results: (a) the treatment groups' gain scores were statistically significantly greater than the control groups' gain scores; (b) "Although the urban treatment group gained the most from this curriculum, the most important finding is that the rural treatment group outperformed the rural control group" significantly (Lipka & Adams, 2004, p. 3); and (c) Lipka and Adams (2004) concluded, "[I]t shows that the treatment effect on Yup'ik students narrows the long-standing academic gap when comparing [the Yup'ik treatment] group's and the Yup'ik control group's relative performance against the urban control group" (p. 3).

The ethnographic side of the evaluation of MCC produced the four case studies mentioned above (Parker Webster et al., 2005; Lipka, Sharp, et al., 2005; Adams et al., 2005; Rickard, 2005). They support the quantitative findings and offer rich detail to help explain those findings and to pinpoint complexities in what occurred in the treatment classrooms. These complexities are very useful to mathematics educators involved in culture-based education of Indigenous students, whether producing teaching materials, teaching with them, conducting teacher professional development programs, or introducing them into teacher education. The MCC case studies included a wide spectrum of teachers, grades, modules, and proportion of Yup'ik students in a classroom (between zero to 100%).

Factors that interacted positively with the cross-cultural teaching materials include *the extent to which teachers*

1. fostered interpersonal relationships (teacher–student and student–student).
2. based their mathematics lessons on local culture and people, including students experienced in the cultural topic, when feasible.
3. ensured that students took ownership of their own personal contributions to group knowledge building (e.g., when reaching a consensus).
4. judiciously balanced an expert–apprentice model and an interactive model based on equality.
5. planned experiential coming-to-know activities and determined how many will occur outside the school.
6. created a harmonious, comfortable, respectful classroom environment.
7. used problem-centered, inquiry-oriented methods.
8. were able to get students to articulate their ideas and the reason behind their answers.

What a teacher does, of course, makes all the difference to the success of a culture-based module. For this reason, professional development programs that begin with a transformative culture immersion experience, especially for non-Indigenous teachers, should be a prime consideration.

Currently, MCC is collaboratively expanding its student assessment item bank "to integrate authentic cultural practice and mathematical knowledge" (Lipka et al., 2013, p. 130). Each question goes through a rigorous R&D process.

University of Hawai'i project

I begin this section with a synopsis of Native Hawaiians' political–social context pertinent to school mathematics. Hawaiians at Waimea Bay, Kaua'i Island, discovered Captain Cook and crew on their beach in 1778—their first encounter with a White person. In the 19th century, American businesses flourished in coexistence with the Kingdom of Hawai'i, until friction heightened over Queen Lili'uokalani's sustainable development policy versus American business interests. It was settled by the U.S. Marines in 1893, who invaded O'ahu, dethroned the Queen, overthrew the monarchy, and gave power to the business groups, who soon helped secure Hawaii's territorial status with the United States in 1898. A familiar story of racism, marginalization, and oppression ensued, thereby colonizing most Native Hawaiians into poverty.

A sovereignty movement in the 1960s initiated the rebirth of the Native Hawaiian language, culture, wisdom, and identity, which continues today. Its initial success was proudly celebrated in 1985 when a replica of an ancient double-hulled Pacific Island canoe, the *Hōkūle'a*, sailed around the Pacific as far as Aotearoa New Zealand and back to Hawai'i, thereby "rekindling the Pacific Island tradition of non-instrument way-finding techniques that include celestial navigation" (Furuto, 2014, p. 113). It has become

“a vehicle to bridge [I]ndigenous models of mathematics at the local and global levels” (Furuto, 2014, p. 113).

The *Hōkūleʻa* voyages, a state treasure to be sure, function as one of many contexts with which to enhance school mathematics culturally. This is the central interest to the Ethnomathematics and STEM (science, technology, engineering, and mathematics) Institute (ESTEMI). This group’s diverse projects began in about 2008 at the University of Hawaiʻi, Oʻahu. The Institute is anchored in college/university-level courses and programs that offer ethnomathematics pedagogies that fit culturally responsive and place-based agendas. As discussed in the Ethnomathematics subsection, ethnomathematics is defined as “the intersection of culture, historical traditions, sociocultural roots and mathematics” (Furuto, 2014, p. 112). ESTEMI has been incredibly successful at increasing student enrollment in undergraduate mathematics courses (by about 70% each year recently) and successful at almost doubling the overall passing rate in these courses, to a level “far above” the average for all campuses in Hawaiʻi (Furuto, 2014, p. 118).

But there is much more to ESTEMI than undergraduate ethnomathematics. Its *educational context* emphasizes school-level international examinations. ESTEMI’s ultimate goal is to strengthen the STEM pipeline from K–12 to college mathematics and on to a career and community readiness. The pipeline’s strength arises from school and postsecondary mathematics courses being attractive throughout students’ education (Furuto, in press).

ESTEMI gives great attention to school teachers’ professional development that features cultural immersions in communities throughout Hawaiʻi. Furuto (2014) describes the experience this way:

Through an orientation series of professional development workshops, and a 1-week summer institute, educators from across the State of Hawaiʻi and [other] Pacific [Islands] design and implement mathematics lesson plans grounded in ethnic, historical and cultural diversities of our island homes. The resulting research and practicum-based textbook is used by current educators to supplement curriculum, and future teachers as training material aligned with MCCSS [Mathematics Common Core State Standards]. (p. 115)

Compared to other major R&D projects, the Institute appears to reach a more equitable balance between its focus on EAM Platonist outcomes and its attention to Indigenous perspectives, due to its cultural immersions, a priority on Native Hawaiian values, attention to Native Hawaiian vocabulary, and the university-level courses that extend that Indigenous focus further, even assessing university students on this cultural content (Furuto, 2013a, in press). This *cross-cultural* project characterizes balance.

The textbooks mentioned in the quotation above refer to Furuto (2012, 2013b), which include some of the module lesson plans found on the ESTEMI (2016) website. These modules are aimed at four levels of education: elementary (8 modules), middle school (7), high school (18), college/university (7 on the website and 3 others in Furuto, 2012). These relate specifically to some facet of Native Hawaiian or other Polynesian cultures. Some attention is given to Indigenous languages. To produce the modules, university students conduct research in a Polynesian community of choice and study history, a Polynesian language, and/or literature (Furuto, in press). Each year more modules become available because of students who enroll in “Math 241” at the university and who choose to develop a module as a major assignment in the course.

Algonquins of Pikwàkanagàn First Nation project

Beatty and Blair (2015) embarked on a long-term goal of decolonizing schools by way of community-focused R&D projects designed to “permeate” the school curriculum with Indigenous perspectives. These projects address “the needs of students from the Algonquins of Pikwàkanagàn First Nation who attend a provincially funded school” (Beatty & Blair, 2015, p. 22) located close to their reserve and to Pembroke, Ontario.

Beatty and Blair’s (2015, p. 4) culturally responsive education means “aligning instruction with the cultural paradigms and lived experience of students.” The projects are also founded upon “a theoretical framework” of ethnomathematics, which differs from D’Ambrosio’s meaning (Ethnomathematics subsection) by fully embracing EAM cultural content (Definitions Clarified and Euro-American Mathematics Cultural Content Made Explicit subsections). In the words of the authors: “From the

ethnomathematics perspective, school mathematics is one of many diverse mathematical practices and is no more or less important than mathematical practices that have originated in other cultures” (Beatty & Blair, 2015, p. 4). Two such cultural knowledge systems were identified: the Ontario mathematics curriculum and “the mathematics inherent in Indigenous cultural practices” (Beatty & Blair, 2015, p. 4). The project adds evidence supporting a critical analysis of school mathematics in an era of reconciliation.

Specifically, Beatty and Blair (2015) reported on teaching “number sense, spatial reasoning, and patterning and algebraic reasoning” (p. 9) by engaging a Grade 2 class (20% Algonquin and 80% non-Indigenous students) in a year-long unit on traditional Algonquin looming. Glass beads and thread were substituted for dyed porcupine quills and sinew. Appropriation of Pikwàkanagàn culture was avoided by assuring that the “core research team was comprised of cultural insiders and outsiders” (Beatty & Blair, 2015, p. 5): two Algonquin teachers, three non-Indigenous teachers, two university academics, and an Algonquin knowledge holder who taught the looming technique and pattern designing to the core research team and then to the Grade 2 students.

The Pikwàkanagàn community became involved by offering guidance and feedback on what should happen and what was happening in the classroom from time to time. This occurred according to a cyclical collaboration model that specified the following sequence: “consult, plan, teach, reflect, and share” (Beatty & Blair, 2015, p. 7). The team’s first consultation meeting heard guidance from three Elders. One spoke of her residential school experience (Consequences of Colonial Genocide subsection) and her struggle to maintain her Algonquin self-identity; “[T]hey couldn’t let us be who we are and just teach us” (Beatty & Blair, 2015, p. 7, emphasis added). Another Elder repeated the wisdom of his father, “[T]ake the best of [Algonquin culture] and the best of the cultures that are out there and marry the two of them together and make them work for you” (Beatty & Blair, 2015, p. 7), advice associated with hybridized knowledge (Enyedy et al., 2011; Jannok Nutti, 2013; Lipka et al., 2007) and two-eyed seeing (Hatcher et al., 2009). The Elders’ initial guidance was seen as a request of the research team to help the community regain its culture that has almost been obliterated by Canadian society.

Parents and grandparents also offered guidance and feedback at several subsequent consultation sessions throughout the year and whenever they visited the classroom or attended evening school events that showed off their children’s accomplishments. Community feedback was “overwhelmingly positive, and community members expressed pride in the mathematical thinking the children were demonstrating” (Beatty & Blair, 2015, p. 19). *This is key evidence* indicating a successful cross-cultural mathematics project, because it comes from some community members who had survived residential school incarceration (Consequences of Colonial Genocide subsection). The fact that residential survivors physically went inside the school is phenomenal.

Importantly, at the end of the year the research team was asked by community members to include more Algonquin language in the mathematics instruction (Centrality of Indigenous Languages subsection). It became an objective for the ensuing year of Beatty and Blair’s long-term project.

In the classroom, Algonquin looming activities were videotaped and analyzed by the core research team to identify “students’ mathematical thinking and the cultural connections” (Beatty & Blair, 2015, p. 12). The mathematical thinking was described in rich enough detail to give me the impression of having watched some of the videos.

The Algonquin knowledge holder taught the cultural connections of looming “effectively, passionately, and authentically” (Beatty & Blair, 2015, p. 10). The researchers did point out that features of Algonquin culture were brought into the classroom in a way that privileged Algonquin culture “alongside the dominant society’s pedagogy and content” (Beatty & Blair, 2015, p. 19). Thus, a more equitable sharing of power was achieved.

A key element of the teacher’s pedagogy was “the numerous opportunities students had to communicate their ideas to one another” (Beatty & Blair, 2015, p. 18). I interpret this as exemplifying two-eyed seeing (Blackfoot First Nations Confederacy Study subsection) applied to pedagogy. Not surprising then, the experience of looming was “more cognitively demanding for students than mathematical instruction that prioritizes algorithmic memorization” (Beatty & Blair, 2015, p. 20). Real life is always messier and more challenging than conventional school lessons.

Beatty and Blair (2015) not only witnessed and documented Pikwàkanagàn students changing their views of what it means to do mathematics, but “this was also important for the non-Native students who gained greater insights into the culture of their classmates, and who extended their own mathematical thinking” (p. 21). In other words, this Grade 2 classroom evolved into a microcosm of reconciliation and increased academic achievement.

Sámi projects

Two independent R&D projects with Sámi Indigenous people took place in Norway and Sweden. Each made a unique contribution to identifying innovative taken-for-granted notions about school mathematics that improve student achievement. These positive features should guide other projects.

But first, a word about the political–social context of the Indigenous Sámi Nation who lives in nine distinct geographic regions across northern Europe. The Sápmelaččat (the Sámi people) first occupied Sápmi (the land occupied by Sápmelaččat) as the ice retreated over 10,000 years ago. They have had consistent contact with Caucazoids over the past 1,100 years, mostly with the mining industry of late. Sápmi has had European settlers arrive, in varying numbers, over the past 700 years. As a result, the Sámi have gone through periods of devastating colonization, similar to what the Turtle Island Indigenous peoples experienced, including boarding schools to assimilate Sámi children into the dominant culture. Resilience and determination of Sápmelaččat have preserved some features of their culture despite attempts to eradicate them. Due to the Sámi demands for education reform and for a degree of sovereignty in the 1960s, the Norwegian Sámi Parliament was established on the authority of Norway’s 1987 Sámi Act, which stipulated its limited responsibilities and powers.

Today, there are two unofficial groups: *Boazosápmelaččat* (Reindeer Sámi) and *Mearrasápmelaččat* (Sámi by the sea). The latter have been closer to, and in longer contact, with Europeans.

Norway

Inspired by Lipka and colleagues’ (2005) *Math in a Cultural Context*, Fyhn (2009) began her research program at Norway’s University of the Arctic to develop “culturally congruent Sámi mathematics” lessons (Fyhn et al., 2011, p. 191), backed by the authority of Norway’s Sámi Language Act and in collaboration with a very experienced Sámi mathematics teacher and school principal. The expression “culturally congruent Sámi mathematics” means that elements of Sámi culture appear in the school mathematics courses. This applies to Sámi immersion schools or to both Sámi and non-Sámi students in regular Norwegian schools.

Ideas, activities, and language from a Sámi local culture become the basis for teaching examples of Sámi mathematizing. In that context, teaching EAM shows Sámi students another (foreign) culture’s way of understanding Sámi ideas, activities, and language; in other words, a cross-cultural or a two-way pedagogy. Because Fyhn forged relationships with Sámi students, teachers, and community members over the years, she has developed a facility with *sámegiella* (the Sámi language) and a deep understanding of Sámi worldviews.

Fyhn’s R&D project explicitly expresses a pluralist notion of mathematics, although EAM’s cultural features are not discussed. The Sámi school mathematics curriculum has always been written in Norwegian until 2010 when the Sámi Parliament funded its translation into *sámegiella*. The translation process was, and continues to be, a quagmire similar to the Māori experience (Early Innovations). Fyhn and colleagues’ (2011) R&D project gave detailed attention to *sámegiella*, carefully analyzing its translation into Norwegian. Currently, high-stake examinations offer a Sámi version, which has raised serious problems concerning its equivalence with the Norwegian versions (Fyhn, 2013).

Culturally congruent Sámi mathematics lessons pay specific attention to language-laden cognition. For instance, the ratios 2:1 and 1:2 are a challenge to learn because in everyday *sámegiella*, *bealli* means half (Fyhn et al., 2011). But the word

is a richer term than just half, because it also is used in different contexts in describing the ratios 2:1 and 1:2. ...

A translation creates an unclear meaning or a misleading expression. Common ways of expression in Sámi are

beali unnit and beli eanet. Unnit means “less” and eanet means “more.” [In a back-translation] beali unnit means “a half less,” while beali eanet means “a half more”; a precise Sámi term sounds odd when translated. The Sámi understanding is that beali eanet means “twice.” (p. 195).

English/French-speaking teachers will be familiar with this type of student confusion over their idiosyncratic everyday language and the technical language found in the culture of EAM.

This demonstrates that everyday mathematics-in-action may have different terminology because its meaning is an analogue to an equivalent abstract Platonist concept (Definitions Clarified and English/French Language-Laden Confusion/Misconceptions subsections). Out-of-school mathematizing can function according to economic values; for example, values not present with in-school mathematizing. As a result, students may experience the two settings as a natural cross-cultural situation, rather than an application of school mathematics to their everyday world.

Another example of culturally congruent Sámi mathematics is evident in the introduction to International System of Units. Sámi body-based measuring units and their accompanying Sámi names are explored. Students learn the Sámi processes and key terms, and they also become acquainted with peripheral concepts that connect with the appropriate contexts of use, the associated cultural values, and the worldview perspectives that wholistically accompany the mathematizing process. The culture-based rationality and linguistics of Sámi proportional reasoning are learned prior to teaching the Euro-American-based rationality and linguistics of school mathematics. Similarly, in an introduction to the study of Euro-American geometry, students learn Sámi perspectives and language concerning what mathematicians call angles and triangles.

The Norwegian Sámi R&D project also explores “culturally valid” assessment items (Lipka & Adams, 2004; Nelson-Barber & Trumbull, 2007) that will help Sámi students “show what they know and can do” (Fyhn, 2013, p. 356), for both Platonist content and Sámi mathematizing. Four Sámi values served as a framework for writing culturally valid test items: reasonableness, cooperation with nature, respect for Sámi traditional culture, and treating Sámi mathematizing as a verb. These values explicitly contextualize both Platonist and Sámi mathematizing test items.

Sweden

Across the border from Norway, and with similar concerns and goals as Fyhn (2013), Jannok Nutti (2010, 2013) carried out a two-part R&D project with Sámi teachers in a Swedish Sámi school for children between the ages of 5 and 12. Jannok Nutti is of Sámi ancestry. Her agenda is the transformation of school mathematics in Sámi schools.

The first part of her project included an action-research professional development program that supported six Sámi teachers in learning their culture in greater depth through seminars, lectures, and contacts with knowledge holders (Jannok Nutti, 2010). To prepare teachers to be agents of transforming school mathematics, she guided them in designing and implementing *culture-based* EAM Platonist content along with “ethnomathematics content” (Jannok Nutti, 2013, p. 57).

Her term *culture-based* signifies her intention of moving beyond a simple translation of the national mathematics curriculum into *sámegiella* (Sámi language) to one that reflects Sámi ontology, epistemology, and axiology. Anticipated student outcomes included a balance between their strengthened Sámi self-identities and their achievement in EAM Platonist content. Jannok Nutti’s plan for the six teachers was to instill in them a sense of empowerment so that they would feel confident in the role of change agent in Sámi school mathematics. This is indeed a key role for both teachers and researchers to play in transforming a 19th-century mathematics curriculum into a culturally enhanced 21st-century curriculum (Innovative Educators and Researchers subsection).

Key Sámi cultural values were emphasized, such as *iešbirgejeaddji* (being independent) and *birget* (to manage). “The concept ‘*birget*’ implies managing to survive and becoming financially self-sufficient, which requires a need for knowledge and skills that enable independence” (Jannok Nutti, 2013, p. 61). These implications associated with *birget* define some of the Sámi term’s *peripheral concepts*. As previously stated, scholarly publications should include important peripheral concepts whenever introducing key Indigenous terms, as Jannok Nutti did. It would even be better to add a back-translation, when appropriate.

Jannok Nutti encouraged the teachers to incorporate Sámi cultural values into both EAM Platonist content and Sámi classroom interactions. These teachers were expected to analyze Sámi artisan handwork, processes, or ideas to find examples of Bishop's (1988b, 1990) six fundamental mathematical processes (Some Fundamentals of Mathematics subsection). Various Sámi people use a range of body-based units when *measuring*. When Boazosápmelaččat (Sámi people who work with reindeer) are *counting* the number of reindeer in a herd, "different names for reindeer herds are based on the approximate number of animals" (Jannok Nutti, 2013, p. 61). Unlike SAE languages such as Swedish, many other languages worldwide have different names for numbers depending on the type of thing being counted (Yousan subsection). In contrast, an SAE number has a *universal* name regardless of what is being counted. This generalizability value exemplifies EAM cultural content that could be taught explicitly.

Part two of Jannok Nutti's (2013) project investigated the teachers' perceptions of their participation in part one, and then she assessed the classroom activities the teachers created. Assessment criteria included how closely their activities reflected local Sámi ontology, epistemology, and axiology.

From extensive teacher interviews, Jannok Nutti (2013) discovered the following:

During the study the teachers changed from a problem-focused perspective to a possibility-focused culture-based teaching perspective characterised by a self-empowered Indigenous teacher role, as a result of which they started to act as agents for Indigenous school change. The concept of "decolonisation" was visible in the teachers' narratives. The teachers' newly developed knowledge about the ethnomathematical research field seemed to enhance their work with Indigenous culture-based mathematics teaching. (p. 57)

These are promising results.

The degree to which the activities represented Sámi ontology, epistemology, axiology, and ideologies was determined by a four-category assessment rubric, which could be adapted to suit other researchers' culturally appropriate teaching materials. Jannok Nutti (2013) adapted hers from Banks's (2004) assessment of multicultural teaching. This decision reflected her wish to emphasize the heterogeneity among Sámi groups within Sweden and Norway (Jannok Nutti, personal communication, May 12, 2016). Her four categories are as follows:

1. Sámi cultural thematic work with ethnomathematical content.
2. Multicultural school mathematics with Sámi cultural elements.
3. Sámi intercultural mathematics teaching.
4. Sámi intercultural education based on Sámi ontology and epistemology.

The term *intercultural* indicates a "transfer of traditions and knowledge between different cultural groups" (Jannok Nutti, 2013, p. 63)—Sámi and EAM, in Jannok Nutti's case. Because the meaning of intercultural epitomizes conversations to the depth of ontological, epistemological, and axiological topics, I interpret it as meaning a "camping spot of dialogue" (Vickers, 2007, p. 592) type of interaction (Appropriation subsection). Details on the four categories can be found in Jannok Nutti (2013) and Aikenhead (2017).

The six teachers' activities fell within the first two categories, which was explained by Jannok Nutti (2013, p. 68): "The teachers wished to implement Sámi culture-based mathematics teaching, but felt that they lacked the knowledge and time to implement Sámi culture-based teaching." This result could be mistaken as a negative outcome. But I realize that transformational changes take time; sometimes measured in years, maybe decades, or even generations. Thus, we all need to conceptualize teachers' efforts as a journey. Perhaps the four-category scheme needs fine-tuning to capture a teacher's journey into Jannok Nutti's category 3 for non-Indigenous and Indigenous teachers alike, along with forays into category 4 led by enriched cross-cultural activities appropriate to a grade level. Jannok Nutti (2013) certainly did acknowledge the six teachers' accomplishments:

After the implementation of Sámi culture-based mathematics activities the teachers still faced the previously described external obstacles, but those that initially seemed to prevent them from adapting to culture-based mathematics teaching no longer stopped them from starting to implement culture-based mathematics activities. The teachers' work with Sámi culture-based mathematics lessons demonstrated their competence in dealing with the described challenges. In addition, *the challenges were now viewed as opportunities*. (p. 68, emphasis added)

Mi'kmaw First Nation projects

Show Me Your Math (2016) in Atlantic Canada engages Mi'kmaw students in exploring their Indigenous communities mathematically. The SMYM fair is, in part, a wonderful out-of-school educational–social event that brings students together once a year. At home, students take a traditional or contemporary artisan handwork, process, or idea found in their community (e.g., a hockey rink's markings, archery, beadwork, snowshoes, and birch bark biting) and learn about its Mi'kmaw mathematizing. Then they analyze it in terms of Platonist content.

Of particular interest to R&D project researchers, the SMYM project evolved out of a familiar strategy by which in 2005 Lunney Borden asked Elders for an artisan handwork or process that represented Mi'kmaw mathematizing; after which she reworked the artisan handwork or processes into content found in the region's Platonist curriculum, followed by her composing contextualized teaching materials for volunteer teachers to try out (Lunney Borden et al., 2017). Not satisfied with the results, she removed herself from this strategy by organizing a direct consultation between students and Elders or knowledge holders. After their consultation, students create a display to show others what they learned about both EAM Platonist content and Mi'kmaw mathematizing. This version of a cross-cultural project became Show Me Your Math in 2007.

Lunney Borden's research methodology (Rigney, 1999) was anchored in “challenging hegemony and overcoming oppression” and was guided by the “interrelated principles of resistance, political integrity, and privileging Indigenous voices” (Lunney Borden, 2013, pp. 5–6). This methodology harmonizes with her research program: “*How can curricula and pedagogy be transformed to support Mi'kmaw students as they negotiate their position between Aboriginal and school-based concepts of mathematics?*” (Lunney Borden, 2013, p. 7, original emphasis).

As a non-Indigenous researcher who undertakes studies with Indigenous people, Lunney Borden (2013) outlined her personal background and relationships with the Mi'kmaw First Nation in order for readers to know that she immersed herself in the Mi'kmaw culture during her 10 years as a teacher and administrator in one Mi'kmaw school. She confronted and deeply reflected upon the complexities of teaching school mathematics from a Mi'kmaw perspective, aided considerably by her functional literacy in Míkmaḡisimk (the Mi'kmaw language).

Lunney Borden's ultimate agenda is to work toward the sovereignty of Mi'kmaq (the Mi'kmaw people). She is explicitly conscious of the present power imbalance between the mathematics curriculum and Mi'kmaw communities. She views SMYM as a mechanism to reduce this imbalance, in particular, and she embraces a decolonizing agenda for Mi'kmaw schools, in general (Lunney Borden et al., 2017).

Some Mi'kmaw schools are bicultural (e.g., an Eskasoni school on Cape Breton Island) due to the community's special efforts to revive Míkmaḡisimk there, as is happening on some reserves across the Maritime Provinces.

Lunney Borden pointed out a unique advantage of collaborating with Mi'kmaw schools (personal communication, March 28, 2016): “[T]he majority of teachers are Mi'kmaq and from the communities in which they teach, so they knew the community context well, and they had ideas about the kinds of things they wanted to do.” Moreover, it is fairly easy to find Elders and knowledge holders to offer guidance and support.

Similar to the MCC project in Alaska, SMYM's capacity development within Mi'kmaw communities is on her agenda for the decolonization of Mi'kmaw schools, as is her allegiance to “making ethnomathematical connections for students” (Lunney Borden, 2015, p. 758).

At first, Lunney Borden's apparent allegiance to ethnomathematics seemed to exclude EAM cultural content, an exclusion both noticeable in her earlier descriptions of SMYM and detrimental to her decolonizing agenda for school mathematics (Ethnomathematics subsection). I appreciate her ambivalence about a truncated EAM school mathematics, an ambivalence that she apparently worked through.

Upon reflection, Lunney Borden (2013, pp. 10–11) summarized four key capabilities with which teachers should be conversant so they can better help Mi'kmaw students make meaningful personal connections to EAM content. These are paraphrased here. Two of them (#2 and #3) definitely address EAM cultural content that ethnomathematics purposefully ignores (Ethnomathematics subsection).

1. Learn *from* students' Indigenous language.

2. Point out to students the *value* differences (i.e., differences in axiological presuppositions) between Indigenous mathematizing and school mathematics, by emphasizing cross-cultural understanding and EAM cultural content that gives a cultural context to the Platonist content in school mathematics.
3. Attend to diverse ways of learning among students (e.g., recurrent learning strengths, Aikenhead et al., 2014, Appendix D) and ways of knowing between the culture of EAM and the local Indigenous culture (e.g., Aikenhead & Michell, 2011, pp. 63–98).
4. Make explicit relationship connections that are physical, intellectual, emotional, and spiritual.

Within SMYM, important relationship dynamics are responsible for negotiating Platonist school mathematics into cross-cultural school mathematics. The Mi'kmaw community becomes a valid source for school mathematics content, which establishes a shared authority with the mathematics curriculum. Students can then enter a discussion about the Mi'kmaw community's knowledge being privileged alongside the textbook's Platonist content, in a coexisting way. Students' roles expand from being just mathematics students to becoming *researchers* of Mi'kmaw mathematizing, *constructors* of EAM Platonist content, and *disseminators* of both (Lunney Borden et al., 2017). The effect on Mi'kmaw students' self-identities seems obvious to educators, as does the opportunity for non-Mi'kmaw students to participate in reconciliation.

A sampling of evidence for this comes from systematically crafted “storylines” (i.e., short narratives) that capture how these new dynamics play out. For example, two Grade 12 students were learning the Mi'kmaw way of replicating an eight-point star design in beadwork, a design of sacred significance to Mi'kmaq (the Mi'kmaw people). The students compared that process with the Euro-American mathematizing found in their textbook that produced the same design. By doing so, these two Grade 12 students illustrated cross-cultural learning in action.

Unlike a number of researchers cited in this article, the two students recognized that the Mi'kmaw beadwork mathematizing occurred with perfect results “*without*” the bead worker knowing any Euro-American geometry (Lunney Borden et al., 2017; the students' actual word). Remarkably, the two young women, after 11 years of schooling, had not been assimilated into a Platonist belief system, which claims that the universalist Platonist geometry was situated or embedded in the Mi'kmaw eight-point star beadwork mathematizing all along.

Lunney Borden's (2013) decolonization agenda for SMYM and the evidence of success provided by Lunney Borden and colleagues (2017) matches her enriched and explicit attention to EAM cultural content.

Other evidence shows that as students forge stronger relationships with Mi'kmaw mathematizing, they tend to improve their relationship with school EAM (Lunney Borden & colleagues, 2017). This helps explain why Indigenous students' mathematics and science achievement increases significantly, on average, when the school subject is taught culturally (Lipka et al., 2005; Meaney et al., 2012; Richards et al., 2008; Sakiestewa-Gilbert, 2011; U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education, 2008). All of this occurs in synchrony with the ever changing role of teachers: as knowledge holders of EAM Platonist content, as sources of information and ideas, as facilitators, as mentor-guides, and, most important, as learners.

How exactly do most students analyze Mi'kmaw artisan handwork, processes, or ideas in order to produce an SMYM display? One answer is superimposing, deconstructing, and reconstructing (English/French Language-Laden Confusion/Misconceptions subsection). Thus, students will need to know specific EAM Platonist content with which to *transform* an Indigenous artisan handwork or process into an SMYM display. Hockey rinks and archery, for instance, were transformed into geometric concepts and vectors, respectively. SMYM's need-to-know feature offers substantial motivation for students.

But students will respond to the challenge in different ways. Students whose worldview harmonizes with a “*quantitative* worldview” endemic to the culture of Euro-American mathematics usually require little guidance (Aikenhead, 2006, pp. 29–31). However, SMYM is less engaging in varying degrees for other students (Projectionism subsection), depending on how severe they experience a culture clash with quantitative abstract thinking in the context of their everyday world (Culture Clashes that Alienate Many Indigenous Students subsection).

Lunney Borden and Wagner (2017) initiated a related and more broadly based R&D project, “Mawk-inumasultinej: Let’s Learn Together!” which grew out of a few excellent SMYM displays. The researchers consulted with more Mi’kmaw Elders, learned the Mi’kmaw mathematizing processes in depth, and then designed a series of *cross-curriculum* and *cross-cultural inquiry* modules that emphasize both EAM and Mi’kmaw cultures. Topics include Kataq/Eels (with a study of Mi’kmaw inherent treaty rights), Quill Boxes (honoring an Elder), and Birch Bark Biting (geometry; Lunney Borden, 2015). These modules are designed according to collaborative inquiry pedagogies. The inquiry projects contain support resources for teachers, including a link to an extensive assessment rubric.

Similar to schools across Canada, most non-Mi’kmaw schools do not find SMYM especially attractive at this time. Based on my experience with enhancing school science with Indigenous perspectives (Aikenhead, et al., 2014), I would suggest that a cascade of reasons for this low interest would begin with the absence of Mi’kmaw perspectives explicitly in the content of the mathematics curriculum. Other reasons would likely include a paucity of resources, few professional development opportunities, the stance to maintain current dogma about Platonist content, and the pervasive racism that exists in the underbelly of Canadian society (Battiste, 2002; Government of Alberta, 2010; St. Denis, 2004).

Optimism, however, resides in the knowledge that Canadian ministries of education are now taking reconciliation very seriously.

University of Alberta project

The University of Alberta project was conducted by three academics, Donald, Glanfield, and Sterenberg (2011), who immersed their project in a purely Indigenous methodological approach described by Kovach (2009). What took place was a shift in research perspectives, from a mainly literacy tradition to a mainly oral tradition that celebrates relationships. Donald is of First Nations ancestry, Glanfield is of Métis ancestry, and Sterenberg is Euro-Canadian.

Donald and colleagues’ (2011, p. 73) “culturally relational” Indigenous inquiry paradigm emerged when Eagle Flight band councillors (named for anonymity) contacted them for help in improving student performance on provincial school mathematics tests. The research team decided against being the experts who would come to the reserve and solve their problem, as the academy is prone to do. Instead, they invited the whole Eagle Flight community to engage in research with them. Most members accepted. Working together, they defined what their community-based research project would be: to explore “the ways in which community members, children in school, school staff, and school and community leadership come to develop a shared understanding of mathematics” (Donald et al., 2011, p. 77). This became a 4-year project at this K–9 reserve school, in which teachers individually created their own personal teaching techniques and materials as a result of Donald and colleagues’ collaborative action research in teacher development.

The first year unfolded with meetings among community members, teachers, and staff, directed at the task of developing

an understanding of the ways in which children in the community know mathematics. ... The teachers identified the need to know what their children could do mathematically so, together with the teachers, we designed a variety of assessment strategies such as performance-based tasks and interviews to develop an understanding of the way in which the children in their classrooms think about mathematical ideas. (Donald et al., 2011, p. 77)

During the second year, the research team

worked alongside three staff-identified lead teachers as [they] engaged children in mathematical interviews and collected data on children’s thinking through video-recordings. The recordings were analyzed by this team of teacher/researchers and then we shared video clips and preliminary results with the rest of the school staff. (Donald et al., 2011, p. 77)

In the third year, “the teachers of the entire school decided to engage their children in paper-and-pencil and performance task assessments. These assessments were designed by the researchers and the initial lead teachers, and then administered by all teachers” (Donald et al., 2011, p. 77).

The fourth year began by all staff analyzing the results of the previous year-end assessment. Then

[T]eachers are using what they are learning about children in their classes to inform planning and classroom practices. Specifically, they are focusing on mathematical vocabulary development and teaching mathematics with manipulatives. Assessments of children's thinking are ongoing and data collected on these assessments is shared with us. During staff in-service meetings, our conversations about what teachers are learning from children when children are asked to explain their mathematical thinking are recorded as data. Insights into the ways in which teachers are learning inform our ongoing work together. (Donald et al., 2011, p. 77)

Donald and colleagues' (2011) culturally relational approach harmonizes strongly with Indigenous perspectives. It is ontologically, epistemologically, and axiologically student and community centered. For instance, it is wholistically organized and profoundly anchored in relationships. Teachers' and students' knowledge was not expressed in written lesson plans or modules for public consumption. Instead it was captured in action; for example, in the teachers' (a) active listening skills (mentored by the researchers); (b) revisions to lessons that took into account differing unique ways students had resolved a mathematics problem based on their Indigenous worldview; and (c) videotaped interactions with students in the classroom. Within a culturally relational research stance, knowledge is action while action is knowledge.

"As educators and researchers, ... we are seeking to honour meaningful engagements with Indigenous philosophies and knowledge systems as they are understood and lived by all in relation" (Donald et al., 2011, p. 80).

Any R&D project that explicitly invests in relationships and in the Indigenous participants' understanding of their own culture will certainly be successful. For non-Indigenous researchers, however, a prerequisite is necessary. Their degree of grounding a R&D project in Indigenous methodologies must correlate with their experiential understanding of an Indigenous community or communities, along with the Indigenous human-resource support found in those communities (Aikenhead, 2002a).

The result is a unique, wholistic, place-based, cross-cultural school mathematics. EAM Platonist content is emphasized because the community's focus was definitely on raising test scores. But the Platonist content did not have its usual neo-colonial influence, an issue worth investigating in the future. Cross-cultural strategies were present and completely in tune with the local Indigenous worldview, although explicit attention to EAM cultural content was not mentioned by Donald and colleagues (2011).

Conclusion

Each R&D project or research study discussed in this section attended to different clusters of pedagogical factors and chose different content features to emphasize when they enhanced their school mathematics culturally. These decisions necessarily involved making trade-offs among a complexity of factors that could help attain their goals for Indigenous students. Much is gained from the *diversity* of projects and studies described in this article.

Although these projects and studies were successful at mitigating culture clashes experienced to various degrees by many students, further mitigation can be accomplished. The following section explains how.

Correcting lingering impediments to student success

European mathematics is offered to [I]ndigenous people around the world, "gift-like", as a passport to success and future development. ... The promotion of mathematics may be ethnocentric, but is it not, we believe, offered with malice. The bearers are probably unaware of the inherent dangers as the receivers.—Bill Barton and Uenuku Fairhall (1995, p. 1)

Throughout this article, I, too, have acknowledged the dangers to Indigenous cultures when students study European mathematics (Platonist school mathematics). And though I also agree with Barton and Fairhall's (1995) assertion that no malice was intended, their assertion strikes me as a red herring to be ignored, because the dangers of which they speak actually emerge from masking power with the innocence of claiming no malice intended. The obstacles that Indigenous students face are

subtle, systemic, institutionalized, and implicit ones. The various species of masking power with innocence noted in this article maintain the neo-colonial impediments to Indigenous students' success in school.

These impediments tacitly undermine the quality of teaching methods and materials because the impediments cause Indigenous students to feel their culture is not respected as it should be (Doolittle, 2006). Student reaction will be tolerance, reluctance, resistance, or rejection. Accordingly, this section answers the following questions: What are the conventional taken-for-granted notions that impede student achievement? Which of these continue to be held by many innovators who have enhanced school mathematics culturally? What practices should be avoided? Exactly how do researchers or teachers “see” school mathematics content “embedded” in an Indigenous piece of artisan handwork or everyday activity?

Some of the answers have been discussed previously in general terms. In order to give greater depth and clarity to them, this section explores them concretely in the context of projects described in the previous section. The issues are organized around four general topics: the altar of Platonist content, myth blindness, projectionism, and misinterpretations of quantified assessment and evaluation.

The altar of Platonist content

Many potential culture-based innovations to school mathematics have been sacrificed on the altar of Platonist mathematical content. Platonist content deserves serious attention, to be sure. But its current excessive emphasis has created (a) myths, (b) beliefs in those myths, (c) social power bestowed upon those beliefs, and (d) privileges gained by that social power (subsections High Status of School Mathematics, Some Fundamentals of Mathematics, and A Hidden Platonist Agenda). Unless broken, this cycle will repeat itself for generations to come. An exclusionary emphasis on Platonist content occurs at the expense of including Indigenous perspectives, an exclusion that assaults Indigenous students' self-identities through cognitive imperialism (Battiste, 1986) and racial discrimination (Battiste, 2002).

The altar of Platonist content comes in direct conflict with educators' efforts to decolonize school mathematics. Classroom incidents of this were identified by Russell and Chernoff (2015), who showed how teachers' Platonist beliefs and Euro-American worldviews prevented many Indigenous students from engaging in activities that had been planned to connect with Indigenous cultures.

The MCC project (University of Alaska Project subsection) appears to assume that Indigenous students' exposure to authentic Yup'ik culture is sufficient to strengthen their cultural self-identities, without having that cultural content assessed by its modules' assessment regime. If it were assessed¹⁰ in a nonquantitative way, however, it would certainly send a clearer message about the importance of Yup'ik mathematizing. In addition, it would tend to open up class discussions comparing features of the Yup'ik culture with the culture of EAM.

This important task seems left to teachers to complete. Some would certainly do it, but many may not. In four case studies (Parker Webster et al., 2005; Lipka, Sharp, et al., 2005; Adams et al., 2005; Rickard, 2005), teachers' attention was only on improving EAM Platonist content outcomes. If “two-way learning” (Lipka, 1994, p. 15) is to reach its full potential, students must experience two-way assessment. The proportion of each culture (Indigenous and EAM) does not have to be *equal*, but Indigenous perspectives need to be noticeable beyond tokenism—in a word, *equitable*.

For example, in addition to asking a Platonist-content question (e.g., “If a fish rack holds 6 King Salmon, how many Red Salmon would it hold”), include a Yup'ik content question (e.g., “What steps do you go through to dry salmon on a fish rack? In your answer, mention the protocols to follow”). A repetition of only contextualized Platonist content questions suggests that MCC is singularly focused on Platonist content outcomes. The altar has been effective here.

As also mentioned by Doolittle and Glanfield (2007) and by Barton and Fairhall (1995), an exclusionary reliance on EAM Platonist examinations is fraught with dangers to Indigenous students' cultural self-identities. From an Indigenous viewpoint, this indicates a degree of tokenism and disrespect (Sternberg, 2013a). In such situations, Elders and Indigenous teachers can experience “a threat towards their culture and language” (Fyhn et al., 2011, p. 191). The issue is this: “How and to what extent can Sámi

mathematizing be included in school mathematics” (Fyhn et al., 2011, p. 190; Jannok Nutti, 2013). “Insofar as the Sámi curriculum claims that the teaching should focus on Sámi values, the [national] examination needs to reflect some Sámi values” (Fyhn, 2013, p. 364). If not, school mathematics will continue its neo-colonizing, systemic discrimination that sustains a political–social power imbalance between the mathematics curriculum and Indigenous communities.

Describing the Algonquins of Pikwàkanagàn First Nation Project (Algonquins of Pikwàkanagàn First Nation Project subsection), Beatty and Blair (2015) state:

As a group, we watched and analyzed video recordings of in-class lessons to identify the mathematical thinking that surfaced during classroom conversations. This process of co-analysis involving Algonquin and non-Native teachers and university faculty ensured that our focus was both on the students’ mathematical thinking and *the cultural connections*. (p. 12, emphasis added)

Ample evidence was presented for claiming Platonist mathematical thinking by students, but little specific evidence, other than assurances, was given for students making connections to Algonquin mathematizing. A reader could wrongly form the opinion that the project’s Platonist content had marginalized features of Algonquin culture. At a Grade 2 level, where is the Algonquin language, values, spirituality, and intergenerational stories that supplement the Algonquin looming project and convey aspects of the community’s culture?

Perhaps such evidence does exist for both the Yup’ik and Algonquin projects but, consciously or unconsciously, the authors excluded it from their manuscripts submitted for review and publication. Like Kovach (2009, p. 28), we could conclude “that Western epistemological privilege pervades the academy.” Is this another privilege-blindness enacted by refereed journal editors and reviewers? If so, mathematics education researchers need to resolve this dominance by the Platonist altar.

The University of Hawai’i R&D projects appear to express an ambivalence over the role of Native Hawaiian mathematizing in their projects (University of Hawai’i Project subsection). On the one hand, the word mathematics refers to Native Hawaiian mathematizing: “This textbook draws on the unique pedagogies, values, and *mathematics of our sacred islands*” (Furuto, 2013b, p. ix, emphasis added), and “The *mathematical ideas of the peoples of the Pacific region* have often been overlooked, particularly in ... mathematics. Every cultural group develops its own ways and styles of explaining, understanding, and coping with their environment” (Furuto, 2013a, p. 39). The concept here is certainly pluralism of mathematics and a legitimacy of Polynesian mathematizing.

On the other hand, EAM Platonist content is clearly central to Furuto’s (2014) allegiance to ethnomathematics (Ethnomathematics subsection). Indigenous mathematizing is acknowledged (especially its values), yet not specifically in comparison to Euro-American mathematizing. This ambivalence is likely caused by unconsciously moving between two different meanings of the word mathematics—its superordinate and subordinate meanings (Some Fundamentals of Mathematics and Definitions Clarified subsections). Without making that distinction, conversations will be laced with confusion and misconceptions, thereby causing the relationship between Polynesian and Euro-American mathematizing to be ambiguous and unbalanced.

However, in some undergraduate mathematics courses at the University of Hawai’i, Polynesian mathematizing is certainly an important explicit feature. As noted in the University of Hawai’i Project subsection, compared to other major R&D projects, the ESTEM Institute at the university level appears to reach a more equitable balance between its focus on EAM Platonist outcomes and its attention to Indigenous perspectives.

Yet the altar of Platonist content, which severely marginalizes Indigenous perspectives, shows up in otherwise excellent lesson plan modules developed by undergraduate students for school-level instruction as well as for undergraduate-level instruction. For example, the undergraduate module “Vectors and Navigating a Voyaging Canoe” (ESTEMI, 2016) details Polynesian mathematizing but does not acknowledge it as mathematizing:

Polynesians used a conceptual star compass to determine direction through the rising and setting of different stars. During the day, close observation of the sun, steady wind patterns and ocean swells were used to navigate. ... In

order to find land, the navigators looked for cloud and swell patterns, as well as for birds that usually fly up to 20 to 30 miles from land. (p. 1)

In the following passage, is Polynesian mathematizing dismissed and replaced by EAM Platonist content by the author?

The Polynesian navigators had an *intuitive sense of math*. Math was needed to estimate speed and time, and to observe the different angles of the stars, planets, and sun. As demonstrated in this lesson plan, math was also needed to understand the different forces that altered the path of the voyaging canoes. (ESTEMI, 2016, p. 2, emphasis added)

If there is any doubt that it has been marginalized, consider the module's conclusion: "Even though the early navigators never took a math course in their entire lives, they were natural mathematicians" (ESTEMI, 2016, p. 4). As discussed in the English/French Language-Laden Confusion/Misconceptions subsection, Platonist power has been masked by an intended compliment that privileges Platonist content at the expense of Polynesian mathematizing (Garrouette, 1999). Subtly, the power imbalance has shifted further toward the colonizers' altar of Platonist content.

Culturally sensitive editing is needed for three reasons: to complement the culturally responsive feature of this ESTEMI module with Polynesian mathematizing and EAM cultural content; to challenge the imbalance of social power between Native Hawaiians and the University of Hawai'i's ethnomathematics curriculum; and to be a model for producing decolonizing teaching materials. All three reasons speak to ESTEMI's espoused goal of social justice.

The altar of Platonist content certainly influences all teachers and researchers grappling with their own specific and illusive balance among Platonist content, Indigenous mathematizing, and EAM cultural content. More examples are found in Aikenhead (2017). There is a strong consensus that all students should come away with at least an appreciation of both the local Indigenous worldview and the worldview endemic to EAM.

The wisdom of consensus making to reach a curriculum balance seems more rational than submitting to the coercion by corporate-economic-political interests that worship at the altar of Platonist content. Their quest to populate the STEM pipeline ignores the development of Indigenous self-identities in school mathematics. Aikenhead (2006, 2017), Charette (2013), and Lunney Borden and Wiseman (2016) present compelling evidence that shows the coercion's rationale to be smoke and mirrors.

Bishop's (1988b) pluralism implies the superordinate designation to the term mathematics, and it causes *all* subordinate mathematics, such as EAM, to exist within a cultural context. Thus, a complete pluralist stance helps to decolonize the political-social power imbalance between Indigenous communities and the mathematics curriculum. However, the altar of Platonist content opposes pluralism, through a process of cognitive imperialism; thereby giving authority to the status of Platonist school mathematics (Ernest, 2016a; High Status of School Mathematics subsection).

Myth blindness

Lipka and colleagues (2005, p. 1) reaffirm that "bringing local knowledge into American Indian/Alaska Native education requires reversing historic power imbalances that continue to separate school knowledge from community knowledge." But how far do the MCC and ESTEMI projects go in reversing historic power relations? Not as far as they could, due to a pervasive blind spot.

As emphatically stated in the University of Alaska Project and University of Hawai'i Project subsections, the MCC and ESTEMI projects *do not* embody privilege-blindness, described in the Introduction as not knowing or not caring that personal privilege and power have accrued at the expense of another group. But many mathematics educators do appear to suffer from a different kind of privilege-blindness: mathematics myth blindness.

Their school subject's privileged status rests on the myth that its Platonist content is acultural, universalist, value free, objective in its use, and nonideological (The Platonist Belief and Yousan subsections). Because these mythical characteristics deny the pluralism of mathematics, the myth denies an equitable legitimacy of Yup'ik and Native Hawaiian mathematizing. The MCC and ESTEMI projects own

a conundrum. Their Platonist content myth masquerades as an acceptable view of school mathematics to the detriment of many Indigenous and non-Indigenous students. The myth has evolved into Platonist dogma. The dogma's unquestioned authority is enshrined in the publication *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) and in the Mathematics Common Core State Standards emulated in the states of Alaska and Hawai'i.

The dogma issue is certainly not one of semantics because it can seriously influence Indigenous students' negativity, as Doolittle (2006) observed (worth repeating one last time): "Students may, implicitly or explicitly, come to question the motives of teachers who lead them away from the true complexities of their cultures" (p. 20). The degree to which Doolittle's position is not held by Indigenous participants involved in MCC's or ESTEMI's development could indicate the extent to which internalized colonization has occurred. The dogma issue is not simply a matter of someone's "vision of mathematics" (Fyhn, 2013, p. 355), because that vision masks power with innocence by impeding the decolonization of school mathematics.

The conundrum remains: popular dogma versus legitimate interests of Indigenous and non-Indigenous students. How do we resolve it?—By explicitly teaching both EAM cultural content and Yup'ik or Hawaiian mathematizing. (EAM cultural content may not be suitable for the intellectual development level of primary-grade students.)

This much needed EAM cultural content has a "myth-busting" quality aimed at the nature of Platonist content and its influence on society. As exemplified throughout this article, EAM cultural content includes (a) ideologies that were formerly implicit in school mathematics, (b) values that were formerly suppressed, and (c) the limitations and consequences of a Platonist belief that were formerly censored. For mathematics educators wanting to make their subject inviting and accessible to students who are anxious or bored over studying school mathematics (Stoet et al., 2016), EAM cultural content can act as a bridge between, on the one hand, students' everyday non-Indigenous or Indigenous lives and, on the other, Platonist content, if and only if Platonist content is treated as another culture's way of mathematizing, a very powerful way indeed.

Projectionism

Euro-American people often assume that their knowledge expressed in their SAE language will have a direct equivalent in an Indigenous language. Many mathematics teachers and researchers also tend to believe the naïve implication that when transforming an Indigenous activity into school mathematics content, nothing important gets lost in that transformation (Historical Appropriation and English/French Language-Laden Confusion/Misconceptions subsections). These assumptions and beliefs occur because researchers (a) do not recognize the unconscious act of projecting their Euro-American presuppositions and concepts onto Indigenous artisan handwork, activities, or ideas and (b) do not understand a mechanistic process of transformation (e.g., superimposing, deconstructing, and reconstructing). In other words, they are blind to this projectionism problem that many Indigenous students intuitively experience daily, in concrete ways and often with negative effects.

As Einstein pointed out (English/French Language-Laden Confusion/Misconceptions subsection), projectionism happens in diverse contexts, many with positive consequences, such as in making inferences in an attempt to understand something in greater depth, exemplified by scientific investigations. But some projections have negative consequences, revealed here by critically analyzing R&D projects to gain a deeper understanding of how subliminal the negative consequences can be.

In documents composed by Alaska's MCC researchers, a key technical expression, *embedded mathematics*, was never defined in the many articles I examined. In Lipka and colleagues (2013), one reads:

- "... the embedded mathematics in [an Elder's] everyday activity. ..." (p. 130)
- "... to become familiar with mathematically embedded processes of body proportional measuring. ..." (p. 139)

In Lipka and colleagues' Figure 1 (p. 132), a Venn diagram shows three overlapping circles: "Culture & Context," "Math Content Knowledge," and "Pedagogy." The expression embedded mathematics is located in the Venn diagram's Culture & Context circle. It is excluded from the Math Content Knowledge and

Pedagogy circles. Thus, embedded mathematics and mathematics content knowledge are logically different but somehow related concepts: “The mathematics embedded in everyday activities *relate directly to ratios and proportional thinking*” (Lipka et al., 2013, p. 132, emphasis added). What is the language-laden cognition that both separates them and relates them? What is the relationship between the two? An answer should be found in a definition of embedded mathematics. If defined, it would have clarified a type of superimposing–deconstructing–reconstructing process carried out in MCC.

Two answers were suggested in Lipka and Andrew-Irhke (2009):

- “... we are increasingly able to understand the mathematical threads *woven into* authentic cultural knowledge and practices.” (p. 8, emphasis added)
- “It’s in the construction of the border patterns on these parkas that the math *is revealed.*” (p. 9, emphasis added)

Two metaphors, “woven into” and “is revealed,” suggest a relationship. Accordingly, the mathematics teacher or researcher’s role seems to be (a) observe Platonist mathematical threads, copy them into a MCC module, and then teach them to students or (b) wait until the Platonist mathematics are revealed and then copy them into an MCC module. In other words, the process that the mathematics teacher or researcher goes through is a passive one, not an active one. When the process is passive, the teacher or researcher has no responsibility in the process. It is not them, it is what is observed or revealed. But when the process is an active one, which it surely is, the educator/researcher bears responsibility for his or her actions. These actions can be interrogated with critical questions. Unconsciously or not, any move *away from* responsible transparency in order to avoid such responsibility exemplifies masking projectionist power with opaque responsibility.

A cross-cultural solution for MCC and other mathematics educators is to be equitable in a pluralist way by recognizing Euro-American mathematics as a cultural-laden human construction just as Yup’ik mathematizing is. From this equitable perspective, we can more clearly answer the question: How are the two related?—Through the transformation process that strips peripheral concepts belonging to a local Indigenous culture and then adds a different set of peripheral concepts belonging to a Euro-American mathematics culture (Historical Appropriation subsection and Avoiding Present-Day Appropriation and Marginalization). Mathematics teachers or researchers actively transform an Indigenous artisan handwork, activity, or idea into a Euro-American Platonist concept or process, for the intended well-being of Yup’ik students. These are students who want to achieve in mathematics in order to walk in both worlds.

In summary, the concept of embedded mathematics is not transparent. It is a cultural projection. According to anthropologist Hall (1976, p. 164): “Cultural projection always has been a stumbling block on the path to better understanding. Yet progress in getting rid of cultural projection has been slow.” The concept of embedded mathematics also exemplifies a “conceptual pitfall” that Garrouette (1999, p. 107) would call “proto-mathematics,” which, as she pointed out, privileges school mathematics at the expense of Indigenous mathematizing.

A transparent transformation process (superimposing, deconstructing, and reconstructing) will confirm for Indigenous students that (a) their culture’s artisan handwork, activities, or ideas remain unchanged despite any superimposition process; (b) their artisan handwork, activities, or ideas continue to possess a cultural integrity in an Indigenous context, even after the deconstruction process by teachers or researchers (Garrouette, 1999); and (c) Indigenous students will *add* another culture’s perspective (a Euro-American perspective) to the students’ intellectual world. Transparency prevents the neo-colonial claim that the student’s Indigenous culture already contained those “foreign” Platonist mental forms in the first place. Garrouette forcefully advocated such transparency in order to achieve authoritative equality between the two ways of mathematizing; thus meeting MCC’s affirmed political–social aim: “bringing local knowledge into American Indian/Alaska Native education requires reversing historic power relations that continue to separate school knowledge from community knowledge” (Lipka et al., 2005, p. 1).

As noted above, these fundamental problems do not cause a project to fail, but they keep the project from reaching its full potential of benefiting more Indigenous students. These problems naturally infiltrate into modules and lesson plans produced by developers of teaching materials, because those developers have been taught (a) the dogma and myths of Platonism that facilitate projectionism and (b) the

historic, self-serving, subjective bifurcation of Euro-American mathematizing into formal versus informal discourse (subsection A Hidden Platonist Agenda). These became unquestioned epistemic presuppositions. One of many examples from the ESTEMI (2016) project, a teaching unit for Grades 9–12, Fractal Landscapes, is analyzed in Aikenhead (2017) to illustrate the problems concretely.

As noted in the subsection Toward a Cultural Belief, teachers often tell students “mathematics is all around you,” implying that students need to learn this year’s content. We are now in a position to take our analysis of the expression to a deeper level, in order to explore its projectionism and to discover it is only one-third true in Canada.

The statement only makes sense to the minority of students whose worldviews harmonize with the worldview endemic to school mathematics and whose intellectual self-identity is similar to their teacher’s (Cobern, 2000). These students, Indigenous or non-Indigenous, easily make a connection between their in-school mathematics and out-of-school home culture. They know the Platonist concepts or conceptual images to project onto their everyday world. In Canada, this minority comprises 34% of high school 15-year-olds who expect to gain employment in the STEM sector (Organization for Economic Cooperation and Development [OECD], 2016). This leaves 66% who, to varying degrees, experience a clash between their own worldview and the worldview endemic to conventional school mathematics (Culture Clashes that Alienate Many Indigenous Students subsection). They tend not to understand Platonist concepts meaningfully enough to project them onto their everyday world. Consequently, they do not “see” (i.e., conceptualize and then project) mathematics all around themselves.

These data roughly define a wide diversity within a high school student population, a continuum from mathematical intellectuals to those students who resist or reject Platonist content due to severe worldview culture clashes. Teachers who wish for all their students to love mathematics do not understand projectionism. They are projecting their own worldviews onto their students.

In their Algonquins of Pikwàkanagàn project, Beatty and Blair (2015) watched and listened to Grade 2 students engaged in traditional looming while talking to each other very naturally (Algonquins of Pikwàkanagàn First Nation Project subsection). These are legitimate raw data from which to infer (to project) what the students were thinking. Beatty and Blair offered very credible inferences. On a cautionary note, however, teachers and researchers need to be mindful of limitations when making inferences because the process does involve mild projectionism (superimposition), according to Einstein. Such issues raise other projection problems (Aikenhead, 2017).

Misinterpretations of quantified assessment

This section deals with three separate issues about misinterpreting standardized test results for purposes of comparing mathematics education systems. In general, the ideology of quantification is explored here by critically analyzing (a) the recent Programme for International Student Assessment (PISA) results produced by the OECD, (b) an earlier *Scientific American* quantitative and qualitative evaluation study based on the Third International Mathematics and Science Study (TIMSS), and (c) the validity of one of three PISA sections acclaimed for its everyday relevance to students. To make the discussion concrete, the University of Hawai’i’s project is chosen as a context (Furuto, 2013b, 2014), but the implications apply to all educators, administrators, politicians, and the general public.

In the Culture Clashes that Alienate Many Indigenous Students subsection, a distinction was made between *intellectually* knowing how to calculate an average and having the *wisdom* of knowing when it is appropriate or inappropriate to do so. An equivalent type of discussion occurs here with real-life examples.

PISA results

Furuto (2014; University of Hawai’i Project subsection) reacted to the international assessment results from PISA by drawing attention to the U.S. average score being below the collective average of all participating countries. However, nothing was mentioned about the plethora of evidence that PISA is essentially a political project masquerading as an educational tool (Sjøberg, 2015). Is this masking power with naïveté? A short critical analysis of the PISA instrument and its data reveals several illusions. All are of

specific interest to EAM educators who write and/or assess R&D projects. (For an in-depth critique of PISA, see Sjøberg, 2016.)

Furuto (2014) insightfully highlighted a key finding in the PISA 2012 report on the 2009 PISA results: “[A]mong data on students, community, and institutional factors that could help explain differences in mathematics performance ... access to *equitable and quality mathematics resources* is key to attaining academic success” (p. 111, emphasis added).

Therefore, we should ask: Can a one-dimensional evaluation (students’ average scores) be trusted to judge the quality of a country’s or a state’s complex mathematics education system? Furuto’s (2014) quote above suggests not; and a resounding “no” comes from a number of other researchers (e.g., Centre on International Education Benchmarking [CIEB], 2015; Gibbs & Fox, 1999; Parkin, 2015; Serder & Jakobsson, 2015; Sjøberg, 2015). For instance, the ranking for the U.S. in the PISA 2012 report on the 2009 test would have changed dramatically if two additional major factors—equity and efficiency—were taken into consideration, according to the CIEB (2015). Social equity in an educational jurisdiction is, in part, “measured by the gap between immigrants and non-immigrants ... between rich and poor children” (Parkin, 2015, p. 1). Equity also encompasses “the distribution of resources within the education system” (Parkin, 2015, p. 1), such as the *quality of mathematics resources* (e.g., projects that enhance school mathematics culturally) and the support offered to teachers, including appropriate professional development programs. The second factor, efficiency, arises from the fact that financial support for education competes with many other social needs. A criterion for a better quality education is “spending that is efficient as opposed to high” (Parkin, 2015, p. 2); in the vernacular, getting the most bang for your buck.

CIEB (2015) ranked countries based on three factors: (a) *mathematics performance*; “average rank on all three sections of PISA”; (b) *equity*; “percent of variation in mathematics performance explained by socio-economic status”; and (c) *efficiency*; “spending per secondary student, U.S. dollars 2011.” CIEB integrated these three factors. One result of CIEB’s analysis was the substantial increase in the trustworthiness of the country’s rankings. A second result was the completely different rankings for each country. For example, the PISA 2012 test report (OECD, 2013) ranked Finland, Estonia, and Canada at 11, 12, and 13 (respectively) on the basis of student performance alone. These rankings were behind many East Asian countries. The much more sophisticated three-factor analysis resulted in Finland, Estonia, and Canada being tied for the *top* PISA ratings overall. Which analysis should be used, PISA’s or CIEB’s? At best, the decision would be a rational one; at worst, entirely political, based on who to blame for low scores—teachers or governments? Furuto (2014) was correct to notice a one-dimensional measure of differences in countries’ mathematics education systems. But she could have critiqued a political instrument’s incursion into mathematics education.

Notably, however, when any measure of achievement focuses on Indigenous versus non-Indigenous students, the resulting large difference is not only unjustifiable on ethical and economic grounds (Parkin, 2015) but it is dramatic evidence for the educational debt created by Canada’s neo-colonial underfunding of education, health, social services, housing, water management, and community development.

Scientific American’s investigation into TIMSS

A second and related issue is the interpretation of TIMSS data. Even though the instrument had more validity problems than today’s PISA, the misinterpretations exposed here are the same for both TIMSS and PISA.

Two science reporters from *Scientific American*, Gibbs and Fox (1999), investigated the “low ranking of American teenagers” (p. 87) on TIMSS. The low rankings had caused a national political crisis at the time, mostly at the expense of mathematics and science teachers. Gibbs and Fox interviewed a wide sample of educational experts and the general public and completed three case studies of typical high school mathematics and science teaching: a high school in Texas (representing a below-average TIMSS score), a high school in Saskatchewan (above-average score), and a high school in Sweden (the highest scoring country).

Among political leaders and the public, Gibbs and Fox (1999) discovered a complete lack of understanding that the testing was actually an elaborate polling project with concomitant statistical confidence limits. Naturally, if countries with different scores have overlapping confidence limits, they were clustered

together as being tied in their ranking. Political leaders and the public were masking the power of Platonist content with ignorance.

I illustrate this by using the U.S. PISA 2012 data (OECD, 2013). The U.S. average score was 481 with a ranking of 29. Its score was a statistical tie with nine other countries whose results ranged from 489 (ranked 23) to 477 (ranked 32). In this context, therefore, the number 29 is no different than the numbers between and including 23 to 32. Interpreting differences in scores is tricky, but there is more.

The U.S. average score (481) is *statistically significantly different* from the OECD average score (494), as Furuto (2014) correctly pointed out. But there is another critical question to pose: Is the difference between 481 and 494 *economically and educationally significant*? How much would it actually cost to bring the U.S. average score up to 494? In what ways and to what extent must the culture of U.S. education change in doing so? Is it worth it for a measly 13 points (on a scale over 600 points) on a test of questionable trustworthiness?

In other words, Furuto's (2014) *intellectual* understanding of statistics (Platonist content) led her to the correct conclusion that the difference of 13 points was statistically significant. However, she tied herself to the altar of Platonist content by ignoring a *wisdom* understanding of that 13-point difference.

A singularly narrow focus on quantitative data's statistical significance masks power with statistical calculations, or with naïveté. Have people been coerced at the altar of Platonist content? This is only one of many ways in which the ideology of quantification asserts itself. This issue is an example of EAM cultural content focused on math-in-action.

Gibbs and Fox (1999) concluded that differences in scores are not really large enough to be concerned about it. In their article, "The False Crisis in Science Education," the smoke-and-mirrors of politically embellished score differences between the average for all countries and the U.S. average created a false crisis. Their article's title could easily have read "The False Crisis in Mathematics Education."

Do these quantitative data reflect Gibbs and Fox's (1999) three case studies, one in each of three countries? No at all. In fact, the quantitative data hide a second false crisis from view. By using qualitative data, the journalists exposed what gets lost when people rely on quantitative data alone. In their Texas case study, Gibbs and Fox (1999) characterized school mathematics and science as memorizing definitions or formulas, copying notes from teachers, and following correct procedures hoping to get the right answer. Innovative courses introduced by teachers were popular with students, but the school administration curtailed them in order to cater to "state-required courses aimed at college-bound kids" (Gibbs & Fox, 1999, p. 90) by maintaining standards for the STEM pipeline.

In the typical, top-scoring, Swedish high school, Gibbs and Fox (1999) discovered that students were "quieter and more attentive than their Texas peers" (p. 90), but classes were a paragon of traditional instruction. Although the content and labs appeared sophisticated in their technology and topics, in the labs students only mimicked the teacher's demonstrations; followed by taking notes. During the teacher's class presentations, a catechism of "Any questions?" Silence. 'Okay on to ...'" (Gibbs & Fox, 1999, p. 90) was observed. This apparently was somniferous, judged by the observation of yawning students in class. The Swedish school's pedagogy was not all that different from the Texas school's pedagogy.

In Saskatchewan, Gibbs and Fox (1999) found a different ethos pertinent to the mathematics classes. Teachers tended to rely more on open-ended assignments and to talk about the social issues raised by science, technology, and their associated mathematics. Students were observed learning from the curriculum on a need-to-know basis. Students asked teachers such questions as, "How do you know that?" (Gibbs & Fox, 1999, p. 91). When a mathematics student wondered: "What's this useful for?" The teacher replied, "I'll give you some electrical engineering problems that use rational functions. How's that?" The student exclaimed, "Cool" (Gibbs & Fox, 1999, p. 91).

What second false crisis in U.S. mathematics and science education arose from these qualitative data?

The false crisis [manufactured by embellishing the significance of small differences between student averages from different countries] masks the sad truth that the vast majority of students are taught [content] that is utterly irrelevant to their lives—and that scientists [and mathematicians] are a major part of the problem" (Gibbs & Fox, 1999, p. 92).

In other words, international test scores, the *quantitative* representations of a country's EAM Platonist education system, are far removed from what actually transpires in that country's classrooms, as indicated

in these systematic *qualitative* case studies. These types of standardized test scores suffer a serious validity problem, no matter how statistically reliable¹¹ they are. When looking at such test results, therefore, mathematics educators' critical vigilance needs to be asserted in order to avoid masking power with systemic naivety.

Organizations that conduct large-scale testing *intellectually* know how to calculate averages and rank them. Mathematics educators and school administrators must have the *wisdom* of knowing how appropriately valid it is to compare those averages and, if so, to proceed with critical vigilance.

Validity of one PISA section

Issues of validity relate directly to writing cross-cultural teaching materials or lesson plans. In one of the three sections of the PISA test, each question begins with a short passage called a "backstory," which is like a newspaper item relevant to a 15-year-old. Students answer questions based on the backstory.

Serder and Jakobsson's (2015) research asked: How relevant are those backstories to the 15-year-olds who take the test? Although their research was on the PISA science test, the issue is identical for the PISA mathematics test. Serder and Jakobsson's (2015) results added to the evidence undermining the trustworthiness and cultural validity of PISA. Many students in the study positioned "themselves as being different from and opposed to the fictional pictured students who appear in the backstories of the test" (Serder & Jakobsson, 2015, p. 833). This happened because of the academic language uttered by the fictional students; the elite "little scientist" images conveyed by them; and their inclination to conduct experiments at home, to which some students reacted, "Why bother so incredibly much?" (Serder & Jakobsson, 2015, p. 848). In other words, many students could not see themselves reflected in the backstories, and their engagement suffered accordingly (Making Connections).

PISA claimed that it monitored students' attainment of scientific knowledge. But for this section of questions, PISA was closer to monitoring students' scientific self-identities. That was not what the instrument intended to measure, thus making the backstory section of PISA invalid. Serder and Jakobsson's (2015) conclusion sums up the problem:

This study adheres to research that advises caution in not over-interpreting [not masking power with systemic naivety] the PISA results and stresses that understanding students' "knowledge" about science [or mathematics] is much more complex than what is communicated by the international assessment organizations. (p. 834)

Writers of cross-cultural modules need to conduct a preliminary study to ensure that their students see themselves reflected in the written materials.

Conclusion

The Platonist ideology of quantification demands that the outcomes of schooling be commodified so that achievement can be measured numerically (Ernest, 2016a). This quantified worth of students, teachers, programs, and educational jurisdictions is so simplistic that it immeasurably distorts reality (Misinterpretations of Quantified Assessment subsection), despite the quantification's false aura of objectivity (Aikenhead, 2008). In other words, political expediency in relying on quantitative results alone trumps quality education defined as "the human dimensions of knowing" (Ernest, 2016a, p. 53). Even worse, the allocation of a government's "resource for testing is the main argument to justify math contents" in curricula (D'Ambrosio, 2016, p. 33).

It appears that a Platonist belief about school mathematics, manifested through several pervasive impediments to student success, is well beyond its "best before date." *Now is an appropriate time to replace it with* a pluralist and cultural understanding of EAM, along with another culture's mathematizing. How will all students benefit? A cross-cultural understanding of Euro-American school mathematics lessens students' reluctance or resistance to understand relevant EAM Platonist content. Moreover, it facilitates students' participation in reconciliation. Today's mathematics curriculum must be revisited accordingly.

Mathematics curricula revisited

Reconciliation is not an event. It's something that needs to enter into the way we do things.—John Ralston Saul (2014, p. 260)

In a research report into what Indigenous parents, teachers, and students say about improving Indigenous student learning outcomes, a mathematics education graduate student responded, “I think that part of my research challenges the colonial, Eurocentric mess of mathematics. I think people need to see that. I think *the people making those curricula decisions* need to see that” (Saskatchewan Instructional Development and Research Unit, 2014, p. 62, emphasis added). When speaking about a group of creative, intelligent, mathematics teachers, the graduate student lamented, “[T]hey also trained in mathematics in a Western way, so they are looking at the universe through a Western mathematics lens” (p. 62); that is, a Platonist lens. Curriculum writers have been challenged similarly over the years but to little or no avail.

Teachers and researchers always refer to their government-sanctioned mathematics curricula, as they should. Some projects expressed the goal of influencing the established curriculum through a natural groundswell of success (Beatty & Blair, 2015; Jannok Nutti, 2013; Lipka, 1994; Lipka et al., 2005; Lunney Borden, 2013). In the end, however, even their documented successes have proven impotent in the face of an entrenched Platonist belief about school mathematics, except for making token changes (e.g., Saskatchewan Curriculum, 2007; Mathematics Curricula subsection).

Two major conclusions are forthcoming in this section: one for school mathematics innovators and another for those who officially set curriculum policy or who lobby those officials.¹²

Innovative educators and researchers

The University of Hawai'i renewed its undergraduate mathematics program so that Polynesian students could see themselves in it (University of Hawai'i Project subsection). The response was dramatic (Furuto, 2014). But this undergraduate renewal did not filter down to influence the school mathematics curriculum explicitly, despite the plethora of mathematics units developed through the ESTEMI (2016). The University of Alaska Fairbanks made inroads at the local level with Ciulistet and MCC (2016), but their successes seem more like maneuvering around the state curriculum by partially ignoring it or supplementing it (Lipka et al., 2013). No evidence was provided by either project to show a change in the state curriculum as a result of their extensive and highly successful R&D projects. The authoritarian ideology of “standards” in the United States is daunting, as are Canadian conventional curricula designed for Canada's 34% minority of students bound for university calculus courses (OECD, 2016; Projectionism subsection).

High-quality student learning of Platonist content relevant to the 66% of Canadians takes more time for students to achieve than does vocabulary and “algorithmic” memorization (Beatty & Blair, 2015, p. 20). This classroom reality is mostly ignored by curriculum developers who cram extraneous EAM Platonist content into the curriculum, thereby subverting teachers' pedagogical flexibility for innovation into EAM cultural content and Indigenous mathematizing (Mathematics Curricula subsection).¹³

A curriculum crammed with content perpetuates the myth that the curriculum is rigorous, even though algorithmic memorization is far from rigorous in terms of meaningful learning—the type of experiential learning expected by the Indigenous “coming to know” (Cajete, 2000, p.110). “How often is our answer to the student's question of ‘Why do I have to know this?’ based upon the superior intellect that abounds from knowing more rather than the relevance to the individual student?” (Russell, 2010, p. 40).

Excessive vocabulary and algorithmic memorization serve as a mythical armor in a fight against curriculum renewal. A curriculum overcrowded with both nonessential and essential content *masks power with innocence* on a grand scale. A required task is to identify nonessential content. Almost anyone with a science or engineering background who has coached or home-schooled their child in EAM Platonist content (Grades 5–12) would most likely make a worthwhile contribution to the task. I am also thinking of people like mathematician Newman (1956), who wrote:

The most painful thing about mathematics is how far away you are from being able to use it after you have learned it. ... [I]n mathematics it is possible to acquire an impressive amount of information as to theorems and methods and yet be totally incapable of solving the simplest problem. (p. 1978)

The feigned academic rigor of nonessential content maintains a neo-colonial treatment of Indigenous communities. An implicit, systemic, race-based policy perhaps?

Curriculum and lesson plan content has a strong ethical dimension to it (Boylan, 2016, p. 399):

[T]he choice in immediate and specific situations as to what curriculum content to include, or not to include, is an ethical choice not only for considerations of social justice but also because of how content may alienate or include learners.

The fear that teaching mathematics culturally would reduce standards is contradicted by numerous research studies cited in this article. Advocates for the Platonist status quo present no such systematic evidence to support their crowded curriculum (High Status of School Mathematics subsection). *Where are their data? Where is their rationale?* They have cleverly masked their power (subsection Hidden Platonist Agenda).

A narrow focus on Platonist content achievement leads to a deficit approach to school mathematics, a neo-colonial pedagogy, and a false assumption that learning mathematics is an acultural process (Bang & Medin, 2010). On the other hand, a relevancy focus on Platonist content harmonizes with the projects described in Examples of Enhancing School Mathematics Culturally. It answers the question: Which notions found in conventional school mathematics continue to serve students' interests?

A plan seems clear: First, base school mathematics on cultural practices, mostly Euro-American mathematizing (e.g., mathematics-in-action), but equitably balanced with local Indigenous mathematizing. Secondly, strip conventional curricula of their extraneous content according culture-practice criteria, judged by people such as Newman (1956) quoted above.

To augment the impact of SMYM on participating schools (Mi'kmaw First Nation Projects subsection), Lunney Borden and colleagues (2017) advised strengthening the connections between students' SMYM projects and the mathematics curriculum outcomes by focusing on new strategies with students.

My advice is quite different. Focus instead on an Atlantic Canada *political project* to officially update the current 19th-century Platonist curriculum to a cross-cultural EAM curriculum, leveraged by the SMYM project's achievements (e.g., class attendance increases, drop-out rate decreases, community members' [voters'] testimonials, test results, etc.). Such an agenda amplifies Lunney Borden's (2013, p. 7) research program ("How can curricula and pedagogy be transformed to support Mi'kmaw students ...?"). Both Mi'kmaw and non-Mi'kmaw students will benefit (Introduction). Updating the current Platonist curriculum requires a sense of empowerment by mathematics education leaders, as Jannok Nutti (2013) demonstrated (subsection Sweden).

In most projects described in Examples of Enhancing School Mathematics Culturally, degrees of decolonization cohabited with revitalizing a marginalized people's culture. This directly benefited Indigenous students' academic success through strengthening their cultural self-identities, while reducing the political-social power imbalance between the curriculum and local Indigenous communities. And, in turn, a decolonized curriculum will help stem the cycle of low graduation rates, unemployment, poverty, and family dysfunctionality. Canada's economy will benefit immeasurably as a consequence (Sharpe & Arsenault, 2009).

By producing teaching materials, innovative researchers and teachers furthered their decolonizing goal to a significant extent (Examples of Enhancing School Mathematics Culturally). But more must be done to scale-up their successes for the benefit all schools. These researchers and teachers need to work out an explicit plan to renew the government sanctioned curriculum. Such a plan could expand the worn-out content/process binary of most curricula standards, by recognizing that reality is first and foremost about *context*, which necessitates a curriculum framework designed with a context-content-process triad. EAM cultural content and local mathematizing are viable contexts that have led to greater student success.

In 2008, the Province of Saskatchewan renewed its science curriculum into a cross-cultural one, along with its production of a Grades 3–9 science textbook series that enhances school science with Indigenous

perspectives (Aikenhead & Elliott, 2010). This happened as a result of a broadly based *political* project that involved a number of independent people diversely situated in various institutions. This informal coalition was initiated in 2005 after a group of high school science teachers participated in a province-funded, year-long in-service program that ended with teachers understanding and accepting the benefits of culture-based teaching. But when asked what changes they would make the next year, all replied, “None,” for the simple reason that culture-based content was not explicitly in the science curriculum. The teachers had the best interests of their students at heart. Their criterion for best was determined by the outcomes and indicators found in the science curriculum. Pervasive general goals in the curriculum did not count.

Among Canadian ministries of education at this time, there is a renaissance of openness to enhancing school subjects with Indigenous perspectives—Indigenous ways of knowing, doing, living, and being. This policy renewal is the direct result of Canada’s Truth and Reconciliation Commission’s (2016) final report. The time is right for substantially reworking mathematics curricula *as an act of reconciliation*.

A cross-cultural mathematics curriculum gives teachers the political power, legal authority, and much encouragement to work toward enhancing school mathematics culturally (Lipka, 1994). The challenges to updating a Platonist mathematics curriculum can be seen as opportunities to mount a political project. Any curriculum is certainly a political document. Accordingly, it needs *political* action to get revisions started. The literature cited in this article supports such a renewal for the benefit of all students. “As human culture changes, so too does some of what we believe we know with certainty. Our ideas and standards of truth ... have changed over the history and development of mathematics” (Ernest, 2016b, p. 390). Even the National Council of Teachers of Mathematics is planning to move from a fairly exclusive stance to a more inclusive one (Larson, 2016).

Where are the Canadian agendas for creating an up-to-date Indigenous cross-cultural, Euro-American mathematics curriculum that harmonizes with reconciliation so Indigenous students can see themselves reflected in it?

Because each educational jurisdiction has its unique political–social context with a unique group of local “movers and shakers,” a political strategy must be mapped out locally by a committed diverse group of innovators, visionaries, Indigenous educators, Elders, selected university academics, and allies with connections to political–social power (Aikenhead, 2002b).

The opposition

I shall first recap from earlier discussions. The decolonization of mathematics education will be seen as a direct assault on

1. the mythical armor of rigor that envelops a Platonist curriculum (Mathematics Clarified and Some Fundamentals of Mathematics subsection).
2. the absolutist philosophies that side with Plato’s world of ideas and demand a Platonist curriculum (The Platonist Belief, A Hidden Platonist Agenda, Historical Appropriation, Marginalization, and Centrality of Indigenous Languages subsections).
3. the supremacist position of mathematics enthusiasts and professional organizations who are enamored with transmitting their crowning achievement of the human intellect (pure abstract decontextualized mathematics) to all students (The Platonist Belief subsection).
4. nationalistic economists who champion corporate profits and global competitiveness and, in turn, finance the intrusion of their STEM movement into school mathematics curricula worldwide (The Altar of Platonist Content subsection).
5. the Western globalization impulse to colonize nations economically (Bishop, 1990; D’Ambrosio, 1991; Mukhopadhyay & Greer, 2012).

In this list I detect a Eurocentric undercurrent of people searching for certainty, in a world composed of varying degrees of uncertainty or flux (Little Bear, 2000).

Einstein (1921) summed up the situation succinctly: “As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.” By drawing upon mathematics, the physical sciences aimed to produce an exact picture of the material world. Göbel’s

incompleteness theorems in mathematics (Stanford Encyclopedia of Philosophy, 2015) and theoretical physics in the 20th century have proven “that aim is unattainable” (Bronowski, 1973, p. 353). Bronowski (1973) explained that “errors are inextricably bound up with the nature of human knowledge” (p. 360). Hence, one epistemic implication for Platonists is clear: their claim to pure knowledge with certainty (Yousan and From Tolerance and Inclusion to Dialogue and Collaboration subsections) is an oxymoron.

These two quotations from Bronowski (1973) come from his chapter entitled “Knowledge or Certainty,” a phrase meant to define a continuum between the two. Certainty is attained at the expense of knowledge, a point of view that resonates with Einstein’s (1921) quotation above. The issue is pursued further in Aikenhead (2017).

“When people believe that they have absolute knowledge, with no test in reality, this is how they behave” in political–social contexts (Bronowski, 1973, p. 374), a context in which mathematics education certainly exists. Bronowski (1973) described such people’s behavior as arrogant, dogmatic, and ignorant.

Making Connections began with mathematician Newman (1956, p. 1614) stating, “Yet no one questions the validity of what you say.” This article seriously questions the validity of what Platonist school mathematics says. Similarly, many students have confronted its power and have reacted both intuitively and negatively, as if a Platonist belief in school mathematics were a deceptive scam to avoid.

Much more than *philosophical tension* exists between an agenda to decolonize the school mathematics curriculum and an agenda to support the Platonist’s status quo. It is the *raw political–social power* that can sweep ethical accountability under the carpet and can rely on pervasive racism in the underbelly of Canadian society (Battiste, 2002; Government of Alberta, 2010; St. Denis, 2004). It is an apparition from Canada’s 19th- and 20th-century nation building achieved at the devastating expense of Indigenous citizens (Consequences of Colonial Genocide subsection). It is the same raw power that incarcerated children in residential schools and that sanctioned national and international government-sponsored human trafficking in infants taken from their mothers for adoption by, or placed in foster homes of, White families in numbers surpassing residential school numbers, now known as “The 60’s Scoop,” even though the practice continues today (Adams, 2016; Thanh Ha & Galloway, 2017).

The poignancy of Canada’s Truth and Reconciliation Commission (2016) is its sharp focus on decolonization through reconciliation. Stakeholders of the mathematics curriculum, especially ministries of education and their curriculum writers, have an ethical choice:

- Support reconciliation by renewing the curriculum to enhance it with Indigenous perspectives exemplified by the decolonizing projects described in this article, or
- Avoid this act of reconciliation and thereby support a 19th-century political–social policy, again at the expense of Indigenous students, families, and communities.

If we do not decolonize of our curricula, we reject reconciliation.

Conclusion

No such thing in the Indyun way as gettin’ wise. Gettin’ wisdom. Wisdom’s a path you decide to take’n follow, not someplace you get to.—Richard Wagamese (1994, p. 189)

This article’s Abstract listed seven general questions. By responding to them, many significant ideas arose, including the foundational importance of paying attention to culture clashes; challenges faced by people moving between the culture of EAM and Indigenous cultures; pluralism; fundamental beliefs about mathematics; the cultural application of mathematics (instead of its scientific application); EAM Platonist content (an intellectual tradition of understanding); EAM cultural content (a wisdom tradition of understanding); the function of language-laden cognition; cognitive imperialism; camping spots of dialogue; the power of cultural self-identities; a notable flaw in ethnomathematics; the systemic neo-colonialism inhabiting a Platonist mathematics curriculum; essential Platonist content taught as Euro-American cultural practice; the close relationship between reconciliation and Platonist or EAM school mathematics; and the inspiration of innovative mathematics researchers and teachers who are pioneering a new future for school mathematics.

The article began by introducing the crucial concept “masking power with innocence” (McKinley, 2001, p. 74). The concept expanded along the way to masking power with convention, ignorance, racism, Platonic innocence, a rhetorical sleight of hand, an intended compliment, opaque responsibility, systemic naïveté, statistical calculations, and feigned mythical innocence of academic rigor. Self-critical analyses on these themes will continue to liberate Euro-American minds from outdated policies for and approaches to school mathematics.

From tolerance and inclusion to dialogue and collaboration

Euclidean geometry was heralded by ancient Greeks as being consistent with Platonic philosophy and the only framework with which to understand the world. Two hundred years ago, however, Gauss, Riemann, Lobachevsky, and Minkowski helped invent non-Euclidian geometries (Director, 2006). A *universalist* geometry gave way to *pluralist* geometries, and humanity *intellectually* evolved as a result. Einstein’s theory of general relativity is but one outcome.

Currently in the domain of mathematics education, the conventional Platonist *universalist* notion of mathematics is being challenged by a contemporary *pluralist* notion of mathematics (the superordinate sense of the word; Mathematics Clarified). Existing outside the domain of philosophy, this challenge reverberates worldwide within the political–social contexts of countries that are home to Indigenous cultures.

A pluralist notion of mathematics generally adheres to Bishop’s (1988b) position: mathematics is a symbolic technology for building a relationship between humans and their environment (subsection Mathematical Pluralism). Indigenous perspectives on building such relationships are being *tolerated* and *included* in some classrooms today, where “learning environments leverage knowledge associated with everyday [cultural] experiences to support subject matter learning” (Bang & Medin, 2010, p. 1014).

Not only does a pluralist view of mathematics acknowledge the mathematizing carried out by Indigenous cultures but it identifies school mathematics as an expression of Euro-American cultures. Put simply, Euro-American mathematics is an amalgam of its cultural content and Platonist content (Definitions Clarified and Historical Appropriation subsections). Its cultural content, however, has been suppressed by a Platonist belief that also marginalizes many Indigenous students and ensures that their high school graduation rates will not reach full potential.

And by what authority?—A self-composed social license to narrowly define the school subject of mathematics, based on an allegiance to an ancient Greek philosophical presupposition; its application to the Eurocentric physical sciences; and its arbitrary, deceptive creation of an epistemic binary composed of *logical–formal* (*formal discourse*) versus *intuitive* (*informal discourse*) (Einstein, 1921; Ernest, 1991; respectively; A Hidden Platonist Agenda subsection).

Ernest (1991) perceived the process of defining the subject matter as a rhetorical sleight of hand (A Hidden Platonist Agenda subsection). This social license was acceptable to a social era involved in colonizing Indigenous people through a residential school system that ensured low graduation rates for Indigenous students (Consequences of Colonial Genocide subsection). Is this social license still acceptable today for school mathematics?

For students coping with any degree of mathematics anxiety (Fowler, 2012; Nasir et al., 2008; Stoet et al., 2016), EAM cultural content and local Indigenous mathematizing serve as bridges between those students and a renewed school mathematics curriculum enhanced in cross-cultural ways. Culture clashes can certainly be reduced (Culture Clashes That Alienate Many Indigenous Students subsection and Examples of Enhancing School Mathematics Culturally).

What does the future hold? Will instances of masking power with innocence be rooted out of school mathematics? Will Platonist curricula evolve into culture-based curricula (Figure 1)? Will neo-colonial power imbalances be renegotiated between Indigenous communities and EAM curriculum developers? Will privilege-savvy be an aim of teacher education? Will humanity eventually evolve as a result, but this time in terms of developing *wisdom*, accrued from newly reconciled relationships between Indigenous and non-Indigenous peoples?

Doolittle and Glanfield (2007, p. 29) point out an Indigenous axiological concept of well-being: the “balance among mind, spirituality, body, and emotion.” An Indigenous ontology reveals a dependent relational spirituality—a web of interrelationships and responsibilities between humans and “the greater-than-human natural world” (Lowan-Trudeau, 2015, p. 653), from which flows the gold standard of sustainability. “[I]t’s not just a question of how Western society can help Indigenous people, but how Indigenous people can help Western society” (Doolittle & Glanfield, 2007, p. 29). Doolittle and Glanfield (2007) are speaking of camping spots of *dialogue* and a commitment to *collaboration*. “Respect is more than tolerance and inclusion—it requires dialogue and collaboration” (8Ways, 2012, p. 4).

At the same time: “[S]ome mathematics is required for those who want to succeed in the society that has risen around us. But to be truly successful as Indigenous people, we must find a balance” (Doolittle & Glanfield, 2007, p. 29). Teachers of *cross-cultural* EAM can contribute to Indigenous communities achieving that pragmatic balance by strengthening students’ Indigenous self-identities through an understanding of EAM Platonist and cultural content. Just as Indigenous mathematizing is a cultural practice, teaching Euro-American mathematics needs to be seen as a Euro-American *cultural* practice (Definitions Clarified subsection).

The conceptualization of mathematics taught in schools is now open for renegotiation. In this context, we must ask: What is mathematics? In 1868, Japanese people called it “Yousan” (Yousan subsection), which I translated to “Euro-American mathematics.” Given the advent of reconciliation, a culture-based contemporary curriculum is now required. Evidence described in this article supports enhancing school mathematics culturally. This means selecting Platonist curriculum content mainly based on Euro-Canadian cultural objects, processes, and ideas.

Four major factors affect Indigenous student achievement: the curriculum; various strategies of instruction; culturally valid assessment; and interpersonal relationships such as a teacher’s respect, firmness, personal warmth, and a sense of caring.

When mitigating the dominant culture’s political–social power imbalance with Indigenous communities, there will be an exchange of gifts:

- From the non-Indigenous people, *a practical intellectual gift of Euro-American mathematics credentials*.
- From the Indigenous people, *a gift of wisdom “to chart the way to a more sustainable society and a more meaningful way of life”* (Kinew, 2015, p. 266).

Willy Alangui’s story

I conclude with a true story (Salleh, 2006). Mathematicians create highly complex sets of equations that model equally complex phenomena in the physical world, such as weather forecasting (i.e., mathematical modeling). So it was with Willy Alangui, a professional mathematician and member of the Indigenous Kankanaey Nation in the Philippines. Large, tall, vertical rice paddy terraces captured his interest. “I’m trying to understand how water is efficiently distributed in all the paddies,” he told Salleh, a reporter from the Australia Broadcasting Corporation.

But every time he tested a revised model of water distribution, his model failed. A non-Indigenous mathematician may have given up perplexed. Willy, however, had two ways of seeing this problem (i.e., “two-eyed seeing”; Hatcher et al., 2009): the perspective of his non-Indigenous mathematics colleagues and his own Indigenous perspective. He decided to observe farmers working on the terrace and forge a relationship with them.

Willy discovered a key variable that determines how the Kankanaey rice irrigation system works: the ethic of cooperation. The community’s social responsibility played a role when water was scarce. The farmers near the top of the terrace would lessen their intake to equally share water with those below. Social responsibility erratically interfered with Willy’s mathematical models. Platonist equations are unable to deal with social responsibility.

This incident caused Willy to broaden his perspective as a mathematician. “My mathematics may be deficient. It’s not the be all and end all of everything. It’s just one way of looking at the world.” Kankanaey

mathematizing includes the value of social responsibility. How can mathematical equations incorporate an ethical value into mathematical modeling? Willy thought it may not be possible. He went on to describe Platonist content as a “powerful” and “arrogant” field that marginalized other mathematical knowledge systems.

Willy’s story strongly suggests to me that cross-cultural school mathematics teaching materials should all include in their titles, “Mathematics: A Cultural Way of Knowing.”

Notes

1. *Critical analysis* in this article broadly refers to (a) judging positive and negative features of an article, idea, or conclusion; (b) identifying and evaluating important evidence, rhetorical moves, and points made or overlooked; and (c) finding implicit assumptions and presuppositions that may lead to insights or to errors, inconsistencies, incompleteness, and other flaws.
2. Details differ regarding Métis and Inuit Nations, but the consequences are very similar.
3. Due to article length limitations, reference will occasionally be made to a much more detailed website discussion paper by Aikenhead (2017).
4. Elder Dr. LeRoy Little Bear (Kainai Nation of the Blackfoot Confederacy) prefers spelling the word *wholistic*, which avoids confounding the sacred *holy* with the secular *holistic*. The colonizer’s English spelling excludes the Indigenous meaning captured by the terms *wholistic* or *wholism*. I follow Elder Little Bear’s spelling convention in this article out of respect for this eminent Elder, scholar, and mentor.
5. This Euro-American moniker recognizes the globalization by European and the United States that spread their academic mathematics worldwide. It reflects those cultures’ histories of appropriation from earlier civilizations discussed in detail in the Historical Appropriation subsection, which are made explicit in the history component of EAM cultural content.
6. Interestingly, Platonists hide the fact that their binary is framed by Greek epistemology (Hall, 1976), thus casting their strategy as being culture laden. Many world cultures, including Indigenous cultures, do not lend much credence to Greek epistemic binaries.
7. For example, mathematics modeling used by airline companies figures out how many seats should be overbooked on a particular flight in order to maximize profits. Mathematical applications in everyday commerce, hobbies, and specific occupations (e.g., architecture) are other examples.
8. Mathematizing in any culture, Canada’s cultures included, involves any of the six foundational processes, cultural practices identified by Bishop (1988b; subsection Mathematical Pluralism).
9. Due to space limitations, Haida Gwaii projects are not described. See Aikenhead (2017).
10. Elders do not normally engage in assessment as understood by schools.
11. *Statistical reliability* refers to consistency of results from one occasion to another. It is a technical term in social science research. This meaning differs greatly from the media’s meaning of a *reliable source*, which is informed and trustworthy. Potentially confusing is the fact that *trustworthy* is close to the statistician’s meaning of valid—it measures what it claims to measure. Culturally valid student assessment is a central concern, for instance, when a national exam is simply translated into a local Indigenous language.
12. A third conclusion concerning the National Council for Teachers of Mathematics is found in Aikenhead (2017).
13. It is essential to offer the minority 34% of students an optional enriched high school program that resembles an International Baccalaureate program. It is beyond the scope of this article to discuss the logistics of various flexible pathways for high school graduation.

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References

- 8 Ways. (2012). *8Ways: Aboriginal pedagogy from western New South Wales*. Dubbo, NSW, Australia: The Bangamalanha Centre.
- Adams, B. A. (2016, March 19). “Serious” cash for [A]boriginal children likely. *Saskatoon StarPhoenix*. Retrieved from <http://thestarphoenix.com/news/local-news/serious-cash-to-correct-discrimination-against-aboriginal-children-likely-in-budget-law-prof-says>
- Adams, B. L., Shehenaz Adam, A., & Opbroek, M. (2005). Reversing the academic trend for rural students: The case of Michelle Opbroek. *Journal of American Indian Education*, 44(3), 55–79.

- Aikenhead, G. S. (1997). Toward a First Nations cross-cultural science and technology curriculum. *Science Education*, 81(2), 217–238.
- Aikenhead, G. S. (2002a). Cross-cultural science teaching: “Rekindling traditions” for Aboriginal students. *Canadian Journal of Science, Mathematics and Technology Education*, 2(3), 287–304.
- Aikenhead, G. S. (2002b). The educto-politics of curriculum development. *Canadian Journal of Science, Mathematics and Technology Education*, 2, 49–57.
- Aikenhead, G. S. (2006). *Science education for everyday life: Evidence-based practice*. New York, NY: Teachers College Press.
- Aikenhead, G. S. (2008). Objectivity: The opiate of the academic? *Cultural Studies of Science Education*, 3(3), 581–585.
- Aikenhead, G. S. (2017). School mathematics for reconciliation: From a 19th to a 21st century curriculum. Retrieved from <https://www.usask.ca/education/documents/profiles/aikenhead/index.htm>
- Aikenhead, G. S., Brokofsky, J., Bodnar, T., Clark, C., Foley, C., ... Strange, G. (2014). *Enhancing school science with Indigenous knowledge: What we know from teachers and research*. Saskatoon, SK, Canada: Saskatoon Public School Division with Amazon.ca. Retrieved from <http://www.amazon.ca/Enhancing-School-Science-Indigenous-Knowledge/dp/149957343X>
- Aikenhead, G. S., & Elliott, D. (2010). An emerging decolonizing science education in Canada. *Canadian Journal of Science, Mathematics and Technology Education*, 10, 321–338.
- Aikenhead, G. S., & Michell, H. (2011). *Bridging cultures: Indigenous and scientific ways of knowing nature*. Toronto, ON, Canada: Pearson Education Canada.
- Aikenhead, G. S., & Ogawa, M. (2007). Indigenous knowledge and science revisited. *Cultural Studies of Science Education*, 2(3), 539–591.
- Alaska Native Knowledge Network. (2016). *Publications*. Fairbanks, AK: Author. Retrieved from <http://ankn.uaf.edu/publications/>
- Alberta Education. (2006). *Common curriculum framework for K–9 mathematics: Western and Northern Canadian protocol*. Edmonton, AB, Canada: Author.
- Anderson, B., & Richards, J. (2016). *Students in jeopardy: An agenda for improving results in band-operated schools* (Commentary 444). Toronto, ON, Canada: C.D. Howe Institute. Retrieved from <https://www.cdhowe.org/>
- Anyon, J. (1980). Social class and the hidden curriculum of work. *Journal of Education*, 162(1), 67–92.
- Ascher, M. (1991). *Ethnomathematics: A multicultural view of mathematical ideas*. New York, NY: CRC Press.
- Ball, P. (2013, December 16). Polynesian people used binary numbers 600 years ago. *Nature*. Retrieved from <http://www.scientificamerican.com/article/polynesian-people-used-binary-numbers-600-years-ago/>
- Bang, M., & Medin, D. (2010). Cultural processes in science education: Supporting the navigation of multiple epistemologies. *Science Education*, 94, 1009–1026.
- Banks, J. A. (2004). Multicultural education: Historical development, dimensions, and practice. In J. A. Banks (Ed.), *Handbook of research on multicultural education* (2nd ed., pp. 32–29). San Francisco, CA: Jossey-Bass.
- Barton, B., & Fairhall, U. (1995, July). *Is mathematics a Trojan horse? Mathematics in Māori education*. Paper presented at the History and Pedagogy of Mathematics Conference, Cairns, Australia.
- Battiste, M. (1986). Micmac literacy and cognitive assimilation. In J. Barman, Y. Herbert, & Y. D. McCaskell (Eds.), *Indian education in Canada: Vol. 1. The legacy* (pp. 23–44). Vancouver, BC, Canada: University of British Columbia Press.
- Battiste, M. (2002). *Indigenous knowledge and pedagogy in First Nations education: A literature review with recommendation*. Ottawa, ON, Canada: Indian and Northern Affairs.
- Battiste, M. (2013). *Decolonizing education: Nourishing the learning spirit*. Saskatoon, SK, Canada: Purich Publishing.
- Battiste, M., & Henderson, J. Y. (2000). *Protecting Indigenous knowledge and heritage*. Saskatoon, SK, Canada: Purich Publishing.
- Beatty, R., & Blair, D. (2015). Indigenous pedagogy for early mathematics: Algonquin looming in a Grade 2 math classroom. *The International Journal of Holistic Early Learning and Development*, 1, 3–24.
- Beaudet, G. (1995). *Nehiyawe mina Akayasimo, Akayasimo mina Nehiyawe ayamiwini masinahigan* [Cree–English dictionary]. Winnipeg, MB, Canada: Wuerz Publishing.
- Belczewski, A. (2009). Decolonizing science education and the science teacher: A White teacher’s perspective. *Canadian Journal of Science, Mathematics and Technology Education*, 9(3), 191–202.
- Bishop, A. J. (1988a). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Bishop, A. J. (1988b). The interactions of mathematics education with culture. *Cultural Dynamics*, 1(2), 145–157.
- Bishop, A. J. (1990). *Western mathematics: The secret weapon of cultural imperialism*. Thousand Oaks, CA: SAGE. Retrieved from http://rac.sagepub.com/search/results?fulltext=Alan+Bishop&x=10&y=8&submit=yes&journal_set=sprac&src=selected&andorexactfulltext=and
- Bolter, J. D. (1984). *Turing’s man: Western culture in the computer age*. New York, NY: Viking Penguin.
- Boylan, M. (2016). Ethical dimensions of mathematics education. *Educational Studies in Mathematics*, 92(3), 395–409.
- Bradley, C., & Taylor, L. (2002). Exploring American Indian and Alaskan Native cultures and mathematics learning. In J. E. Hanks & F. R. Fast (Eds.), *Changing the faces of mathematics: Perspectives on Indigenous people of North America* (pp. 49–56). Reston, VA: National Council of Teachers of Mathematics.
- Bronowski, J. (1973). *The ascent of man*. Toronto, ON, Canada: Little, Brown and Company.
- Cajete, G. A. (2000). *Native science: Natural laws of interdependence*. Santa Fe, NM: Clear Light.

- Centre on International Education Benchmarking (CIEB). (2015). *Performance, equity and efficiency: Top ten PISA performance*. Washington, DC: Author. Retrieved from <http://www.ncee.org/2015/01/statistic-of-the-month-education-performance-equity-and-efficiency/>
- Charette, R. N. (2013, August 30). The STEM crisis is a myth. *IEEE Spectrum*. Retrieved from <http://spectrum.ieee.org/at-work/education/the-stem-crisis-is-a-myth>
- Chinn, P. W. U. (2007). Decolonizing methodologies and Indigenous knowledge: The role of culture, place and personal experience in professional development. *Journal of Research in Science Teaching*, 44(9), 1247–1268.
- Coburn, W. W. (2000). *Everyday thoughts about nature*. Boston, MA: Kluwer Academic.
- Collins English Dictionary*. (3rd ed.). (1994). Glasgow, Scotland: HarperCollins Publishers.
- Corrigan, D., Gunstone, R., Bishop, A., & Clarke, B. (2004, July). *Values in science and mathematics education: Similarities, differences and teacher views*. Paper presented at the 35th annual meeting of the Australasian Science Education Research Association, Armidale, NSW, Australia.
- Cuthand, D. (2012, September 7). Ottawa spin cannot ease growing resentment. *The Saskatoon StarPhoenix*. Retrieved from <http://www.pressreader.com/canada/the-starphoenix/20120907/281711201833692>
- D'Ambrosio, U. (1991). *On ethnoscience*. Campinas, Brazil: Interdisciplinary Center for the Improvement of Science Education.
- D'Ambrosio, U. (2003). Stakes in mathematics education for the societies of today and tomorrow. *Monographie de L'Enseignement Mathématique*, 39, 301–316.
- D'Ambrosio, U. (2006). *Ethnomathematics link between traditions and modernity* (A. Kepple, Trans.). Rotterdam, The Netherlands: SensePublishers.
- D'Ambrosio, U. (2007). Peace, social justice and ethnomathematics. In B. Sriraman (Ed.), *The Montana Mathematics Enthusiast, Monograph 1* (pp. 25–34). Butte, MT: Montana Council of Teachers of Mathematics. Retrieved from <https://www.google.ca/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwjH4YXU0bjTAhVKw4MKHWozDX0QFgglMAA&url=http%3A%2F%2Fciteeex.ist.psu.edu%2Fviewdoc%2Fdownload%3Fdoi%3D10.1.1.503.9296%26rep%3Drep1%26type%3Dpdf&usq=AFQjCNHkqgqQ4SshCEo7HRgYmMonnSO7wg&sig2=haGIsImAYyVv4TOjbnND2qQ>
- D'Ambrosio, U. (2016). Ethnomathematics: A response to the changing role of mathematics in society. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis and reality* (pp. 23–34). Charlotte, NC: Information Age Publishing.
- Daschuk, J. (2013). *Clearing the plains: Disease, politics of starvation, and the loss of Aboriginal life*. Regina, SK, Canada: University of Regina Press.
- Davison, D. M. (2002). Teaching mathematics to American Indian students: A cultural approach. In J. E. Hankes & F. R. Fast (Eds.), *Changing the faces of mathematics: Perspectives on Indigenous people of North America* (pp. 19–24). Reston, VA: National Council of Teachers of Mathematics.
- Deloria, V. (1992). Relativity, relatedness and reality. *Winds of Change*, 7, 35–40.
- Director, B. (2006). On the 375th anniversary of Kepler's passing. *FIDELIO Magazine*, 15(1–2), 98–113. Retrieved from http://www.schillerinstitute.org/fid_02-06/2006/061-2_375_Kepler.html
- Donald, D., Glanfield, F., & Sterenberg, G. (2011). Culturally relational education in and with an Indigenous community. *in education*, 17(3), 72–83.
- Doolittle, E. (2006). Mathematics as medicine. In P. Liljedahl (Ed.), *Proceedings of the annual meeting of the Canadian Mathematics Education Study Group* (pp. 17–25). Calgary, AB, Canada: University of Calgary.
- Doolittle, E., & Glanfield, F. (2007). Balancing equations and culture: Indigenous educators reflect on mathematics education. *For the Learning of Mathematics*, 27(3), 27–30.
- Einstein, A. (1921, January). *Geometry and experience*. Paper presented to the Prussian Academy of Science, Berlin, Germany. Retrieved from http://todayinsci.com/E/Einstein_Albert/EinsteinAlbert-MathematicsAndReality.htm
- Einstein, A. (1930, November 9). Albert Einstein über Kepler. *Frankfurter Zeitung*.
- Elmore, R. F. (2003, March). *Large-scale improvement of teaching and learning: What we know, what we need to know*. Paper presented at the annual meeting of the National Association for Research in Science Teaching, Philadelphia, PA.
- Enyedy, N., Danish, J. A., & Fields, D. A. (2011). Negotiating the “relevant” in culturally relevant mathematics. *Canadian Journal of Science, Mathematics and Technology Education*, 11(3), 273–291.
- Ernest, P. (1988). The impact of beliefs on the teaching of mathematics. Retrieved from <http://webdoc.sub.gwdg.de/edoc/e/pome/impact.htm>
- Ernest, P. (1991). *The philosophy of mathematics education*. London, England: Routledge-Falmer. Retrieved from <https://p4mriunpat.files.wordpress.com/2011/10/the-philosophy-of-mathematics-education-studies-in-mathematicseducation.pdf>
- Ernest, P. (2013). What is “first philosophy” in mathematics education? *The Philosophy of Mathematics Education Journal*, 27. Retrieved from <http://people.exeter.ac.uk/PErnest/pome27/index.html>
- Ernest, P. (2016a). Mathematics education ideologies and globalization. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis and reality* (pp. 35–79). Charlotte, NC: Information Age Publishing.
- Ernest, P. (2016b). The problem of certainty in mathematics. *Educational Studies in Mathematics*, 92, 379–393.
- Ethnomathematics and STEM Institute. (2016). University of Hawai'i at Mānoa and West O'ahu. Retrieved from <http://ethnomath.coe.hawaii.edu/index.php>

- First Nations Education Steering Committee. (2011). *Teaching mathematics in a First Peoples context: Grades 8 and 9*. Vancouver, BC, Canada: Author. Retrieved from <http://www.fnesc.ca/wordpress/wp-content/uploads/2015/05/PUB-LFP-Math-First-Peoples-8-9-for-Web.pdf>
- Fisher, D. (2017). Reorganizing algebraic thinking: An introduction to dynamic system modeling. *The Mathematics Enthusiast*, 14, 347–370.
- Fowler, H. H. (2012). Collapsing the fear of mathematics: A study of the effects of Navajo culture on Navajo student performance in mathematics. In S. T. Gregory (Ed.), *Voices of Native American educators* (pp. 99–129). Lanham, MD: Lexington Books.
- François, K., & Van Kerkhove, B. (2010). Ethnomathematics and the philosophy of mathematics (education). In B. Löwe & T. Müller (Eds.), *Philosophy of mathematics: sociological aspects and mathematical practice* (pp. 121–154). London, England: College Publications.
- Furuto, H. L. (2012). *Ethnomathematics curriculum textbook: Precalculus, trigonometry, and analytic geometry*. Honolulu, HI: University of Hawai'i SEED Office and the National Science Foundation.
- Furuto, H. L. (2013a). Bridging policy and practice with ethnomathematics. *Journal of Mathematics & Culture*, 7, 31–57.
- Furuto, H. L. (2013b). *Ethnomathematics curriculum textbook: Symbolic reasoning and quantitative literacy*. Honolulu, HI: University of Hawai'i SEED Office and the National Science Foundation.
- Furuto, H. L. (2014). Pacific ethnomathematics: Pedagogy and practices in mathematics education. *Teaching Mathematics and Its Applications*, 33(2), 110–121.
- Furuto, H. L. (in press). Mathematics education on a worldwide voyage. *Cultural Studies of Science Education*, 11.
- Fyhn, A. B. (2009, January). Sámi culture and algebra in the curriculum. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the annual meeting of the European Society for Research in Mathematics Education 6* (pp. 489–498). Lyon, France.
- Fyhn, A. B. (2013). Sámi culture and values: A study of the national mathematics exam for the compulsory school in Norway. *Interchange*, 44, 349–367.
- Fyhn, A. B., Sara Eira, E. J., & Sriraman, B. (2011). Perspectives on Sámi mathematics education. *Interchange*, 42(2), 185–203.
- Garrouette, E. M. (1999). American Indian science education: The second step. *American Indian Culture and Research Journal*, 23(4), 91–114.
- Gibbs, W. W., & Fox, D. (1999, October). The false crises in science education. *Scientific American*, 87–93.
- Government of Alberta. (2010). *Connecting the dots: Aboriginal workforce and economic development in Alberta*. Edmonton, AB, Canada: Author. Retrieved from <https://work.alberta.ca/documents/connecting-the-dots-aboriginal-workforce.pdf>
- Greer, B., Mukhopadhyay, S., Powell, A. B., & Nelson-Barber, S. (Eds.). (2009). *Culturally responsive mathematics education*. New York, NY: Routledge.
- Greer, B., & Skovsmose, O. (2012). Introduction: Seeing the cage: The emergence of critical mathematics education. In O. Skovsmose & B. Greer (Eds.), *Opening the cage: Critiques and politics of mathematics education*. Boston, MA: Sense Publishers.
- Hall, E. T. (1976). *Beyond culture*. Toronto, ON, Canada: Doubleday.
- Hatcher, A., Bartlett, C., Marshall, A., & Marshall, M. (2009). Two-eyed seeing in the classroom environment: Concepts, approaches, and challenges. *Canadian Journal of Science, Mathematics and Technology Education*, 9, 141–153.
- Hogue, M. (2011). *Narratively speaking: Oscillating in the liminal space of science education between two worlds* (Unpublished doctoral dissertation). University of Calgary, Calgary, AB, Canada.
- Hogue, M. (2013). Building bridges: Teaching science through theatre. *Education Canada*, 53(4), 1–3.
- Hough, L. (2015, Fall). There is no average. *Harvard Ed. Magazine*, 21–27.
- Irvine, J. (2017). Problem posing in consumer mathematics classes: Not just for future mathematicians. *The Mathematics Enthusiast*, 14, 387–412.
- Ishimaru, A. M., Barajas-López, F., & Bang, M. (2015). Centering family knowledge to develop children's empowered mathematics identities. *Journal of Family Diversity in Education*, 1(4), 1–21.
- Jannok Nutti, Y. J. (2010). *Grouse steps towards front line knowledge in Sámi mathematics—Teachers' perspective on transformations activities in Sámi preschool and Sámi school* (Unpublished doctoral thesis). Luleå University of Technology, Department of Education (in Norwegian), Luleå, Sweden.
- Jannok Nutti, Y. J. (2013). Indigenous teachers' experiences of the implementation of culture-based mathematics activities in Sámi schools. *Mathematics Education Research Journal*, 25(1), 57–72.
- Jorgensen, R. (2016). The elephant in the room: Equity, social class, and mathematics. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis and reality* (pp. 127–145). Charlotte, NC: Information Age.
- Jorgensen, R., & Wagner, D. (2013). Mathematics education with/for [I]ndigenous peoples. *Mathematics Education Research Journal*, 25(1), 1–3.
- Kawasaki, K. (2002). A cross-cultural comparison of English and Japanese linguistic assumptions influencing pupils' learning of science. *Canadian and International Education*, 31(1), 19–51.

- Keene, A. (2016). Exploring the fine line between appreciation and appropriation [podcast]. Retrieved from <http://www.cbc.ca/radio/popup/audio/listen.html?autoPlay=true&clipIds=&mediaIds=2685100624&contentarea=radio&subsection1=radio1&subsection2=currentaffairs&subsection3=unreserved&contenttype=audio&title=2016/03/13/1.3485476-exploring-the-fine-line-between-appreciation-and-appropriation&contentid=1.3485476>
- Kinew, W. (2015). *The reason you walk*. Toronto, Canada: The Penguin Group (Viking).
- King, T. (2012). *The inconvenient Indian: A curious account of Native people in North America*. Toronto, ON, Canada: Doubleday Canada.
- Kovach, M. (2009). *Indigenous methodologies: Characteristics, conversations, and contexts*. Toronto, ON, Canada: University of Toronto Press.
- Larson, M. (2016, September 15). A renewed focus on access, equity, and empowerment. Retrieved from <https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Matt-Larson/A-Renewed-Focus-on-Access,-Equity,-and-Empowerment/>
- Lipka, J. (1994). Culturally negotiated schooling: Toward a Yup'ik mathematics. *Journal of American Indian Education*, 33(3), 14–30.
- Lipka, J., & Adams, B. (2004). *Culturally based math education as a way to improve Alaska Native students' math performance* (Working Article No. 20). Athens, OH: Appalachian Center for Learning, Assessment, and Instruction in Mathematics.
- Lipka, J., & Andrew-Irhke, D. (2009). Ethnomathematics applied to classrooms in Alaska: Math in a Cultural Context. *NASGE Newsletter*, 3.1, 8–10.
- Lipka, J., Mohatt, G., & The Ciulistet Group. (1998). *Transforming the culture of schools: Yup'ik Eskimo examples*. Mahwah, NJ: Lawrence Erlbaum.
- Lipka, J., Sharp, N., Adams, B., & Sharp, F. (2007). Creating a third space for authentic biculturalism: Examples from math in a cultural context. *Journal of American Indian Education*, 46(3), 94–115.
- Lipka, J., Sharp, N., Brenner, B., Yanez, E., & Sharp, F. (2005). The relevance of culturally based curriculum and instruction: The case of Nancy Sharp. *Journal of American Indian Education*, 44(3), 31–54.
- Lipka, J., Webster, J. P., & Yanez, E. (2005). Factors that affect Alaska Native students' mathematical performance. *Journal of American Indian Education*, 44(3), 1–8.
- Lipka, J., Wong, M., & Andrew-Irhke, D. (2013). Alaska Native Indigenous knowledge: Opportunities for learning mathematics. *Mathematics Education Research Journal*, 25(1), 129–150.
- Lipka, J., Yanez, E., Andrew-Irhke, D., & Adam, S. (2009). A two-way process for developing effective culturally based math: Examples from math in a cultural context. In B. Greer, S. Mukhopadhyay, A. B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 257–280). New York, NY: Routledge.
- Little Bear, L. (2000). Jagged worldviews colliding. In M. Battiste (Ed.), *Reclaiming Indigenous voice and vision* (pp. 77–85). Vancouver, BC, Canada: University of British Columbia Press.
- Lowan-Trudeau, G. (2015). Contemporary studies in environmental and Indigenous pedagogies: A curricula of stories and place. *Environmental Education Research*, 21, 652–653.
- Lunney Borden, L. (2013). What's the word for ... ? Is there a word for ... ? How understanding Mi'kmaw language can help support Mi'kmaw learners in mathematics. *Mathematics Education Research Journal*, 25, 5–22.
- Lunney Borden, L. (2015). Learning mathematics through birch bark biting: Affirming Indigenous identity. In S. Mukhopadhyay & B. Greer (Eds.), *Proceedings of the 8th international Mathematics Education and Society conference* (Vol. 3, pp. 756–768). Portland, OR.
- Lunney Borden, L., & Wagner, D. (2017). *Mawkinumasultinej: Let's learn together!* Antigonish, NS, and St. John, NB, Canada. Retrieved from <http://showmeyourmath.ca/>
- Lunney Borden, L., Wagner, D., & Johnson, N. (2017). Show me your math: Mi'kmaw community members explore mathematics. In C. Nicol, S. Dawson, J. Archibald, & F. Glanfield (Eds.), *Living culturally responsive mathematics curriculum and pedagogy: Making a difference with/in Indigenous communities*. Rotterdam, The Netherlands: Sense Publishers.
- Lunney Borden, L., & Wiseman, D. (2016). Considerations from places where Indigenous and Western ways of knowing, being, and doing circulate together: STEM as artifact of teaching and learning. *Canadian Journal of Science, Mathematics and Technology Education*, 16(2), 140–152.
- Martin, D. B. (2006). Mathematics learning and participating as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. *Mathematical Thinking and Learning*, 8(3), 197–229.
- Maryboy, N., Begay, D., & Nichol, L. (2006). Paradox and transformation. Retrieved from <http://www.indigenousedu.org/WINHEC%20Journal%203-29-06%20Final%20c.pdf>
- Math in a Cultural Context. (2016). Math in a cultural context. Retrieved from <http://www.uaf.edu/mcc/>
- McKinley, E. (2001). Cultural diversity: Masking power with innocence. *Science Education*, 85(1), 74–76.
- McMurphy-Pilkington, C., & Trinick, T. (2002). Horse power or empowerment? Mathematics curriculum for Māori—Trojan horse revisited. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics education in the South Pacific: Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia* (pp. 465–472). Sydney, Australia: Merga.
- Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in Indigenous mathematics classrooms*. Dordrecht, The Netherlands: Springer.
- Medin, D. L., & Bang, M. (2014). *Who's asking? Native science, Western science, and science education*. Cambridge, MA: The MIT Press.

- Michell, H., Vizina, Y., Augustus, C., & Sawyer, J. (2008). *Learning Indigenous science from place*. Retrieved from <http://iportal.usask.ca/docs/Learningindigenousscience.pdf>
- Mukhopadhyay, S., & Greer, G. (2012). Ethnomathematics. In J. A. Banks (Ed.), *Encyclopedia of diversity in education* (pp. 857–861). Thousand Oaks, CA: SAGE.
- Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between “cultural” and “domain” knowledge in the mathematics classroom and beyond. *Review of Research in Education*, 32(1), 187–240.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nelson-Barber, S., & Trumbull, E. (2007). Making assessment practices valid for Indigenous American students. *Journal of American Indian Education*, 46(3), 132–147.
- Nespor, J. (1994). *Knowledge in motion: Space, time and curriculum in undergraduate physics and management*. Philadelphia, PA: Falmer Press.
- Newman, J. R. (Ed.). (1956). *The world of mathematics*. New York, NY: Simon and Schuster. Retrieved from <http://math.furman.edu/~mwoodard/ascquon.html>
- Nikolakaki, M. (2016). Mathematics education and citizenship: Critical dimensions. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis and reality* (pp. 273–286). Charlotte, NC: Information Age Publishing.
- NOVA. (2016). *Ethnomathematics*. Retrieved from <http://www.nova.org.au/everything-else/ethnomathematics>
- Ogawa, M. (1995). Science education in a multi-science perspective. *Science Education*, 79(5), 583–593.
- Ojalehto, B., & Medin, D. (2015). Emerging trends in culture and concepts. In R. Scott & S. Kosslyn (Eds.), *Emerging trends in the social and behavioral sciences*. New York, NY: John Wiley & Sons. doi:10.1002/9781118900772.etrds0064
- Organization for Economic Cooperation and Development. (2013). *PISA 2012 results: What students know and can do—Student performance in mathematics, reading and science* (Vol. 1). Paris, France: OECD Publishing. Retrieved from <https://www.oecd.org/pisa/keyfindings/pisa-2012-results-volume-I.pdf>
- Organization for Economic Cooperation and Development. (2016). *PISA 2015 results: Excellence and equity in education* (Vol. 1). Paris, France: OECD Publishing. Retrieved from <http://dx.doi.org/10.1787/9789264266490-en>
- Parker Webster, J., Wiles, P., Civil, M., & Clark, S. (2005). Finding a good fit: Using MCC in a “third space.” *Journal of American Indian Education*, 44(3), 9–30.
- Parkin, A. (2015). *International report card on public education: Key facts on Canadian achievement and equity*. Toronto, ON, Canada: The Environics Institute.
- Philosophy Department. (2017). *Informal fallacies*. Texas State University. Retrieved from <http://www.txstate.edu/philosophy/resources/fallacy-definitions.html>
- Proust, M. (1923). *Remembrance of things past: Vol. 5. The captive* (C. K. Scott Moncrieff, Trans.). Project Gutenberg Australia. Retrieved from <https://clearingcustoms.net/2013/12/17/what-marcel-proust-really-said-about-seeing-with-new-eyes/>
- Richards, J., Hove, J., & Afolabi, K. (2008). *Understanding the Aboriginal/non-Aboriginal gap in student performance: Lessons from British Columbia* (Commentary No. 276). Toronto, ON, Canada: C.D. Howe Institute.
- Rickard, A. (2005). Constant perimeter, varying area: A case study of teaching and learning mathematics to design a fish rack. *Journal of American Indian Education*, 44(3), 80–100.
- Rigney, L. (1999). Internationalisation of an indigenous anti-colonial cultural critique of research methodologies: A guide to Indigenist research methodology and its principles. *WICAZO SA Review*, 14(2), 109–121.
- Rudd, K. (2009, February 26). Closing the Gap report speech to Parliament. *The Australian*. Retrieved from <http://www.theaustralian.com.au/archive/apology/kevin-rudds-closing-the-gap-speech/news-story/5ed69819ecb6a42f4fd28e76dceb02a6>
- Russell, G. L. (2010). Racism by numbers. *vinculum - Journal of the Saskatchewan Mathematics Teachers' Society*, 2(2), 36–44. Retrieved from <http://www.smts.ca/wordpress/wp-content/uploads/2014/07/vinculum2-compressed.pdf>
- Russell, G. L. (2016). *Valued kinds of knowledge and ways of knowing in mathematics and the teaching and learning of mathematics: A worldview analysis* (Unpublished doctoral dissertation). University of Saskatchewan, Saskatoon, SK, Canada.
- Russell, G. L., & Chernoff, E. J. (2013). The marginalisation of Indigenous students within school mathematics and the math wars: Seeking resolutions within ethical spaces. *Mathematics Education Research Journal*, 25(1), 109–127.
- Russell, G. L., & Chernoff, E. J. (2015). Incidents of intrusion: Disruptions of mathematics teaching and learning by the traditional Western worldview. In M. V. Matinez & A. Castro Superfine (Eds.), *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1018–1025). Chicago, IL: University of Illinois at Chicago.
- Sakiestewa-Gilbert, W. (2011). Developing culturally based science curriculum for Native American classrooms. In J. Reyhner, W. Sakiestewa-Gilbert, & L. Lockard (Eds.), *Honoring our heritage: Culturally appropriate approaches for teaching Indigenous students* (pp. 43–56). Flagstaff, AZ: Northern Arizona University.
- Salleh, A. (2006). *Maths “needs to listen” to other cultures*. Ultimo, NSW, Australia: Australian Broadcasting Corporation. Retrieved from <http://www.abc.net.au/science/articles/2006/02/17/1571700.htm>
- Saskatchewan Curriculum. (2007). *Grade 6 mathematics (outcomes and indicators)*. Retrieved from <https://www.curriculum.gov.sk.ca/webapps/moe-curriculum-BBLEARN/index.jsp?lang=en&subj=mathematics&level=6>

- Saskatchewan Instructional Development and Research Unit. (2014). *Seeking their voices: Improving Indigenous student learning outcomes*. Regina, SK, Canada: Author.
- Saul, J. R. (2014). *The comeback*. Toronto, ON, Canada: Penguin Canada Books.
- Serder, M., & Jakobsson, A. (2015). "Why bother so incredibly much?": Student perspectives on PISA science assignments. *Cultural Studies of Science Education*, 10(3), 833–853.
- Sharpe, A., & Arsenault, J.-F. (2009). *Investing in Aboriginal education in Canada: An economic perspective* (CPRN Research Report). Ottawa, ON, Canada: Canadian Policy Research Networks. Retrieved from http://www.cprn.org/documents/51980_EN.pdf
- Show Me Your Math. (2017). Antigonish, NS, Canada: Author. Retrieved from <http://showmeyourmath.ca/>
- Sjøberg, S. (2015, August–September). PISA—A global educational arms race? *The PISA science assessments and the implications for science education: Uses and abuses* (J. Osborne, Chair). Symposium conducted at, Helsinki, Finland.
- Sjøberg, S. (2016). OECD, PISA, and globalization: The influence of the international assessment regime. In C. H. Tienken & C. A. Mullen (Eds.), *Education policy perils: Tackling the tough issues* (pp. 102–133). New York, NY: Routledge.
- Skovsmose, O. (2016). Mathematics: A critical rationality? In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis, and reality* (pp. 1–22). Charlotte, NC: Information Age.
- St. Denis, V. (2004). Real Indians: Cultural revitalization and fundamentalism in Aboriginal education. In C. Schick, J. Jaffe, & A. Watkinson (Eds.), *Contesting fundamentalisms* (pp. 35–47). Halifax, NS, Canada: Fernwood.
- Stanford Encyclopedia of Philosophy. (2015). Gödel's incompleteness theorems. Retrieved from <https://plato.stanford.edu/entries/goedel-incompleteness/>
- Sterenberg, G., & Hogue, M. (2011). Reconsidering approaches to Aboriginal science and mathematics education. *Alberta Journal of Educational Research*, 57(1), 1–15.
- Sterenberg, G., & McDonnell, T. (2010). Indigenous and Western mathematics. *vinculum - Journal of the Saskatchewan Mathematics Teachers' Society*, 2(2), 10–22.
- Sterenberg, G. (2013a). Considering Indigenous knowledges and mathematics curriculum. *Canadian Journal of Science, Mathematics and Technology Education*, 13(1), 18–32.
- Sterenberg, G. (2013b). Learning Indigenous and Western mathematics from place. *Mathematics Education Research Journal*, 25, 91–108.
- Stoet, G., Bailey, D. H., Moore, A. M., & Geary, D. C. (2016). Countries with higher levels of gender equality show larger national sex differences in mathematics anxiety and relatively lower parental mathematics valuation for girls. *PLoS ONE*, 11(4), e0153857. doi:10.1371/journal.pone.015
- Thanh Ha, T., & Galloway, G. (2017, February 14). Ontario judge sides with Sixties Scoop survivors. *The Globe and Mail*. Retrieved from <http://www.theglobeandmail.com/news/national/ontario-judge-sides-with-60s-scoop-survivors-damages-to-be-decided/article34015380/>
- Truth and Reconciliation Commission. (2016). *A knock on the door*. Winnipeg, MB, Canada: University of Manitoba Press.
- Uegaki, W. (1990). A historical research on the definition of Wasan and Yōsan. *Bulletin of the Faculty of Education, Mie University, Educational Science*, 50, 13–29.
- U.S. Congress House of Representatives Subcommittee on Early Childhood, Elementary and Secondary Education. (2008). *Challenges facing bureau of Indian education schools in improving student achievement*. Washington, DC: U.S. Government Printing Office.
- Venville, G. J., Wallace, J., Rennie, L. J., & Malone, J. A. (2002). Curriculum integration: Eroding the high ground of science as a school subject? *Studies in Science Education*, 37(1), 43–83.
- Verhulst, F. (2012). Mathematics is the art of giving the same name to different things: An interview with Henri Poincaré. *NAW*, 5/13(3), 154–158.
- Vickers, P. (2007). Ayaawx: In the path of our ancestors. *Cultural Studies of Science Education*, 2, 592–598.
- Wagamese, R. (1994). *Keeper'n me*. Toronto, ON, Canada: Anchor Canada.
- Watson, H., & Chambers, D. W. (1989). *Singing the land, signing the land*. Geelong, VIC, Australia: Deakin University Press.
- White, L. A. (1959). *The evolution of culture*. New York, NY: McGraw-Hill.
- Whorf, B. L. (1959). *Language, thought, and reality*. New York, NY: John Wiley & Sons.
- Wikipedia. (2016). Japanese mathematics. Retrieved from https://en.wikipedia.org/wiki/Japanese_mathematics
- Wikipedia. (2017). History of mathematics. Retrieved from https://en.wikipedia.org/wiki/History_of_mathematics#Prehistoric_mathematics
- Wilder, R. L. (1981). *Mathematics as a cultural system*. Oxford, England: Pergamon Press.
- Wilson, S. (2008). *Research is ceremony: Indigenous research methods*. Halifax, NS, Canada: Fernwood.
- Woolford, A., Benvenuto, J., & Hinton, A. L. (Eds.). (2014). *Colonial genocide in Indigenous North America*. London, England: Duke University Press.