Can. J. Sci. Math. Techn. Educ. (2021) 21:475–492
https://doi.org/10.1007/s42330-021-00152-8

Resolving Conflicting Subcultures Within School Mathematics: Towards A Humanistic School Mathematics

Glen S. Aikenhead[1]

Accepted: 20 May 2021 / Published online: 15 July 2021
© Ontario Institute for Studies in Education (OISE) 2021

Abstract Informed by a cultural understanding of human sense-making, the mathematical identities of Grades 7–12 learners, teachers, and conventional curricula are explored. Due to a clash of values between mathematics and the humanities, a majority of learners do not achieve their mathematical potential. This frustrates both these learners and their teachers. A resolution to the conflict emerges from scrutinizing the conventional Plato-based (Platonist) mathematics' ontology, epistemology, and axiology that convey anti-humanistic images of school mathematics, the antithesis to the majority of learners' humanistic-oriented self-identities. Platonist mathematics is philosophically critiqued, revealing a choice amongst it being cultural, spiritual, or simply opportunistic. Its nineteenth-century anti-humanist façade is replaced by evidence-based humanistic features that the original façade was meant to hide from learners and the general public for politically inspired reasons. These humanistic-oriented features of mathematics are transposed into a proposal for a humanistic school mathematics program that will engage a large proportion of learners (Grades 7–12) in the Western mathematics actually used by adults not employed in occupations requiring advanced Platonist mathematics with its highly hypothetical and abstract reasoning. A Platonist school mathematics program, however, will play an even a greater role in preparing the minority of learners for the advanced mathematics employment sector, some of whom may enrol in both programs. The article concludes with examples of humanistic mathematics lessons and modules.

Résumé En nous appuyant sur une compréhension culturelle du processus de construction du sens chez l'être humain, nous explorons les identités mathématiques des apprenants de la 7e à la 12e année, des enseignants et des programmes d'études traditionnels. En raison du conflit de valeurs entre le domaine des mathématiques et celui des lettres et sciences humaines, une majorité d'apprenants n'atteignent pas tout leur potentiel en mathématiques. Cette situation est frustrante pour ces apprenants comme pour leurs enseignants. Une solution à ce conflit se dessine dans l'examen ontologique, épistémologique et axiologique de la logique mathématique traditionnelle fondée sur Platon (platonicienne)

✉ Glen S. Aikenhead
glen.aikenhead@usask.ca

1 College of Education, University of Saskatchewan, 538 Blackburn Cres, SK S7V 1E8 Saskatoon, Canada

montrant les mathématiques scolaires sous un angle anti-humaniste; ceci allant à l'encontre de l'identité personnelle orientée vers l'humanisme de la majorité des apprenants. D'un point de vue philosophique, les mathématiques platoniciennes sont critiquées, ce qui révèle un choix soit culturel, spirituel ou simplement opportuniste. Sa façade anti-humaniste datant du 19^{e} siècle est remplacée par des éléments humanistes reposant sur des données probantes que la façade originelle devait dissimuler aux apprenants et à l'ensemble de la population pour des motifs politiquement motivés. Ces éléments humanistes reposant sur des données probantes des mathématiques se retrouvent dans une proposition en faveur d'un programme scolaire humaniste des mathématiques qui mobilisera un grand nombre d'apprenants (de la 7^{e} à la 12^{e} année) des mathématiques en Occident qui sont en fait utilisées par des adultes qui n'occupent pas des professions exigeant des compétences avancées en mathématiques platoniciennes, celles-ci requérant un haut niveau de raisonnement hypothético-déductif et abstrait. Cependant, un programme scolaire de mathématiques platoniciennes jouera un rôle encore plus grand dans la préparation de la minorité d'apprenants destinés au secteur de l'emploi nécessitant des mathématiques avancées, dont certains pourraient s'inscrire dans les deux programmes. Nous concluons l'article avec des exemples de cours et de modules de mathématiques humanistes.

Keywords Humanistic · School mathematics · Culture conflicts · Values · Self-identities

Introduction

C. P. Snow (1953) famously identified two academic solitudes, the sciences and the humanities. Mathematics' dictionary definition is "a group of related sciences, including algebra, geometry, and calculus; concerned with the study of number, quantity, shape, and space; and their interrelationships, by using a specialised notation" (Makins, 1994, p. 964). Snow urged the two academies to build bridges between their diverse ways of knowing, but he neglected to acknowledge those who embrace degrees of both ways. Transposed into a 2021 educational vernacular, Snow's two solitudes could be thought of as *STEM* (science, technology, engineering, and mathematics) and *non-STEM,* respectively.

The sciences were shown to be broadly based subcultures within a nation's mainstream culture (Aikenhead, 1996). Dorce (2020) correctly points out that for learners, the history of mathematics, for example, "can supply human roots to the subject" (p. 3). This coincides with one of the four Saskatchewan mathematics curriculum goals: understanding mathematics as a human endeavour. Students learning some historical human roots to mathematics sounds like a prime candidate for one of many bridges that C.P. Snow envisioned.

Drawing on Gilligan's (1982) axiological theory, Ernest (2018) went into greater depth by contrasting her two clusters of values: "separation versus connection" (p. 194–195)—the sciences and humanities, respectively.

> The *separated* position valorises rules, abstraction, objectification, impersonality, unfeelingness, dispassionate reason and analysis, and tends to be atomistic and thing-centred in focus. The *connected* position is based on and valorises relationships, connections, empathy, caring, feelings and intuition, and tends to be holistic and human-centred in its concerns. (pp. 194–195, emphasis added)

In the context of school mathematics, Aikenhead (2017) explored the culture of Western (Platonist) mathematics compared to Indigenous mathematical systems and reviewed the literature on combining the two into "Indigenous culture-based school mathematics" (Aikenhead, 2020, p. 687).

In contrast, this article investigates a *mainstream* culture-based school mathematics in terms of learners embracing Gilligan's separated or connected value clusters, regardless of their ancestry; in other words, (1) those learners whose self-identities (Darragh & Radovic, 2018; Ishimaru et al., 2015; Ruef, 2020) harmonise with a mathematician's perspective, to varying degrees, and (2) those learners having a humanistic perspective, more or less. The ways of viewing and understanding their worlds usually differ noticeably, although some will possess some combination of both perspectives. As explained in detail below (subsection Learners' Mindsets), a large majority identifies with a humanistic predisposition, whilst a small minority is most comfortable with a mathematical predisposition.

This article begins by clarifying the terms "humanistic school mathematics" and "culture" as used in this article. Their meanings frame the development of a proposal to renew the mathematics program for the majority of, but not all, learners in Grades 7 to 12. Today's curriculum has not changed fundamentally since its inception in about 1850 (details are described below), but most teachers' pedagogy is definitely twenty-first-century modern (Furr, 1996). A nineteenth-century school mathematics curriculum could be revised to serve today's future adults living in the twenty-first-century's Digital Age.

The discussion that follows interrogates claims that learners hear from their mathematics program that eventually causes about 37% of high school graduates to "hate" mathematics (Ipso, 2005), which in turn (a) causes them to shy away from using it as they try to function effectively in society and (b) as parents, they negatively influence their children's predispositions about success at school mathematics, as any elementary teacher knows only too well.

This article concerns itself with the current mathematics program that creates conflicting subcultures within school mathematics detrimental to many learners reaching their potential for success. This is not about the teachers; it is about the "highly restrictive, over-crowded, outdated curriculum" (Duchscherer et al., 2019, p. 63) that handcuffs teachers from innovating for the twenty-first century.

Reasons and research are laid out to support a humanistic mathematics program for about 70% of learners in Grades 7–12 (a calculation explained in subsection "Learners' Mindsets"), for whom the current Plato-based (Platonist) mathematics program is generally obsolete, over-crowded, and irrelevant (Borovik, 2017; Noddings, 2017).

For the 30% of learners interested in considering future STEM employment, the high school's precalculus program would not change significantly, other than adopting some aspects of the International Baccalaureate and including some individual or group projects on out-of-school mathematics-related topics such as employment: What mathematics do "the vast majority of scientists, engineers, and actuaries" actually use on the job (Edwards, 2010)?

Humanistic Mathematics Education

"Those who have experienced mathematics as a depersonalised, uncontextualised, non-controversial, and asocial form of knowledge might very well consider the expression humanistic mathematics education to be the epitome of an oxymoron" (Brown, 1996, p. 1289). Over the past 50 years or so, clusters of invisible colleges have been formed by mathematicians and educators who share one common viewpoint: their opposition to Platonist mathematicians' absolutist philosophy of pure mathematics that eschews any human dimensions to its knowledge system. Their critiques of Platonist mathematics are represented in this article. The principal groups involved are the following (within each group listed, the founders are cited first, followed by a recent publication of the group, and then an example):

- Mathematics education culturalists: Wilder (1965, 1981), Aikenhead (2021a), and Larvor (2016); for example, workplace mathematics.
- Humanistic mathematicians: White (1974, 1976), Brown (1996), Skrivanos & Zhang (2013), and Sriraman (2017); for example, a history of key mathematicians. A *Humanistic Mathematics Network Newsletter* began circulating in 1987. The *Journal of Humanistic Mathematics* began publishing in 2001.

- Critical mathematics educators: Skovsmose (1985, 1994), Ernest (2019), and Ernest et al. (2016); for example, a critique of mathematician's ethics and politics in their mathematical modelling.

Some members of each group attend other groups' meetings and contribute to their literature. Therefore, humanistic school mathematics does not have clearly defined borders. The citations in this article come from advocates in all three groups. The article ends with specific examples of humanistic mathematics lessons.

Culture

The language we personally use serves as an encyclopaedia of our lived dynamic cultures. Based on the work of Gutiérrez and Rogoff (2003), Bang and Medin (2010) provide an action-oriented notion of the term "culture":

> Although the construct of culture is problematic, people nevertheless "live culturally" … [by way of] a wide repertoire of sense-making *practices* that people participate in, particularly in every-day contexts. …This understanding of culture implies that there is no cultureless or "neutral" perspective any more than a photograph…could be without perspective. In this sense, everything is cultured. (pp. 1014–1015, emphasis added).

Culture is a verb here rather than a noun (i.e., a classic noun rendition of culture is "the collection of customs, beliefs, rituals, tools, traditions, etc., of a group of people…" [Wilder, 1965, p. 282]). The verb form characterises humanistic school mathematics, and the verb form is prescient to resolving conflicts between the two clusters of subculture identities within school mathematics. For learners to experience mathematics as a humanistic endeavour, they must engage in a repertoire of its sense-making cultural practices (Boylan, 2016). This view lends itself to a school mathematics in which learners negotiate amongst multiple ways of understanding, dependent upon a learner's self-identities (Darragh & Radovic, 2018; Ishimaru et al., 2015; Nasir, 2002; Ruef, 2020). Ruef (2020) wrote.

> To be successful in mathematics classes, students must negotiate and navigate the normative identity of the class—what counts as being "good at math." Within the constraints of normative identity, students must also negotiate a personal doer-of-math identity: who they are within the context of this particular mathematics class. (p. 22).

This short broad-brush-stroke description of the complexities of cultures and subcultures in a mathematics classroom sets the context to analyse deeper into the identities of the "conflicting subcultures within school mathematics" stated in this article's title.

Two Conflicting Subcultures

In North America, a *large majority* of learners and their teachers (Grades 7–12) become frustrated over a subliminal conflict between two different general sense-making practices: Western mathematics and the humanities. *This is not a dichotomy*. People can possess various degrees of both, and not at the expense of the other necessarily. Mathematics and the humanities tend to use a different language and implicit presuppositions. Therefore, frustration certainly arises, to differing degrees, between those whose general sense-making practices differ widely, enough to cause clashing subcultures within the mathematics-humanities non-dichotomy.

Resolving such clashes promises to improve many learners' mathematical self-identities. Joseph (2011) identified aspects of the two clashing subcultures of interest here: "[pure mathematics] engenders a sense of remoteness and irrelevance associated with the subject in *many who study it* and an ingrained elitism in *many who teach it*" (p. 75, emphasis added).

Mathematicians' Mindset

There is a language of gatekeeping power that dictates pure mathematics is value-free, non-ideological, culture-free, purely objective in its use, and perfectly certain and accurate in descriptions of reality (Kennedy, 2018; Nasir et al., 2008). This makes pure mathematics to be non-humanistic (Corrigan et al., 2004; Mukhopadhyay & Greer, 2001). Its globalised language controls learners through curriculum documents; graduation criteria; school examinations; politically motivated international tests such as PISA; and pervasively, mathematics' high status as the general public's indicator of smart people. This status arises from "the ideology of European superiority" (Joseph, 2011, p. 4) associated with Eurocentrism, for instance, "the supremacist position maintained by many mathematician educators who regard abstract mathematics as the crowning achievement of the human intellect, and school mathematics as the transmission of its products" (Mukhopadhyay & Greer, 2012, p. 860). For Indigenous culture-based school mathematics, this ideology had to be unlearned by teachers before they could be successful with Indigenous learners (Aikenhead, 2020).

Importantly, the "mathematically inclined" (Borovik, 2017, p. 323) naturally see abstract pure mathematics in their world all around them. Because this makes common sense to mathematicians, they assume anyone else can see what they see as well, if they work hard enough. However, most mathematicians do not understand the mechanism by which their mind applies abstract mathematical presuppositions and concepts to the everyday world around them. According to Einstein (1930; quoted in Director, 2006), these abstractions are associated with images or forms that the mind *first* creates to represent the abstraction:

> It seems that the human mind has first to construct forms independently, before we can find them in things. Kepler's marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone, but only from the comparison of the inventions of the intellect with observed fact. (p. 113).

We now have a four-part mechanism to explain mathematicians' sense-making practices, such as how they apply pure mathematics:

1. When viewing an object or event, a person's mind recalls an invented image or form that represents an abstract concept for that person, as Einstein explained.
2. Their mind projects or superimposes that image or form onto a concrete object or event.
3. If there is a good enough fit, their mind deconstructs it by both focusing on features that best fit the image or form and ignoring other features that do not seem to conform to the image or form.
4. Their mind judges the best balance between the focussing and ignoring, thereby reconstructing a conformity between an abstract mathematical concept and an everyday object or event.

In short, the mechanism is image recall, projection, deconstruction, and reconstruction. It is usually an unconscious mental act. No wonder it seems easy to mathematicians and to people whose worldview harmonises to a significant degree with a mathematicians' worldview.

Learners' Mindsets

A large majority of learners intuitively feels that the language of school mathematics tends to be foreign to their lived sense-making experiences (Barta et al., 2014). Teachers tell them that mathematics is all around them, but they do not see it, as required by the academic standards of abstract school mathematics. These learners' sense-making practices entail a cultural language contextualised in social relationships or humanities-related cultural experiences. For them, "Mathematics becomes best understood by how it is used" (p. 3). For example, this is the mathematics used by the general public who are *not* employed in advanced mathematical sectors of society.

The self-identities or worldviews of these learners do not harmonise with those of mathematicians' worldviews to any significant degree (Cobern, 2000). Due to their humanities-oriented sense-making practices, these learners tend to find it extremely difficult or impossible to apply abstract pure mathematics to their everyday world as described by the four-part mechanism described above. As a result, conventional school mathematics in Grades 7 to 12 "is commonly seen as consisting essentially of computation and formulas, yielding exact and infallible answers, without relevance to everyday life, accessible only by experts, and not open to criticism" (Mukhopadhyay & Greer, 2001, p. 297), simply put, non-humanistic. "Many of the points raised in this chapter, and elsewhere in this book, are about mathematics education having a dehumanising effect on people" (Greer & Mukhopadhyay, 2012, p. 245). These perceived features help explain the results of an Ipsos (2005) poll that found that 37% of adults in their 20 s and 30 s "hated" school mathematics.

The same Ipsos poll recorded the adults' favourite school subjects. Mathematics was 31% compared to the humanities combination (English, History, Social Studies, Arts, Foreign Languages, and Religion/Philosophy) at 69%. This clearly suggests that the humanities' "connected values" (Ernest, 2018, p. 194; Gilligan, 1982) would enrich a mathematics program's context for this larger group of learners with their degrees of connected values.

Learners' mindsets encompass their mathematics self-identities, which Aikenhead (2021a) represented on a continuum of six categories: the math-oriented, math-curious, math-interested, math-disinterested, math-shy, and math-phobic. These categories must be treated as tentative because a learner's classification will depend on many changeable variables, such as the teacher, the particular topic, the success learners have enjoyed recently, and the time of year. Under no circumstance should these categories be used for streaming learners. Distinctions between adjacent categories are not made in this article.

According to the Ipsos poll, the first three categories comprise 31% of Grade 12 graduates whilst the last three categories make up 69%. These two figures were closely corroborated by Meyer and Aikenhead (2021) who synthesised the data on learners' STEM orientation reported in three major projects: National Bureau of Economic Research STEM project (Card & Payne, 2017), U.S. Office of Technology Assessment (Frederick, 1991), and PISA (OECD, 2019).

Their synthesised results showed that a small *minority* of about 26 and 30% of Grade 12 graduates in the provinces of Saskatchewan and Ontario (respectively) see themselves as "math-interested, math-curious, or math-oriented learners" (Meyer & Aikenhead, 2021, p. 104). They feel comfortable, to varying degrees, believing that Platonist mathematics is value-fee, culture-free, decontextualised, and purely objective in its use.

This leaves a large *majority* of about 74 and 70% for Saskatchewan and Ontario, respectively. The *30 and 70% figures* will be used throughout this article. The continuum's six categories have the following percentages of learners in each category: math-phobic (20), math-shy (24), math-disinterested (26), math-interested (20), math-curious (6), and math-oriented (4). This continuum of mathematics and humanities values and learners' self-identities is a more realistic representation that avoids simplistic dichotomies such at STEM versus non-STEM learners. The 70% group often copes with school mathematics by either dropping out of school or by playing Fatima's rules.

Fatima's Rules

Fatima's rules are a set of coping strategies used by learners. They were originally identified by science education researcher Larson (1995) in a Grade 11 chemistry class. The strategies equally apply to mathematics classes. Fatima was the most articulate amongst the students who explained to Larson how to pass Chemistry with a minimum of effort. Fatima's rules explain the way many learners are able to pass their mathematics course, and some even with high grades. Learners become adept at making it appear as if meaningful learning has occurred, when it has not. It has been called "playing the system" in the vernacular, or "instrumental understanding [by the academy], defined as knowing how to carry out procedures without understanding, versus 'relational understanding,' which includes, in addition, knowing how and why such procedures work" (Ernest, 2018, p. 193).

Fatima's rules are characterised by the language of these students' pervasive cultural norm: "just deal with it" (Wood et al., 2009, p. 437). This means jumping through a series of hoops, mostly by rote memorisation, but also by such "passive-resistance mechanisms as accommodation, ingratiation, evasiveness, and manipulation" (Aikenhead, 2011, p. 114). Tobin and McRobbie (1997) documented a teacher's complicity in playing Fatima's rules: "There was a close fit between the goals of Mr. Jacobs and those of the students and satisfaction with the emphasis on memorisation of facts and procedures to obtain the correct answers needed for success on tests and examinations" (p. 366). "Any curriculum policy that inadvertently, but predictably, leads students or teachers to play Fatima's rules is a policy difficult to defend educationally, even though the policy flourishes for *political reasons*" (Aikenhead, 2006, p. 28, original emphasis).

An Implication

Hence, within school mathematics we can identify the following two types of repertoires of sense-making practices—two conflicting subcultures:

1. The public face of pure mathematics (a.k.a. Platonist mathematics) held by almost all mathematics teachers and is attractive, in varying degrees, to math-oriented, math-curious, and math-interested learners whose mathematical self-identities harmonise to some degree with their teacher's, because of their mutual degrees of preference for "separated values" (Ernest, 2018, p. 194; Gilligan, 1982).
2. The collective repertoires of diverse sense-making practices of the math-disinterested, math-shy, and math-phobic learners whose mathematical self-identities do not harmonise with their mathematics teacher's, to differing degrees because of their preference for humanists' "connected values" (Ernest, 2018, p. 194; Gilligan, 1982).

This second subculture of learners requires humanistic mathematics learning experiences that exemplify mathematics is a symbolic technology for humans to forge relationships between themselves and their social and physical environments (Bishop, 1988). "Self-identity," in terms of mathematics, is defined as "a socially produced way of being, as enacted and recognised in relation to learning mathematics. It involves stories, discourses and actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities" (Darragh & Radovic, 2018). This is consistent with Ruef's (2020) detailed descriptions.

These learners' cultural self-identities must be engaged in order for non-superficial learning to take place. This usually occurs when they learn mathematics conveyed as being predominantly value-laden, culturally contextualised, ideology-related, and authentically subjective in its use. That is their everyday world. Such a humanistic mathematics course tends to engage this majority of learners to a great extent (Barta et al., 2014), because it significantly reduces learners' cultural self-identity conflicts with conventional mathematics classes (Aikenhead, 2017; Gijsbers et al., 2020). "We suggest that the alienation that many children in school, and adults out of school, feel towards mathematics is partly the result of the lack of connexions between their experience in mathematics classrooms and their experiences out of school" (Greer & Mukhopadhyay, 2012, p. 244).

Conflict between the two subcultures also leads to serious miscommunication problems between mathematics teachers and their math-disinterested, math-shy, and math-phobic learners. Mathematics teachers have always applied mathematics implicitly to the world around them, so they have not experienced their large majority of learner's consistent breakdowns when attempting to apply abstract mathematics to concrete events in the everyday world. As a result, the teachers' language and expectations are at odds with the lived reality of the majority of learners. This is a serious blind spot in many teacher's mindsets (Aikenhead, 2021b). It prevents teachers from understanding these learners when they ask in frustration, "When will I ever use this stuff?" So a teacher's response, composed from a Platonist standpoint, does not make common sense to these learners.

These conflicts within mathematics classrooms lead to low engagement and consequently lower achievement (Ruef, 2020). Learners' proficiencies can be raised somewhat by specific pedagogical interventions, but never as high as the popular, slogan-like expectations that bombard mathematics teachers; for example, "All students can learn math to the highest levels" (Boaler, 2020, p. 1). This ideology is welcomed by the 30% of learners, however, the future employees of mathematically rich workplaces—the potential STEM people.

What happens to these high school graduates when enrolled in university science and engineering programs? The U.S. Office of Technology Assessment conducted a 16-year longitudinal study, beginning with four-million Grade 10 students and following up to PhDs (Frederick, 1991). During their last 3 years in high school, 19% lost interest in their STEM subjects. During undergraduate university science and engineering programs, 39% lost interest—twice the proportion of STEM students in high school. For governments and industry concerned with their country's economic future reliant on STEM employees, they need to intervene at the university level where the major problem lies. The number of STEM graduates from high school is not the problem.

The Platonist dominance over a large majority of learners (Grades 7–12) and the general public requires scrutiny. This article does so by exploring Platonist mathematics' cultural nature, first historically (What should we know about the decisions that led to the first school mathematics curriculum?) and then philosophically (Is Platonist mathematics really what mathematicians claim it is?).

Establishing the First School Mathematics Curriculum

I strongly suggest that the math-phobic, math-shy, and math-disinterested students' generally low proficiencies are not the root problem to be solved by educators. Instead, a more basic underlying problem is an inappropriate school mathematics curriculum anchored to standards defined by nineteenth-century university mathematicians and designed strictly for math-oriented learners (Aikenhead, 2017; Barta et al., 2014). These mathematicians apparently, but wrongly, either assumed that the majority of students would possess a mathematical worldview that harmonised with their own, or assumed that these students' minds were a tabula rasa. These stances are ego-centred and outdated, respectively.

The problem of conflicting subcultures, defined above, can be traced to the decision that produced the first mathematics curriculum for public schools. A thorough in-depth description can be found in Aikenhead (2017). A synopsis is offered here.

Europe's post-Industrial Revolution created the need to establish public school systems in Europe and North America during the first half of the nineteenth century (Furr, 1996; Nikolakaki, 2016; Willoughby, 1967). The decision over its mathematical content was highly controversial between (a) the elite university's Platonist mathematicians anticipating students proficient in abstract mathematics and (b) the local merchants and emerging industries that needed a labour force proficient at practical mathematics.

The Platonist mathematicians, channelling ancient Greek beliefs in the superiority of pure abstract thought ("Plato's World of Ideas;" Kawasaki, 2002, p. 93), insisted on abstract *decontextualised* content for the curriculum. Platonists eschewed worldly practical knowledge ("Plato's Phenomenal World"; Kawasaki, 2002, p. 94) connected with human activity, which would produce a curriculum of *contextualised* content dealing with the people.

The Platonists won the politically charged curriculum debate using deceptive strategies that were rendered invisible to future generations (Ernest, 1991). These strategies ensured there would be no fundamental change to the curriculum over the ensuing 170 years. This article proposes that our twenty-first-century culture requires a fundamental revision to school mathematics' repertoire of sense-making practices for the 70% but not much revision for the 30% minority of math-oriented, math-curious, and math-interested learners in junior and senior secondary school.

The nineteenth-century Platonists' surreptitious strategies and rhetorical sleights of hand were analysed by Ernest (1991) who concluded, "[T]he values of the absolutists are smuggled into mathematics, either consciously or unconsciously, through *the definition of the field*" (p. 259, emphasis added). He is referring to the Platonists' transformation of the ancient Greek binary, logical versus irrational, into their newly invented binary, formal mathematical discourse versus informal mathematical discourse. Then the Platonists defined school mathematics as formal mathematical discourse (Plato's World of Ideas) and any content that suggested mathematics was a human endeavour (Plato's Phenomenal World) was jettisoned into the *informal* mathematical discourse category and out of sight, unworthy of curriculum content.

Simply put, the fact that nineteenth-century Platonists' rhetoric can reach into the twenty-first century to stifle much needed innovation today speaks to the urgency to challenge their non-ideological ideologies. Borovik (2017) documented quantitatively the disparity between the Victorian nineteenth-century life and the digital age of the twenty-first century with its sophisticated algorithms that substantially decrease the mathematics needed by the general public, and at the same time, it decreases the need for PhD mathematicians due to artificial intelligence replacing them in high-technology fields. Borovik concluded, "[T]he current crisis in school-level mathematics education is a sign that it reaches a bifurcation point and is likely to split into two streams" (p. 309).

Challenges to Platonist Mathematics

Platonists claim their pure mathematics is value-free, non-ideological, culture-free, purely objective in its use, perfectly certain, and accurate in descriptions of reality. As it turns out, this is a façade that masks Platonists' power with the claimed innocence of "absolute objectivity and neutrality" (Ernest, 1991, p. 259). One purpose for challenging Platonist mathematics is to replace its non-humanistic façade with a nuanced humanistic Western mathematics perspective. The adjective "Western" is an important reminder that mathematics is pluralistic amongst many cultures, past and present.

Fundamentals of Platonist Mathematics

In the *Stanford University Encyclopedia of Philosophy*, Linnebo (2018) summarises the fundamentals of Platonist mathematics as:

> *Existence* [ontology]: There are mathematical objects.
>
> *Abstractness* [epistemology]: Mathematical objects are abstract.
>
> *Independence* [axiology]: Mathematical objects are independent of intelligent agents and their language, thought, and practices. (website quote)

In other words, Plato's "abstract mathematical objects" comprise the concrete universe as we know it. Accordingly, mathematicians do not create or invent an abstract object; they *discover* it (Kessler, 2019). The expression "abstract objects" seems to be an oxymoron. If these abstract objects are neither physically concrete (because they are abstract) nor of an intelligent agent, what are they?

Aikenhead (2021a) investigated this issue of abstract mathematical objects but changed the ontological question (What are they?) to an epistemic one: "How can we explain their concrete appearance?" (in press). The result was a narrow dichotomy:

1. They might arise from "divine inspiration," described by, but not endorsed by Sriraman (2004, p. 22). This spiritual characterisation of Plato's abstract mathematical objects was plausible to some writers (e.g., Kessler, 2019; Nightingale & Sedley, 2010; Phillips, 2009).
2. They are an intellectual mirage: "What has been thought of as the mind is actually internalised culture" (anthropologist Hall, 1976, p. 192, original emphasis). Thus, Plato's source of his abstract objects is his culture, which causes his abstract objects to be *cultural* entities rather than *spiritual* entities, which in turn means that Platonist mathematics is fundamentally cultural.

A third position, of course, supports mathematical Platonism, explained by Gordon (2019) in the context of Ancient Greek history, and thoroughly documented by Cole (2009, n.d.), who reported, "Platonists take the mathematical realm to be quite distinct from the spatial–temporal realm" (n.d.). Importantly, Cole's quote indicates that the Platonists have introduced the language of two different "realms" into a dichotomous axiom which leads directly to a theorem-like, logically tight, deductive argument.

It all looks very familiar: the language of "discourses" introduced into a dichotomous axiom, formal versus informal mathematical discourses (i.e., logical versus irrational). In the nineteenth century, this rhetorical tactic powerfully and politically legitimised the Platonist school curriculum we have today for Grades 7–12. On the surface, the tactic suggests opportunism. Beneath it, however, lies raw social-political power, masked by the innocence of its public façade of an intellectual, value-free, culture-free game called "Platonist Mathematics" (McKinley, 2001). The tactic of constructing two-different-realms may provide Platonists with a defence against a claim it is spiritual. A deductive argument will always produce a "truth" as long as it is crafted to be self-consistent.

Culture-Laden Platonist Mathematics

Based on his research interviewing mathematicians, Kessler (2019) stated, "My conclusion is that there is indeed *something* of a spiritual nature inherent in mathematics itself" (p. 59, original emphasis). Fundamentally, therefore, Platonist pure mathematics is either spiritual or cultural or engaged in another opportunistic power move. The choice of Western mathematics being *cultural* would present school jurisdictions with far fewer political issues to handle, I suggest.

A *cultural* mathematics is the reasoned conclusion by critical mathematics educators (e.g., Bishop, 2016; Ernest, 1988, 2016a, b; Larvor, 2016; Sriraman, 2017; Skovsmose & Greer, 2012). It is rooted in the culture of those who created the knowledge, or modified it by the culture of whoever appropriates it, as Western mathematics did with Islamic and Ancient Greek mathematics. Some of these cultural attributes have contributed to Western mathematics' humanistic dimensions, which can be related meaningfully to learners' cultural self-identities (Darragh & Radovic, 2018; Ishimaru et al., 2015; Nasir, 2002). Bishop (1988) recognised this double function when he stated that mathematics is a product of its developer's culture and "as a cultural product, [it] is now strongly shaping Western culture as a whole" (p. 155).

Value-Laden and Ideology-Laden Platonist Mathematics

This issue is especially critical for the 70% of high school graduates: the math-disinterested, math-shy, and the math-phobic. Their usual perception of mathematics is "the antithesis of human activity—mechanical, detached, emotionless, value-free, and morally neutral" (Fyhn et al., 2011, p. 186), as well as being decontextualised, abstract, and rigid. This anti-humanistic school mathematics makes the content seem foreign to this significant majority of learners' self-identities, and therefore, difficult to learn (Aikenhead, 2017; Ruef, 2020). A humanistic mathematics program would replace the Platonist value-free ideology-free façade. Ernest (2016a) pointed out that:

> [S]ome of mathematics is contingent on human history and culture and thus it allows that mathematics itself can be *imbued with the values of the culture of its human makers*. Overall, a consequence of redefining objectivity as cultural is that mathematics can be understood as being laden with values from human culture. (p. 193, emphasis added).

Platonists' assertions that their mathematics is value-free and ideology-free were also critically challenged by Bishop (2016) and Corrigan and colleagues (2004). Together all three research groups, amongst others, identified and articulated the following *values*: truth, certainty, universalism, mystery, beauty, purity, mathematical realism, provability, and absolute accuracy in descriptions of reality. A sampling of these is described here to establish the statement: Platonist mathematics is value-laden and ideology-laden.

The Platonist values of certainty, truth, and purity lay claim to absolute pure mathematics knowledge. However, Ernest (2016a) warned, "prized as it is the attainment of the value of truth in mathematics is not without difficulty" (p. 194). Gödel's Incompleteness Theorems in mathematics has also proven that this aim is unattainable (Raatikainen, 2020).

Similarly, Einstein (1921) stated, "As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality" (website quotation). The absolutist language of the Platonist expression "pure knowledge with certainty" amounts to another oxymoron, a façade behind which to treat people according to several of its ideologies, thereby masking Platonist mathematics' social-political power with its presumed innocence of possessing certain pure knowledge (McKinley, 2001).

Ideologies denied by Platonists but identified by the mathematics research literature include (1) rationalism, objectivism, empiricism, progress, openness, mystery, and control (Bishop, 2016; Corrigan et al., 2004) and (2) purism, quantification, dominance, instrumentalism, and foundationalism (Ernest, 2016b).

The ideology of purism (Ernest, 2016b), for instance, is described in more detail here. The expression "pure mathematics" implicitly suggests the category "*impure* mathematics," on the strength of mathematicians' ubiquitous use of dichotomies. Subtly, a hierarchy of status is established by this dichotomous language: pure or impure. This was played out, for example, at some universities when their Department of Mathematics renamed itself Department of Mathematics and Statistics. The ideology of purism "denigrates applied mathematics [such as statistics] and calculation as technical and mechanical, pertaining to the utilitarian, practical, applied, and mundane, understood as the lowly dimensions of existence" (Ernest, 2016a, p. 209). "Purism is an intellectual strategy serving social goals including the demarcation of knowledge and defending the pursuit of knowledge for its own sake from outside interests" (p. 210). *This defence includes opposing innovative educators wishing to humanise school mathematics curricula for a majority of learners—the 70% defined above*. "Purism…emphasises boundaries that strongly demarcate disciplines and social groups, pure mathematics, and the community of pure mathematicians" (p. 210). Purism "facilitates purist values by locating mathematical objects in a pure and ideal realm disconnected from the material world we inhabit" (p. 209).

The ideology of *quantification* "valorises the measurable outputs of knowing over those that are less easily measured" (Ernest, 2016b, p. 52). Quantification can harmfully lead to "the objectification and commodification of knowledge" (p. 52), so that knowledge *appears to be* objective, but turns out not to be when we consider its context. This is illustrated by Boylan (2016):

> A significant capitalist response to the environmental crisis has been to enlist mathematics in the search for market solutions. Under the banner of green capitalism, mathematics is being

> used as a means to extend the commodification of natural resources in new ways… The value and worth of the natural world and our relationship to it are transmuted into valorisation; everything—water, trees, clean air, biodiversity and ecosystems—can be given a price. (p. 402).

In Ernest's (2016b, 2019) view, whenever the content of mathematics wanders beyond its intellectual domain in the human mind (e.g., when someone projects a concept onto the physical world, teaches a concept to someone, and plans or undertakes a course of action based on that concept), this Platonist mathematics beyond the mind was never pure in the sense of being ethics-free. The entwined combination of mathematics and a user of mathematics must take responsibility for the consequences of using Platonist mathematics and not hide behind its façade of innocent objectivity (Ernest, 2019). For instance, when you hear someone claim, "The numbers speak for themselves," be particularly vigilant in analysing whether or not they are attempting to identify with mathematics' purism ideology.

To summarise, this interrogation into the nature of Platonist mathematics has provided evidence and explanations for knowing it as a human endeavour, characterised by various ontological and epistemological beliefs (some of which are controversial), and by values and ideologies. *No longer tenable or truthful is its façade of being* value-free, non-ideological, culture-free, purely objective in its use, and perfectly certain and accurate in its descriptions of reality.

Humanistic Curricula

Voices from the mathematics education culturalists (e.g., Larvor, 2016; Pierce & Stacey, 2006) and the critical mathematics educators (e.g., Ernest et al, 2016; François & Van Kerkhove, 2010) offer support for learners to experience and understand "mathematics as a human endeavour" (Saskatchewan Curriculum, 2008, p. 9). "We take it as self-evident that mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context" (Hersh, 1997, as quoted by Skovsmose & Greer, 2012a, p. 379). Ernest (1991) pointed out that mathematics is the product of mathematicians' human creativity, and as a result, it bears some cultural features of its inventors—a very humanity-like stance to take.

"Humanistic" and "culture-based" school mathematics could be understood as two sides of the same coin, differing only in degrees of emphasis and often overlapping to a large extent. Meyer and Aikenhead's (2021) mathematics research dealt with enhancing school mathematics with Indigenous mathematizing. They named it "Indigenous culture-based school mathematics" (p. 100). It, too, is a humanistic mathematics applicable to countries with Indigenous citizens. Culture-based school mathematics' content emphasise the needs of non-STEM adults, as judged by those adults, not by mathematics educators.

In general, any mathematics curriculum or lessons will earn the moniker "humanistic," if

- They clearly specify or exemplify that Western mathematics is a human endeavour.
- Their mathematical symbols and manipulations reveal or forge relationships between learners and their social, political, and physical environments.
- Their mathematics content is enhanced by "the history and *nature* of mathematics" (Panasuk & Horton, 2012, p. 17, original emphasis) as well as by its interactions with societal issues and cultural perspectives.
- They convey their school mathematics as being predominantly value-laden, culturally contextualised, ideology-related, and/or authentically subjective in its use.
- They avoid a disconnect with learners whose "minds are grounded in systems of social relations… and [mathematics] concepts must be understood as elements of those systems" (Ojalehto & Medin, 2015, p. 9). Learners need to connect personally with what they learn.

However challenging, teachers need to connect with learners whose worldviews do not include, to varying degrees, the facility to abstract mathematical symbolic technologies and apply them to a concrete situation or event.

Examples of Humanistic Lessons and Modules

The first example is a pair of different trigonometry units: one for a non-STEM group and the other for a STEM group (Bressoud, 2010).

A. A while back, studying trigonometry was most likely for navigation and surveying, when defining these functions as ratios made sense. An authentic-like story of survival at sea due to navigating accurately would make a humanistic context for the 70% group.
B. Today, STEM learners are likely to study trigonometry's sine and cosine as periodic functions. "Biological, physical, and social scientists use them more often to model periodic phenomena" (p. 112). A social science theme could be composed to create a humanistic context.

A country's Remembrance/Memorial Day celebration is another venue for humanistic mathematics lessons that were developed by K.G.B. Secondary College (n.d.) for Grade 9:

1. Pythagoras theorem to calculate the size of trenches; estimate means and medians.
2. Perimeter and area of WW I trenches through partitioning composite shapes.
3. Using ratio factors to create a down-sized replica of WW I trenches.

A twenty-first century commerce-related humanistic module for the 70% will look very different than a typical high school textbook unit on financial literacy. For example, Saskatchewan's textbook, *Foundations of Mathematics 12* (Canavan-McGrath, 2012), devotes 141 pages to "Unit 1: Financial Mathematics," comprised mostly around algebraic formula manipulations and calculations concerning compound interest related to savings and loans. It does offer savvy advice on credit cards. Two subconcepts are actually associated with this textbook unit: (a) Platonist *algebraic* literacy and (b) a relevant context into which abstract algebra is *applied*. For these two reasons, Unit 1 is not an example of a humanistic Western mathematics unit.

To be authentic, students would learn the deep meaning of all the important financial concepts and their consequences, but in the authentic context of the twenty-first-century financial industry, by using a computer algorithm on the internet. If feasible, learning is identified with a local context, such as specific banks or a learner's parent, thereby *connecting* their learning with real humans, rather than hypothetical humans. Some learners experience their role as participating in an introductory internship to the industry, or as problem solving and problem posing in a project-based learning setting (Boaler & Selling, 2017).

This is qualitatively different than forming an abstract concept from a textbook and then applying it hypothetically. That two-step process usually separates the math-phobic, math-shy, and math-disinterested learners from the math-interested, math-curious, and math-oriented learners. An unpublished module equivalent to the textbook's Unit 1 took one third the time to complete, which left time for students to explore some history of mathematics: Where did all this financial mathematics come from?—an excellent context for humanistic content.

Some teachers have their favourite activities that meet the criteria of a humanistic lesson or a set of lessons, which result in students' enthusiastic engagement. The following unpublished modules will help clarify further what a humanistic mathematics lesson or module looks like.

In a module "Music in the Numbers," Grade 7 learners are introduced to or re-enforced by the addition and subtraction of fractions. They are taught by their musical peers: (a) to identify musical beats in several different songs by clapping the beats (experiencing full notes to sixteenth notes); (b) to learn the symbols for each type of beat length; (c) to learn that a 4:4 signature bar on a musical staff contains the equivalent of one full beat; (d) to practice making several combinations of different beats that could fit into one bar; and (e) to solve the problems when a bar has too few beats in it, and needs someone to fill it accurately.

The lesson ends with the teacher writing the fraction symbols in the bottom empty row of a chart of musical beat symbols, and then repeating the very same problems but this time with the fraction symbols. Learners *transfer* the concepts learned with concrete musical fractions to abstract mathematical fractions. The abstract mathematical meaning of fractions may mature with similar problem solving in other everyday situations (not hypothetical situations) that involve fractions.

Another example of a humanistic mathematics module is entitled “Why Is Math So Abstract?” At a Grade 10 interest and reading level, it presents a historical synopsis focussed on Thales of Miletus. Learners engage in activities, such as writing their date of birth using one of the several Ancient Greek numeral systems, which does not have a zero, of course. Some learners will ask, “Who invented zero?” They have just initiated an authentic historical investigation for the whole class to conduct out of school. Their post-investigation class discussion will naturally include the question, “Why are there so many different answers to: Who invented zero?” The teacher can then introduce learners to mathematics’ interaction with society’s culture, such as Who gets to write history? and What influence does culture have on the development of mathematics today? (e.g., the industrial and military demand for artificial intelligence algorithms). Simply put, the module provides insights into humanistic features of Western mathematics related to societal issues. Often, Western mathematics does have important roles to play in social/political/ethical issues.

In this next example, learners interact with the Hollywood movie *Hidden Figures* to witness and discuss the development of a sophisticated algorithm that brought astronaut John Glenn safely back to earth in 1962, the first American in space and the first human to circle the Earth in an orbit. The module introduces learners to exponentials, scientific notation, some authentic problem-solving using scientific formula (not to manipulate, but to act as scaffolding for calculations), and the need to know some analytical geometry in order to follow the action in the movie. Three Black employees of NASA, under the duress of systemic racism, were instrumental in the project’s success. Mathematics played several key roles in the societal issue of systemic racism, as it does today (Bellringer, 2019).

The last example, “Logical Thinking,” a module for Grade 11, has activities that draw on topics such as court cases related to drinking and driving, symbolic logic for analysing everyday conversations or arguments with family or classmates, and some common Aristotelian fallacies of arguments for students to hone their reasoning skills in terms of everyday events. Within these contexts, the following Western mathematics topics are explored: error of measurement, an intellectually honest simplification of confidence intervals, the public’s misinterpretations of polls and PISA scores (including the politics of international testing), artificial intelligence, and a critical logical analysis of the mathematics theorem concerning the sum of the interior angles of a triangle—find the ambiguity in it.

In all the examples above, learners are deeply engaged using symbolic technologies in developing relationships between themselves and their social and physical environments (Bishop, 1988).

However, a major roadblock to humanistic school mathematics is “the highly restrictive, overcrowded, outdated curriculum” (Duchscherer et al., 2019, p. 63). This prevents teachers from having the time to innovate with humanistic lessons and at the same time, complete teaching their mandated Platonist curriculum. The Conference Board of Canada (2020) advised, “Some [mathematics] content may be removed to make room for more innovative material” (p. 15). As mentioned earlier, the problem is not the learners; it is the adults who determine the curriculum content, processes, and most importantly, *the context of learning* that exacerbates the two conflicting subcultures detrimental to a large majority of learners.

"New curriculum reform projects will have to find different ways to create and distribute the innovative materials that can drive change" (Conference Board of Canada, 2020, p. 27). A detailed template for doing so is available: "The Ministry of Education could establish cadres of developers of teaching materials" (Duchscherer et al., 2019, p. 71). A number of other challenges need to be addressed, such as mathematics teachers' reluctance to learn some of the history of the subject they teach. These challenges were identified by Panasuk and Horton's (2012) research.

Conclusion

To paraphrase Bang and Medin (2010) quoted in the section "Culture:" life in a *humanistic* mathematics classroom will be comprised of cultural sense-making practices in which a majority of learners naturally participate, because the content supports their understanding of mathematics as a human endeavour in *their* world. This resolves, or at least ameliorates, a conflict that learners otherwise encounter to varying degrees, a conflict between, on the one hand, learners' humanities-oriented self-identities that embrace a host of connective values, and on the other, a non-humanistic Platonist school mathematics curriculum emphasising separative values (Ernest, 2016a, b; Gilligan, 1982). These learners are in a much better position to succeed in reaching their full mathematical potential for adulthood.

Extensive international research on *Indigenous* culture-based school mathematics has demonstrated a consistent increase in learner proficiencies measured by standardised mathematics tests (reviewed in Aikenhead, 2017). To produce similar results for humanistic school mathematics is a research program worth pursuing.

References

Aikenhead, G. S. (1996). Science education: Border crossing into the subculture of science. *Studies in Science Education*, *27*, 1-51.

Aikenhead, G. S. (2006). *Science education for everyday life*. New York: Teachers College Press.

Aikenhead, G. S. (2011). Towards a cultural view on quality science teaching. In D. Corrigan, J. Dillon, & R. Gunstone (Eds.), *The professional knowledge base of science teaching* (pp. 107–127). Springer.

Aikenhead, G. S. (2017). Enhancing school mathematics culturally: A path of reconciliation. *Canadian Journal of Science, Mathematics and Technology Education*, *17*(2), 73-140.

Aikenhead, G. (2020). School science and mathematics storylines. *Canadian Journal of Science, Mathematics and Technology Education*, *20*(4), 682-699.

Aikenhead, G. S. (2021a). A 21st century culture-based mathematics for the majority of students. *Philosophy of Mathematics Education Journal*. No. 37, in press.

Aikenhead, G. (2021b). Out-of-school, applied, in-school, and Indigenous mathematics. *The Variable*, *6*(2), in press.

Bang, M., & Medin, D. (2010). Cultural processes in science education: Supporting the navigation of multiple epistemologies. *Science Education, 94*, 1009-1026.

Barta, J., Eglash, R., & Barkely, C. (2014). The crossroads of mathematics and culture. In J. Barta, R. Eglash & C. Barkeley (Eds.), *Math is a verb* (pp. 1–7). Reston VA: National Council of Teachers of Mathematics.

Bellringer, C. (2019). *Progress audit: The education of Aboriginal students in the B.C. public school system.* Government of British Columbia, Office of the Auditor General of British Columbia.

Bishop, A. J. (1988). The interactions of mathematics education with culture. *Cultural Dynamics*, *1*(2). 145-157.

Bishop, A. (2016). What would the mathematics curriculum look like if instead of concepts and techniques, values were the focus? In B. Lavor (Ed.), *Mathematical cultures* (181–188). Cham, Switzerland: Springer International Publishing.

Boaler, J. (2020). Setting up positive norms in math class. youcubed at Stanford University. http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf. Accessed 3 June 2020.

Boaler, J., & Selling, S. (2017). Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education, 48*(1), 78-105.

Borovik, A. V. (2017). Mathematics for makers and mathematics for users. In B. Sriraman (Ed.), *Humanizing mathematics and its philosophy* (pp. 309-327). Cham, Switzerland: Springer International Publishing.

Boylan, M. (2016). Ethical dimensions of mathematics education. *Educational Studies in Mathematics*, *92*, 395-409.

Bressoud, D. M. (2010). Historical reflections on teaching trigonometry. *Mathematics Teacher*, *104*(2), 107-112.

Brown, S. (1996). Towards humanistic mathematics education. In A. J. Bishop et al. (Eds.), *International handbook of mathematics education* (pp. 1289 – 1321). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Canavan-McGrath, C., et al. (2012). *Foundations of mathematics 12*. Toronto: Nelson Education.

Card, D., & Payne, A. A. (2017, September). *High school choices and the gender gap in STEM* (Working Paper 23769). Cambridge, MA: National Bureau of Economic Research. http://www.nber.org/papers/w23769. Accessed 15 July 2019.

Cobern, W. W. (2000). *Everyday thoughts about nature*. Dordrecht, The Netherlands: Kluwer Academic.

Cole, J. (2009). Creativity, freedom, and authority: A new perspective on the metaphysics of mathematics. *Australasian Journal of Philosophy*, *87*, 589–608.

Cole, J. (n.d). Mathematical platonism. *The Internet Encyclopedia of Philosophy*. https://iep.utm.edu/mathplat/. Accessed 8 December, 2020.

Conference Board of Canada. (2020, October 13). Curriculum and reconciliation: Introducing Indigenous perspectives into K-12 science (Impact paper). Author. https://www.conferenceboard.ca/e-library/abstract.aspx?did=10772. Accessed 20 October 2020.

Corrigan, D., Gunstone, R., & Bishop, A. (2004, July 4–7). *Values in science and mathematics education: Similarities, differences and teacher views*. [Paper presentation]. Austalasian Science Education Research Association; Armidale, NSW, Australia.

Darragh, L., & Radovic, D. (2018). Mathematics learner identity. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Cham, Switzerland: Springer International Publishing.

Director, B. (2006). On the 375th anniversary of Kepler's passing. *FIDELIO Magazine*, *15*(1–2), 98–113. http://www.schillerinstitute.org/fid_02-06/2006/061-2_375_Kepler.html. Accessed 17 July 2018.

Dorce, C. (2020). History of mathematics as an integration and social cohesion tool in the classroom, *International Journal of Mathematical Education in Science and Technology*, *51*(8), 1-14.

Duchscherer, K., Palmer, S., Shemrock, K., et al. (2019). *Indigenous culture-based school mathematics for reconciliation and professional development*. Report 287. Stirling McDowell Foundation. http://mcdowellfoundation.ca/research/culture-based-school-mathematics-for-reconciliation-and-professional-development/. Accessed 20 June 2020.

Edwards, D. (2010, August). Revolutionary implications. *The Notices of The American Mathematical Society, 57*(7) 822. http://www.ams.org/notices/201007/rtx100700822p.pdf. Accessed 23 April 2021.

Einstein, A. (1921, January 27). Geometry and experience. [Paper presentation]. Prussian Academy of Science. Berlin, Germany. http://todayinsci.com/E/Einstein_Albert/EinsteinAlbert-MathematicsAndReality.htm. Accessed 17 July 2018.

Einstein, A. (1930, November 9). Albert Einstein über Kepler. *Frankfurter Zeitung*. Frankfurt, Germany.

Ernest, P. (1988). The impact of beliefs on the teaching of mathematics. http://webdoc.sub.gwdg.de/edoc/e/pome/impact.htm. Accessed 21 April 2019.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Routledge-Falmer. https://p4mriunpat.files.wordpress.com/2011/10/the-philosophy-of-mathematics-education-studies-inmathematicseducation.pdf. Accessed 17 Mar 2016.

Ernest, P. (2016a). Mathematics and values. In B. Larvor (Ed.), Mathematical cultures (pp. 189-214). Cham, Switzerland: Springer International Publishing.

Ernest, P. (2016b). Mathematics education ideologies and globalization. In P. Ernest, B.Sriraman & N. Ernest (Eds.), Critical mathematics education: Theory, praxis and reality(pp. 35-79). Charlotte, NC, USA: Information Age Publishing.

Ernest, P. (2018). The ethics of mathematics: Is mathematics harmful? In P. Ernest (Ed.), *The philosophy of mathematics education today* (pp. 187-216). Cham, Switzerland: Springer International Publishing.

Ernest, P. (2019). Privilege, power and performativity: The ethics of mathematics in society and education. *Philosophy of Mathematics Education Journal*, No. 35(December), 1-19.

Ernest, P., Sriraman, B., & Ernest, N. (Eds.) (2016). *Critical mathematics education: Theory, praxis and reality* (pp. 35-79). Charlotte, NC, USA: Information Age Publishing.

Frederick, W. A. (1991). Science and technology education: An engineer's perspective. In S. K. Majumdar, L. M. Rosenfeld, et al. (Eds.), *Science education in the United States: Issues, crises and priorities* (pp. 386–393). Easton, PA: The Pennsylvania Academy of Science.

Furr, J. (1996). A brief history of mathematics education in America. University of Georgia. http://jwilson.coe.uga.edu/EMAT7050/HistoryWeggener.html. Accessed 8 December 2020.

Fyhn, A. B., Sara Eira, E. J., & Sriraman, B. (2011). Perspectives on Sámi mathematics education. *Interchange, 42*(2), 185-203.

Gijsbers, D., de Putter-Smits, L., & Pepin, B. (2020). Changing students' beliefs about the relevance of mathematics in an advanced secondary mathematics class. *International Journal of Mathematical Education in Science and Technology, 51*(1), 87–102.

Gilligan, C. (1982). *In a different voice*. Cambridge, MA: Harvard University Press.

Gordon, M. (2019). Identity, culture, and pedagogical challenge: The presentation of mathematics. *Philosophy of Mathematics Education Journal*, No. 35, 1-15.
Greer, B., & Mukhopadhyay, S. (2012). The hegemony of mathematics. In O. Skovsmose & G. Greer (Eds.), *Opening the case: Critique and politics of mathematics education* (pp. 229-248). Rotterdam, The Netherlands: Sense Publishers.
Gutiérrez, K. D., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice. *Educational Researcher*, *32*(5), 19-25.
Hall, E. T. (1976). *Beyond culture*. Toronto, ON, Canada: Doubleday.
Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.
Ipsos. (2005, August 16). Press release. https://www.ipsos.com/en-us/apaol-poll-people-have-love-hate-relationship-math-most-unpopular-school-subjects-especially-among. Accessed 22 June 2018.
Ishimaru, A. M., Barajas-López, F., & Bang, M. (2015). Centering family knowledge to develop children's empowered mathematics identities. *Journal of Family Diversity in Education, 1*(4), 1–21.
Joseph, G. G. (2011). *The crest of the peacock: Non-European roots of mathematics* (3rd Ed.). Princeton: Princeton University Press.
Kawasaki, K. (2002). A cross-cultural comparison of English and Japanese linguistic assumptions influencing pupils' learning of science. *Canadian and International Education*, *31*(1), 19–51.
Kennedy, N. S. (2018). Towards a wider perspective: Opening a philosophical space in the mathematics curriculum. In P. Ernest (Ed.), *The philosophy of mathematics education today* (pp. 309-320). Cham, Switzerland: Springer International Publishing.
Kessler, V. (2019). Spirituality in mathematics. *Journal for the Study of Spirituality, 9*(1), 49-61.
K.G.B. Secondary College. (n.d.). World War I curriculum—year 9. https://ww1curriculum.weebly.com/maths.html. Accessed 26 December 2020.
Larson, J. O. (1995, April 15–19). Fatima's rules and other elements of an unintended chemistry curriculum. [Paper presentation]. American Educational Research Association Annual Meeting, San Francisco.
Larvor, B. (Ed.) (2016). *Mathematical cultures*. Cham, Switzerland: Springer International Publishing.
Linnebo, Ø. (2018). Platonism in the philosophy of mathematics. In W. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy*. https://plato.stanford.edu/archives/spr2018/entries/platonism-mathematics/. Accessed 14 August 2020.
Makins, M. (1994). *Collins English Dictionary.* (3rd ed.). Glasgow: HarperCollins Publishers.
McKinley, E. (2001). Cultural diversity: Masking power with innocence. *Science Education*, *85*, 74-76.
Meyer, S., & Aikenhead, G. (2021). Indigenous culture-based school mathematics in action: Part I: Professional development for creating teaching materials. *The Mathematics Enthusiast*, *18*(1&2), 100-118.
Mukhopadhyay, S., & Greer, B. (2001). Modelling with purpose. Mathematics as a critical tool. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research in mathematics education. An international perspective* (pp. 295–311). Lawrence Erlbaum.
Mukhopadhyay, S., & Greer, G. (2012). Ethnomathematics. In J. A. Banks (Ed.), *Encyclopedia of diversity in education* (pp. 857-861). Thousand Oaks, CA: SAGE Publication.
Nasir, N. S. (2002). Identity, goals and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning*, *4*(2 & 3), 213–247.
Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. *Review of Research in Education*, *32*, 187-240.
Nightingale, A., & Sedley, D. (2010). *Ancient models of mind: Studies in human and divine rationality*. Cambridge University Press.
Nikolakaki, M. (2016). Mathematics education and citizenship in capitalism. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: Theory, praxis and reality* (pp. 273-286). Charlotte, NC: Information Age Publishing.
Noddings, N. (2017). A gift to teachers. In B. Sriraman (Ed.), *Humanizing mathematics and its philosophy* (pp. 347-363). Cham, Switzerland: Springer International Publishing.
OECD. (2019). What can students do in mathematics? in *PISA 2018 Results* (Vol. I): *What students know and can do*. OECD Publishing. DOI: https://doi.org/10.1787/f649d3c2-en. Accessed 16 June 2019.
Ojalehto, B., & Medin, D. (2015). Emerging trends in culture and concepts. In R. Scott & S. Kosslyn (Eds.), *Emerging trends in the social and behavioral sciences*. John Wiley & Sons.
Panasuk, R. M., & Horton, L. B. (2012). Integrating history of mathematics into curriculum: What are the chances and constraints? *International Electronic Journal of Mathematics Education*, *7*(1), 3–20.
Phillips, S. (2009). *The mathematical connection between religion and science*. Chippenham, Wiltshire, UK: CPI Antony Rowe.
Pierce, R., & Stacey, K. (2006). Enhancing the image of mathematics by association with simple pleasures from real world contexts. *ZDM, 38*(3), 214-225.

Raatikainen, P. (2020). Gödel's incompleteness theorems, *The Stanford encyclopedia of philosophy* (Winter 2020 Edition), Edward N. Zalta (Ed.). https://plato.stanford.edu/archives/win2020/entries/goedel-incompleteness/. Accessed 26 Dec 2020.

Ruef, J. L. (2020). What gets checked at the door? Embracing students' complex mathematical identities. *Journal of Humanistic Mathematics, 10*(1), 22-38.

Saskatchewan Curriculum. (2008). *Mathematics 8*. Regina, Saskatchewan, Canada: Ministry of Education.

Skovsmose, O. (1985). Mathematical education versus critical education. *Educational Studies in Mathematics, 16*, 337-354.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Skovsmose, O., & Greer, B. (2012a). Opening the cage? Critical agency in the face of uncertainty. In O. Skovsmose & B. Greer (Eds.), Opening the cage: Critique and politics of mathematics education (pp. 369-385). Rotterdam, The Netherlands: Sense Publishers.

Skovsmose, O., & Greer, B. (Eds.) (2012b). Opening the cage: Critique and politics ofmathematics education. Rotterdam, The Netherlands: Sense Publishers.

Skrivanos, C., & Zhang, Q. (2013). Humanistic mathematics network newsletter: A bibliographic Report. *Journal of Humanistic Mathematics*. DOI: https://doi.org/10.5642/jhummath.201301.05.

Snow, C. P. (1956, October 6). The two cultures. *New Statesman*, London, UK.

Sriraman, B. (2004). The characteristics of mathematical creativity. *The Mathematics Educator, 14*(1), 19-34.

Sriraman, B. (2017). *Humanizing mathematics and its philosophy*. Cham, Switzerland: Birkhäuser.

Tobin, K., & McRobbie, C. (1997). Beliefs about the nature of science and the enacted science curriculum. *Science and Education*, *6*, 355-371.

White, A. (1974). Humanistic mathematics: An experiment. *Education*, *95*(2, Winter).

White, A. (1976). A new paradigm for the mathematics classroom. *International Journal of Mathematical Education in Science and Technology*, *7*(2), 243-246.

Wilder, R. L. (1965). *Introduction to the foundations of mathematics* (2nd Edition). New York: John Wiley.

Wilder, R. L. (1981). *Mathematics as a cultural system*. Elmsford, NY: Pergamon Press.

Willoughby, S. S. (1967). *Contemporary teaching of secondary school mathematics*. John Wiley & Sons.

Wood, N. B., Lawrenz, F., & Haroldson, R. (2009). A judicial presentation of evidence of a student culture of "dealing." *Journal of Research in Science Teaching*, *46*, 421-441.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.