SCHOOL MATHEMATICS: TOWARDS ENDING ITS CYCLE OF MYTHS

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Abstract

This article interrogates a cycle of myths perpetuated by Plato-based (Platonist) mathematics. Emphasis is placed on Grades (Years) 7 to 12 (ages 13-18). The cycle follows four stages: (1) Platonist mathematics' controversial ontological axioms lead to deducing its mythical images of itself, maintained by privileged social power. (2) Widespread beliefs in those myths. (3) Socio-politico-economic power bestowed on those beliefs. (4) Privileges gained by that social power. The myth cycle pertains to: (a) the ontology, epistemology, and axiology of Platonist mathematics; and (b) its socio-politico-economics capital. The interrogation's purposes are: (i) to expose myths and their current negative impact on a large majority of learners – those not interested in pursuing STEM (science, technology, engineering, mathematics) employment; and (ii) to counter some of mathematics' educational propaganda aimed at teachers. The article's purpose is to question the ethics of Platonist mathematics education dictating that *all* learners must enroll in a *Platonist* mathematics program imbued with its cycle of myths, when alternative mathematics programs are viable.

Key Words: School mathematics · Anti-Platonism · Platonist mathematics myths · Societal contexts of mathematics

1. Introduction

A summary of this article might look something like this:

Question: What happens when public school mathematics is highjacked by a philosophically inconsistent, perhaps spiritually-based, movement promoting a slogan of market fundamentalism?

Answer: A myth cycle of inequities is perpetuated and supported by myth blindness – not knowing or not caring that personal privilege and power have accrued at the expense of others. The result is detrimental to a large majority of learners, but it is attractive to a small high-status minority.

Problem: The myth cycle maintains the privileged status enjoyed by a Plato-based (Platonist) mathematics curriculum, established as public education in a 19th century Victorian society. But today this curriculum's philosophy is fundamentally unchanged, thereby causing a relevancy gap of crisis proportions (Borovik, 2017).

The world that a majority of learners experience differs from today's culturally obsolete curriculum forced on these learners. Consequently, they do not reach their mathematics education potential at being prepared to deal with the Digital Revolution's way of using mathematics in the 21st century. As adults, they will be faced with social policy decisions informed by mathematical modelling, and concerned with the climate-change crisis exacerbated by "market fundamentalism" (Carney, 2020b, lecture transcript, 17 min. 17 sec.). Understanding the social functions and limitations of mathematics will become critical knowledge.

Solutions: For the *majority of learners* in Grades 7-12 who will graduate without wanting the credentials to pursue STEM (science, technology, engineering and mathematics) postsecondary programs, there are broadly based sets of curricular solutions (e.g., culture-based and humanistic mathematics), details of which have already been published in some detail elsewhere (Aikenhead, 2021a,c,d; Brown, 1996; Dorce, 2020; Gainsburg, 2008; Ishimaru et al., 2015; O'Brien, 2021; Panasuk & Horton, 2012; Nicol, 2002; Resnick, 1987; Stephan et al., 2020). They all establish a clear and urgent need to update a 19th century Victorian era curriculum to be relevant in today's Digital Revolution era for the majority of learners.

The *minority of learners* requires a precalculus program: (a) enriched to fit the 21st century Digital Age, (b) extended to include some International Baccalaureate exercises, and (c) enhanced by individual and group projects that legitimize mathematics as a human endeavour.

This article addresses the Answer and the Problem. The Question and Solutions provide the context only.

2. The Myth Cycle

School mathematics' socio-politico-economic capital is associated with its "value in exchange" but not its "value in use," described by economist Mark Carney (2021, p. 22), former governor of the Bank of Canada, and then the Bank of England. Mathematics professor Paul Ernest (2019) prefers the synonyms "*exchange value*" and "*use value*" (p. 4). The exchange value of mathematics refers to the benefits someone can accrue in *exchange* for appearing to have a high interest and proficiency in Platonist mathematics. Carney (2021) illustrated both concepts by noting that, from an economist's perspective, "gold has no value in use, but it has great value in exchange because it can be used to command other goods" (p. 22). Davis (1996) pointed out an "exchange value" of mathematics education: "The number of mathematics courses taken by a student is commonly regarded as an indicator of his or her potential and ability, not in the least because mathematics wears the mask of impartiality so effectively" (p. 145). "Use value" is the value of applied mathematics to someone or to society.

Underlying conventional school mathematics' exchange value, however, there is a complexity of interacting myths, found in the myth cycle (Figure 1). It comprises a dynamic set of beliefs initiated by an ontological stance of Plato-based (Platonist) mathematics (a.k.a. conventional or Western mathematics).

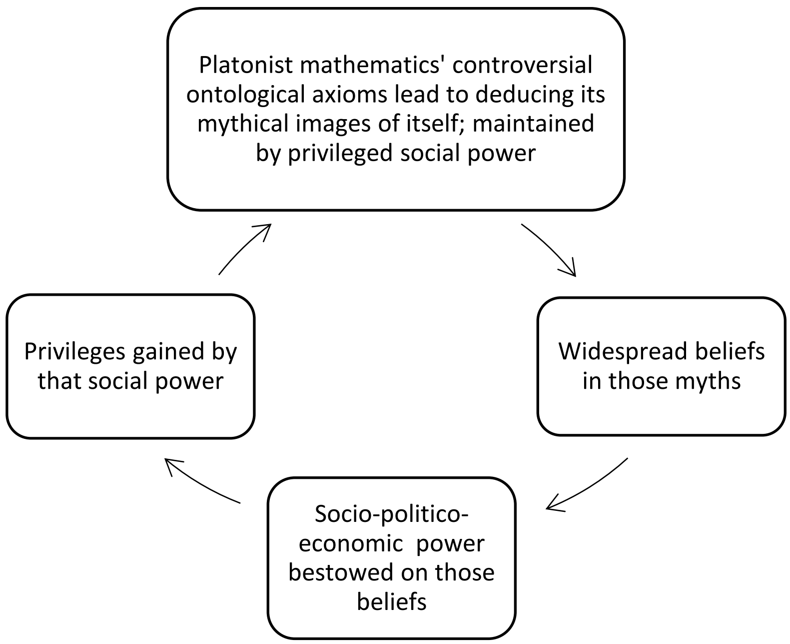

Figure 1.
The myth cycle and Platonist mathematics' socio-politico-economic capital.

In Western cultures, prowess in mathematics leads to various privileges (e.g., studying at better postsecondary institutions, people assuming you are brilliant, etc.). Exactly how and why this occurs is one focus of this article. Another focus is the combination of both harmful and supportive effects that Platonist myths have on school mathematics. Inequities are exacerbated by: (a) the deep-seeded self-identity clashes, to varying degrees, with the nature of Platonist mathematics experienced by a *majority* of learners; and (b) the deep-seeded self-identity supports for a *minority* of learners who embrace Platonist mathematics, to varying degrees, because that is the way they see their world

(Aikenhead, 2021c). These clashes and supports reveal how the myth cycle plays out in mathematics classrooms, regional politics, and international economics.

3. The Nature of Platonist Mathematics within the Myth Cycle

The myth cycle is ultimately initiated by three foundational axioms, the core of Plato's philosophy of mathematics: its ontology, epistemology, and axiology; in short, *the nature of Platonist mathematics*. Therefore, this topic is explored first.

Normally, this topic is debated as, "Is mathematics discovered or invented?" (e.g., PBS, n.d.; Wigner, 1960; Wolfram & Weinstein, n.d.). However, this article's question regarding the nature of Platonist mathematics is posed more specifically: Is Platonist mathematics purely philosophical, or spiritual-based, or from an alternate universe, or cultural-based? Each is a stance expressed in the academic literature.

3.1 The Philosophical Case

The *Stanford Encyclopedia of Philosophy* lists three fundamental axioms of Platonist mathematics philosophy:

1. "Existence: There are mathematical objects;"
2. "Abstractness: mathematical objects are abstract;" and
3. "Independence: Mathematical objects are independent of intelligent agents and their language, thought, and practices" (Linnebo, 2018, website quotes).

The universe is composed of abstract mathematical objects (AMO) that existed before humans walked the Earth.

This metaphysical view of mathematical Platonism "has been among the most hotly debated topics in the philosophy of mathematics over the past few decades" (Linnebo, 2018, website quote). In my view, the philosophical literature concerning this ongoing debate (e.g., Balaguer, n. d.; Burges, 2001; Frege, 1884/1950; Linnebo, 2018) is a quagmire of reductionist categories punctuated with imaginative and nuanced interpretations and reinterpretations of language and concepts – a philosophical Wild West event, if you will. For example, in reaction to critiques aimed at the notion of AMO, some Platonists changed their definition of "objects" to include ideas in one's mind (Balaguer, n. d.): "In this view, numbers and circles and so on do exist, but they do not exist independently of people; instead, they are concrete mental objects – in particular, ideas in people's heads" (website quote). This rhetorical move, of course, animates Aristotle's fallacy of equivocation.

Burges' (2001) review of Balaguer (1998) pointed out:

> Mark Balaguer demonstrates that there are no good arguments for or against mathematical Platonism (i.e., the view that abstract, or *non-spatio-temporal*, mathematical objects exist), Balaguer does this by establishing that both Platonism and anti-Platonism are defensible positions (p. 79, emphasis added).

The topic concerning non-spatio-temporal objects resurfaces in Subsection 3.3.

This impasse seems like the norm in the philosophy of mathematics. When Hersh (2017) wrote about the pluralism of mathematics, he stated:

> Although in mathematics complete consensus is the norm, the philosophy of mathematics, I have been told, it's just the opposite way; you never convince anybody to give up their position. …The practice of the philosophy of mathematics seems to be to a large extent choosing a position and fighting for it. (p. 20)

This suggests that the pro-Platonist perspective on the nature of mathematics engrained in Grades 7-12 mathematics curricula is not on firm ground, and therefore, the issue is open to further scrutiny – the topic of the next three subsections.

The Platonist stance seems at best weak; hence it
is possibly disposable as the most rational answer.

3.2 The Spiritual-Based Case

Gordon (2019) pointed out that Plato:

> very cleverly invented the World of Forms [i.e., world of ideas], where all mathematics resided. With mathematical formulations based in the World of Forms, it was for the human enterprise to recover/discover but not create the pristine expressions of mathematical knowledge. Sense impressions caught up in the vagaries of existence could not be trusted to determine what was Real [for example, fool's gold]. (pp. 1-2)

Gordon added:

> Plato's formulation that mathematical truth resides in the World of Forms was inspired by Parmenides' metaphysics and has determined to a most significant extent the presentation format of mathematics up to today. In its pristine finished expression, the elegance of mathematics argument is made evident. (p. 1)

This view continues to be expressed today; for example, noted but not endorsed by, Mukhopadhyay & Greer, 2012: "the supremacist position maintained by many mathematician educators who regard abstract mathematics as the crowning achievement of the human intellect, and school mathematics as the transmission of its products" (p. 860).

Plato's rational line of reasoning came to another specific conclusion. He assumed that abstract mathematical objects (AMO) (i.e., mathematical ideas) gave the universe its structure. And Plato wrote, "[T]he number of beings [objects] in the universe is the greatest possible, because of the goodness …of the creator" (Grabiner, 2021, p. 57). This strongly suggests the Ancient Greek's creator created everything in the universe. Thus, Plato taught that his creator designed the structure of the universe mathematically, out of AMO. Ergo, the nature of Platonist mathematics is spiritual (Phillips, 2009).

Skovsmose (2019) added a historical insight to this conclusion. Platonist mathematics' current performative power operates "as a political pacifier by making controversial readings and handlings appear neutral and objective" (p. 1).

> The pacification caused by mathematics emerges from the glorification of mathematics, which has deep historic roots back to the so-called scientific revolution. The people engaged in this revolution – Copernicus, Kepler, Galilei, Newton, etc. – were all deeply religious. The universe was considered God's creation. They believed that we can come to understand this creation by means of mathematics, for it appeared that mathematics captures the rationality of God. (p. 10)

For example, when Newton signed his autograph, it was accompanied by the message (written in Latin), "From this fountain (the free will of God) …the laws of nature have flowed" (Today in Science History, 2021, website quote). Skovsmose (2019) concluded that during the ensuing Age of Reasoning (a.k.a. The Enlightenment) "the belief in God was removed from the scientific outlook, but the glorification of mathematics continued" (p. 11).

This glorification continued in the 20th and now into the 21st centuries by several well-known theoretical physicists, such as Paul Dirac, and more recently, Neil Turok. Dirac (1963) wrote:

> It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of a mathematical theory of great beauty and power, needing a quite high standard of mathematics for one to understand it. You may wonder: why is nature constructed along these lines? One could perhaps describe the situation by saying that God is a mathematician of a very high order and He used very advanced mathematics in constructing the universe" (as quoted by Turok, 2012, p. 186).

Turok went on to comment on Dirac's passage: "Dirac's God was, I believe, the same one that Einstein or the ancient Greeks would have recognized: …whose works epitomizes the very best in rationality, order and beauty" (p. 186-187). There seems to be a scientific mathematics theology circulating beneath the surface.

When Sriraman's (2004) analyzed reasons for mathematicians' creativity, he pointed out that a Platonist ontology implies "that mathematical objects exist prior to their discovery and that any meaningful question about a mathematical object has a definite answer, whether we are able to determine it or not" (p. 21). In other words, the question, "What are abstract mathematical objects?" has an unknown answer now, but, according to Platonists, the answer may likely be beyond anyone's comprehension. This dismissive *rhetorical move* by the Platonists seems even more suspicious than their invention of the dichotomy: the mathematical realm versus the spatial-temporal realm, discussed in Subsection 3.3.

Kessler (2019) examined the literature by asking and answering the following question:

> How great is the distance between spirituality and mathematics? In the Classic Greek tradition [such as with Pythagoras and Plato], spirituality and mathematics were close to each other (e.g., Phillips, 2009). Although both Pythagoras and Plato regarded mathematics as a spiritual path leading to the Divine, their approaches were different. Pythagoras was convinced that numbers and formulae had an inherent spiritual meaning, and his school at Croton was 'more a religious brotherhood than an academy' (p. 5). (p. 52)

Kessler (2019) devised "four aspects of mathematics that might be a potential gateway for spiritual experiences" (p. 51) that mathematicians may encounter while doing mathematics. These four aspects are: "(1) Pragmatic Platonism, (2) Aesthetics, (3) The study of patterns, and (4) Inner vision" (p. 52). In addition, Nightingale and Sedley (2010) published an extensive study on human and divine rationality, with an emphasis on Ancient Greece.

Today, the spirituality of mathematics is taken seriously,
either publicly or privately, by a number of mathematicians.

However, none of the researchers cited offered a quantitative estimate on the proportion of mathematicians who embrace this stance.

Expressing nuances in Kessler's (2019) subculture of "pragmatic Platonism," mathematicians will say both, "I *discovered* this mathematical law" and "I *invented* this algorithm" (p. 54, original emphasis). "The majority of mathematicians exercise a 'practical Platonism;' and Platonism always has a spiritual connotation" (p. 54). However, it "probably has little effect on the practice of mathematics" (p. 54).

Yes, but it does have a negative influence on mathematics *education*. For instance, school mathematics received a 37 percent failure rating from adults (aged 20 to 39) who reported they "hated" mathematics (Ipsos, 2005). No reasons were reported. The topic is explored below in Section 8.

Based on his research interviewing mathematicians, Kessler (2019) wrote, “My conclusion is that there is indeed something of a spirituality nature inherent in mathematics itself” (p. 59). Therefore, fundamentally Platonists’ pure mathematics can be conceived as being spiritual; more in the Ancient Greek sense than a contemporary spiritual sense. How many school boards are aware that this spirituality subliminally infiltrates Platonist mathematics classrooms as a peripheral concept identified by Kawasaki’s (2002) domain of language-laden cognition (Aikenhead, 2017)?

3.3 An Alternate Universe Case

Some Platonist mathematicians who disagreed with Kessler’s stance invented a dichotomy with which to clarify Plato’s AMO: “the mathematical realm versus the spatial-temporal realm” (Cole, n.d., website quote). They assigned Plato’s AMO to *the* non-spatial-temporal realm. The term “spatial-temporal” refers to space-time. For instance, “In everyday life, spatial-temporal reasoning includes: …(a) merging in traffic while driving, and (b) determining how many objects can fit in a box” (Omni-Sci, n.d., website quote). Simply put, the spatial-temporal realm focuses on the accessible everyday world – the reality of the world or universe. Platonists seem to characterize AMO as belonging to another world – the “non-spatial-temporal realm” (website quote).

Parallel to Cole (n.d.), Ravn and Skovsmose (2019) focus on AMO as being beyond our *senses*, yet these mathematicians state, “We can grasp them by means of our rationality” (p. 5). Rationality seems like another metaphor among a litany of metaphors used to locate AMO throughout history, a topic which Ravn and Skovsmose reviewed thoroughly.

The tactic of constructing a dichotomy can always produce a self-consistent deductive argument to “prove” that Platonist mathematics is neither philosophical, nor spiritual, nor cultural; but instead a fourth category: a non-spatial temporal entity – a rhetorical gimmick that does not pass as a rational argument. Deductive arguments, like theorems, will produce a “truth” as long as it is crafted in a self-consistent way with sound premises.

When justifying the independence of AMO from humans, Linnebo (2018) wrote, “Just as electrons and planets exist independently of us, so do numbers and sets” (Linnebo, 2018, website quote). However, Linnebo’s scientific literacy is wanting: electrons are theoretical inferences while planets are observable objects. His comparison between Platonists’ AMO and planets fails on epistemic grounds.

Moreover, Cole’s (n.d.) spatial-temporal versus non-spatial-temporal dichotomy is a somewhat tenuous and arbitrary distinction. Much less arbitrary, but certainly as controversial as the theoretical physicists’ string theory that predicts “adding extra dimensions of spacetime” (Moskowitz, 2018, website quote); that is, resulting in multiple universes. The Platonists’ non-spatial-temporal realm seems to be less grounded compared to the physicists' string theory's prediction of alternate universes, even though the two fields differ in their epistemologies. Could AMO have inhabited an alternate universe and somehow crossed over to Plato's goodness-of-the-creator's universe? That would not be a rational position, in my view.

Because there is no consensus in the physics and mathematics communities over string theory or non-spatial-temporal entities, respectively, there is no reason to explore any further an alternative universe as being the sources of, or analogues to, AMO.

3.4 The Culture-Based Case

In keeping with their pristine image of pure mathematics, Platonists constructed deductive arguments that described Platonist mathematics' main features (Steel, 2010); that is, its *public image* of being: value-free, non-ideological, culture-free, purely objective in its use, perfectly certain and accurate in descriptions of reality, etc. (Ernest, 2018; Kennedy, 2018). Platonists deduced these images from the three axioms summarized above as "the universe is composed of abstract mathematical objects [AMO] that existed before humans walked the Earth."

Each image, however, has been shown to be false; not on logical grounds but on empirical grounds by identifying values, ideologies, cultural practices, etc. that Platonists actually follow. In 1991, Ernest had expressed Platonist mathematical values as dyads: "Abstract is valued over concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, general over particular, theory over practice, the work of the brain over the work of the hand, and so on" (p. 259). Platonist values that mathematicians embrace include: truth, provability, universalism, objectivism, rationalism, and formalism (Ernest 2016a, 2020). Its ideologies include: purism, quantification, universalism, objectivism, and rationalism (Bishop, 2016; Ernest, 2016b). For example, Skovsmose (2019) suggested that the public's glorification of Platonist mathematics can be explained in terms of it embracing an *ideology of purity*" (p. 11, emphasis added).

Anthropologist Hall (1981) asserted that Plato's philosophical concept "purity of mind" (p. 192) and Plato's idea that the universe is made up of AMO, amounted to an intellectual mirage: "*What has been thought of as the mind is actually internalized culture*" (p. 192, original emphasis). Accordingly, Plato's source of his abstract objects was his culture, which causes his AMO to be cultural artefacts, rather than either artefacts from an alternate universe or spiritual entities.

From a human-centred stance, Ravn and Skovsmose (2019) reinterpreted the traditional notion of the philosophy of mathematics by adding social and ethical dimensions to it. I subsume these two dimensions within the category of culture – defined broadly as: "a wide repertoire of sense-making practices that people participate in, particularly in everyday contexts" (Bang & Medin, 2010, p. 1014).

I am persuaded that Platonist mathematics is fundamentally cultural rather than mathematically philosophical, spiritual, or from an alternate universe. Hall's conclusion is supported by critical mathematics educators, for example, Bishop (2016), Ernest (1988, 2016a,b), Larvor (2016), Ravn and Skovsmose (2019), Skovsmose and Greer (2012), and Sriraman (2017).

These scholars demonstrated that Platonist mathematics is surreptitiously value-laden, ideological, cultural, subjective in its use, its certainty is at the expense of its accuracy, as Einstein (1921) explained, "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" (website quote). Platonists' purity-of-the-mind images of mathematics are not necessarily true outside the purview of the mind.

In keeping with the explanation that Plato's AMO are cultural, Ernest (2016a) contended, "mathematics itself can be imbued with the values of the culture of its human makers" (p. 193).

The Platonists' deductive arguments are
false when claiming certain idealistic
features of their nature of mathematics.

4. High Jacking the Curriculum

When, where and how were these false features insinuated into school mathematics? – In the post-Industrial-Revolution era during the reign of Queen Victoria beginning in 1837. There was a need to educate workers for the new industrial economy with its market fundamentalist philosophy; that is, Carney's (2020a) profit society. This need necessitated a public education system. The merchants and industrialists at the time wanted concrete practical business mathematics taught.

Here, "practical mathematics" does not necessarily mean applied mathematics. It means the concrete equivalent or analogue that is usually learned on the job to get things done mathematically without necessarily applying an abstraction like a mathematician would (Aikenhead, 2021b; Nicol, 2002). This type of mathematics has been called "concrete procedural mathematics" (Aikenhead, 2005, 2021b), and is always tied to getting a job done well.

In opposition to the merchants and industrialists, elite university mathematics professors wanted their future students to be well grounded in Platonist mathematics, a position that was justified and empowered by the politics of national economic progress and territorial expansion (Nikolakaki, 2016; Willoughby, 1967).

Aikenhead (2017) described in detail the politically charged debate that ensued. The following summary provides a highlight. In essence, Platonists' arguments rested on inventing a theoretical binary: *formal* mathematical discourse versus *informal* mathematical discourse, and cleverly associating it with the binary "logical versus irrational, dating back to Socrates, Plato, and Aristotle" (Hall, 1981, p. 213). The Platonists denigrated the commercial interests by assigning them to "the informal discourse" (irrational) category for being impure; in contrast to exalting Plato's logically pure mathematics. Ernest (1991) hints at a magic trick: "[A]t the heart of the absolutist neutral view of mathematics is a set of values and a cultural perspective, as well as an ideology which renders [Western mathematics' values, ideologies and culture] invisible" (p. 260) – a deception, to be sure. This ideology arose from the Greek-based European-adopted impulse to reify, essentialize, and vigorously promote arbitrary binaries that only exist in the mind. In summary, one might call the Platonists' strategy for winning the curriculum debate "hijacking the mathematics curriculum." In any case, "the glorification of mathematics" (Skovsmore, 2019, p. 1) continues.

Rhetorical tactics by an elite university power group,
backed by investors in national progress and expansion,
were the weapons of choice in the hijacking process.

5. Widespread Beliefs within the Myth Cycle

The Platonists' mythical idealized images of their mathematics have, over about 170 years, evolved into Platonist dogma, mainly due to successfully silencing its critics when they wrote or taught content Platonists' classified as "informal discourse;" for instance, the interactions between mathematics and society. The dogma's unquestioned authority is ensconced in influential sources, such as university classes taken by future mathematics teachers, and by material published by such organizations as the (American) National Council of Teachers of Mathematics, which has a strong political influence on many, but not all, provincial mathematics teachers' professional organizations in Canada.

After frequenting mathematics classrooms for 12 years, most learners have unconsciously absorbed the Platonist dogma. "The manner in which the [Platonist] mathematical way of seeing is integrated into schooling, society and above all into the interpersonal and power relations in society results in the *transformation of the human outlook*" (Ernest, 2018, p. 195, emphasis added). In the parlance of Educational Foundations, this is not education but indoctrination (Clabaugh, 2007).

In any case, the Platonist dogma became society's mainstream culture's "widespread beliefs in the myths" (Figure 1) that comprise the false façade of Platonist mathematics (i.e., it being value-free, non-ideological, culture-free, purely objective in its use, perfectly certain and accurate in descriptions of reality, etc.).

This *false* façade aligns with a domain of knowledge that *eschews* human dimensions in the foundations of mathematics, while the *true images* of Platonist mathematics are humanistic: value-laden, ideological, cultural, etc. (Ernest, 2016a,b). In other words, Platonists *reject* the reality of their type of knowledge in order to embrace Plato's purity-of-the-mind type of knowledge. Simply stated, *reality is being rejected* in order to maintain Plato's *imagined* world of ideas. In a phrase, this situation merits the expression "epistemic schizophrenia."

Ernest (2016a) summed up many learners' outcomes from having studied mathematics in junior and senior high school levels: "The net result of extended exposure to and practice in mathematics is a social training in obedience, an apprenticeship in strict subservience to the text [as a consequence of] its imperative-rich and rule-governed character" (p. 192). This is the antithesis to a humanistic view that a majority of learners tend to hold, to varying degrees. To experience mathematics as a human endeavour is to engage in a repertoire of mathematics' sense-making cultural practices (Boylan, 2016; Sriraman, 2017).

The Platonists' false façade myth can be added to their potential myths regarding the philosophical, or spiritual, or alternate-universe natures of AMO. Their false-façade myth makes Platonist mathematics look absolutely certain when it is not (Einstein, 1921; Ernest, 2016c). Epistemically schizophrenic Platonists have created curricula about which a majority of learners have complained bitterly for some time – "It's irrelevant and boring." Reality is not.

Through years of Platonist mathematics schooling, the public psyche has been indoctrinated into accepting the superiority of Platonists' epistemic dogma; thereby transforming the dogma into widespread beliefs within the myth cycle.

6. An Educational Implication Arising from the Nature of Platonist Mathematics

Learners in Grades 7-12 tend to protest because they experience the Platonist version of mathematics as being "mechanical, detached, emotionless, value-free, and morally neutral – the antithesis of human activity" (Fyhn et al., 2011, p. 186); in a word, *non-humanistic* (Aikenhead, 2021d). Psychologist Gilligan (1982) identified this Platonist perspective as "valorising: rules, abstraction, objectification, impersonality, unfeelingness, dispassionate reason and analysis" (quoted in Ernest, 2018, p. 194); as opposed to a humanistic stance of living by valuing "relationships, connections, empathy, caring, feelings and intuition" (p. 195). Examples of humanistic mathematics programs were cited in this present article's Section 1.

7. Socio-Politico-Economic Power Bestowed

The power wielded by Platonist mathematics draws its authority from: (a) its legitimate positive use value; (b) its illegitimate public façade of being non-ideological and culture-free, for example; yet (c) treating people ideologically according to its claimed certainty and objectivity when applied. Consequently, it has become the power of authority within mainstream Western cultures.

A "formal/informal mathematical discourse" binary politically legitimized the first Platonist school curriculum, which is fundamentally unchanged today for Grades 7-12 (Nikolakaki, 2016) because it continues to be *Platonist* mathematics. "Each new version of the curriculum [over at least a hundred years] is presented as objective, neutral and final reflection of mathematics" (Ernest, 2020, p. 7).

This is particularly evident in large scale student assessments whose statistical analyses of assessment data "provide the objectivity effect that one's assessment or research is actually measuring something objective" (Harris, 2017, p. 118), when it is not. Greer and Skovsmose (2012) called it the "tantalizing illusion of certainty" (p. 6). Thus, on the one hand, learner assessment is a major strategy in maintaining Platonist's socio-politico-economics power in the myth cycle. On the other hand, we are dealing with a public institution, Platonist mathematics, whose track record for playing lose and spurious with dichotomies and false images of itself.

Consider the validation of standardized mathematics tests. Standardization procedures are typically reported in the language of conventional procedures, statistical calculations, and approval by academic mathematicians. The basic goal of standardized mathematics tests is to produce winners and losers. This severely narrows the validation process to the test's *exchange value* (Carney, 2021; Ernest, 2019). High marks by winners are exchanged for social and economic privileges. Consequently the validation of standardized tests ignores the *use value* of the test's contents.

A possible use-value research question would be: How do learners' mathematics scores compare with the scores of specified groups of adults (e.g., 20-49 years of age) on the same test? This type of use-value validation is employed by a cluster of college entrance tests, in which items are selected by their predictive power to distinguish between successful and unsuccessful college students. A use-value of test standardization measures the relevancy of the content being tested. To what extent is the test content familiar to the adult taking the test? Highly relevant content would tend to be familiar to adults, and so the marks would tend to be higher than when the test content was unfamiliar to them.

A major logistical problem, however, has prevented such research from occurring. Finding volunteer subjects who are in public positions of authority, and would agree to having their test score anonymously included in their group's average score that would be made public.

But there has been at least one exception. Two Washington Post journalists each wrote an article about a very successful business man and Florida State school board member "of one of the largest school systems in America" (Brady, 2011, website quote). One article is about his use-value validation of a standardized mathematics assessment, the Grade 10 Florida Comprehensive Assessment Test. "Rick Roach was in his fourth four-year term representing his Orange County District 3 on the Board of Education" (Strauss, 2021, website quote). Brady (2021) reported that his academic background included a bachelor of science degree, two masters degrees, and was an active doctoral candidate at the time. "He said he'd make his scores public" (website quote). He kept his promise. In Roach's own words (Brady, 2011):

> The math section had 60 questions. I knew the answers to none of them, but managed to guess ten out of the 60 correctly. … I have a wide circle of friends in various professions. … Not a single one of them said that the math I described was necessary in their profession. … It might be argued that I've been out of school too long, that if I'd actually been in the 10th grade prior to taking the test, the material would have been fresh. *But doesn't that miss the point?* A test that can determine a student's future life chances should surely relate in some practical way to the requirements of life. I can't see how that could possibly be true of the test I took. (website quote, emphasis added)

He went on to describe the consequence of a 10th grader in his position (Brady, 2021):

> It makes no sense to me that a test with the potential for shaping a student's entire future has so little apparent relevance to adult, real-world functioning. Who decided the kind of questions and their level of difficulty? Using what criteria? He emphasized the point that the education testing industry has no accountability to the educators it serves. …*To whom did they have to defend their decisions?* (website quote, emphasis added)

The board member was up against the powers of the myth cycle; for example, the Platonist ideology of quantification and the political ideology that there must be winners and losers. Perhaps it was out of frustration that he concluded, "Those decisions are shaped not by knowledge or understanding of educating, but by ideology, politics, hubris… " (Brady, 2011, website quote). He surmised that education administrators like himself who follow through on such decisions "*are doing something ethically questionable*" (website quote, emphasis added).

In his chapter "Do Mathematicians Have Responsibilities," mathematics professor Harris (2017) stated, "[B]eing entrusted with power by virtue of our role in the transmission of mathematical knowledge imposes the responsibility to insist on the limitations of that knowledge" (p. 118). Harris added: "The ideology of certainty and objectivity [should not] be used to undermine democracy" (p. 123); "Mathematical modelling is not objective or scientific just because it is mathematical" (p. 123).

A Platonist stance appears to resist the crucial move from a profit society to a sustainable society because of the latter's initial uncertainties and unknowns (Barwell, 2018).

Objective certainty is
the opiate of Platonists.

On the surface, the Platonists' rhetorical tactics of creating binaries may suggest opportunism. But underneath the rhetorical tactics lie its raw socio-politico-economic power, masked by the innocence of its public false façade of being: value-free, ideology-free, acultural, and an intellectual game to play (Zeilberger, 2017) called "Platonist Mathematics." Similar to standardized testing, its false façade masks the school's role as an apparatus of the state that generally maintains a person's current socio-economic strata of society (Andrews, 2016). Pais (2012) explored this idea in greater detail:

> This concealment is essential to maintain the role of school as an ideological state apparatus. Seeing school as a place free of ideology disables bringing ideological struggles to school. All enterprises undertaken by teachers to unmask the "invisible" ideology are immediately accused of being ideological acts. In this way, the dominant ideology ensures that no ideology is present in school except, of course, the dominant one. The dominant one is precisely the one that presents itself as ideologically free, by positing the importance of mathematics as knowledge and competence [for citizens]. (p. 70)

Then there is the strong belief in mathematics' high-status function to serve as the key gatekeeper for: high school graduation, acceptance to post secondary education, and the quality of employment. This function simultaneously performs an ideology of exclusion to such an extent that the current high school mathematics curriculum exacerbates the "systemic racism" in countries with Indigenous citizens (Aikenhead, 2017, p. 74).

Ernest (2020) operationalizes the "social power bestowed on those beliefs" (Figure 1):

> I argue that because of the ubiquity and importance of mathematical applications there is an overvaluation of mathematics that distorts education for all. *The needs of the few are generalised*, resulting in a demand for high stakes *test certification from the many*. Such mathematics testing operates as a critical filter for entry to further study and almost all professions, and this is class reproductive in distributing social advantage along the lines of social capital. …Mathematicians and mathematics-related professionals, such as myself, are complicit and undoubtedly gain from the overvaluation of mathematics in society" (p. 21, emphasis added).

And finally, Pais (2012) questioned "the ideological injunction that you really need mathematics to attain [full] citizenship" (p. 65). Ernest (2019), however, recognizes the injunction as "a fallacious argument" (p. 4) based on an embellished utility of mathematics for society that gets translated into an educational policy that *all* students need to be certified in mathematics to the highest possible level. Teachers tend to repeat this policy many times to justify their subject to their learners, the majority of whom knows the claim is either incredulously hypothetical or simply an exaggeration. Greer and Mukhopadhyay (2012) go a step further in dismissing the policy: "Really? Think about people you know. Aren't there many who do not have a solid grounding… in mathematics that are living full and productive lives? Isn't it offensive to tell such people that they are dysfunctional?" (pp. 239-240). Recall Rick Roach of the Florida school board (Brady, 2011). It would seem probable that these mathematics teachers' claim is another myth to add to the myth cycle.

These examples reveal the mechanisms by which subliminal beliefs and dogma bestow power to a minority of people who are highly proficient on Platonist mathematics standardized tests. This power leads directly to privileges such as gaining entry into social networks among business and political leaders. This clearly illustrates the conventional exchange value of Platonist mathematics that keeps the myth cycle revolving.

Subliminal, exaggerated false beliefs are the currency
for the exchange value of Platonist school mathematics.

8. Privileges Denied by the Myth Cycle's Power

Platonist mathematics has "a hidden dark side" (Ernest, 2019, p. 16). Its social-politico-economic power "in education and society has costs and harmful outcomes, as well as the more widely acknowledged benefits. Only when we acknowledge this dark side can we start to plan and act on ameliorating, reducing and countering the harm done, the algorithmic injustices" (p. 7).

From the perspective of a majority of learners, high school mathematics does not relate to the 21^{st} century Information Revolution. High school mathematics turns out to be simply "a hoop to jump through just to prove you could…a false boundary in one's life" (Russell, 2017, p. 25); "a formidable rite of passage into adulthood" (Aikenhead, 2021a, in press).

"The net effect of mathematical examinations is the grading of students into a hierarchy with respect to life chances" (Ernest, 2018, p. 201). In his article "Privilege, Power and Performativity: The Ethics of Mathematics in Society and Education," Ernest (2019) provides important details and a general solution.

> Thus the overvaluation of mathematics, its exaggerated exchange value, leads to the wastage of human power in the workforce, contributes to the reproduction of social inequality, and leads to negative attitudes and reduced self-confidence with regard to mathematics, as well as, for some, reducing full participation in our democratic society. An ethical appraisal must surely acknowledge the harm done to *some students and citizens* by imposing the mandatory study of abstract mathematics beyond its use value because of its inflated exchange value. (p. 7, emphasis added)

By giving attention to "*some* students and citizens," Ernest raises the quantitative question: How many?

Results from three major research projects (Card & Payne, 2017; Frederick, 1991; OECD, 2019) and a professional poll (Ipsos, 2005), all converged on the figure 70 percent of high school graduates in North America who are mentally, socially, or physiologically harmed in various ways to varying degrees (Aikenhead, 2021a). It would be a gross oversimplification to dichotomize them as 70 percent non-STEM versus 30 percent STEM students. Instead, Aikenhead (2021a) created a continuum between two extremes: from math-phobic to math-oriented learners. The continuum was structured by the PISA's (Program for International Student Assessment) six proficiency categories (OECD 2019). The full continuum is mentioned here, along with the approximate percentages of Canadian Grade 12 graduates in each category (Aikenhead, 2021a): *math-phobic (20%), math-shy (24%), math-disinterested (26%), math-interested (20%), math-curious (6%), and math-oriented (4%).* This continuum represents the variations in learners' mathematical self-identities, from low to high (Darragh & Radovic, 2018; Ishimaru, Barajas-López, & Bang, 2015; Ruef, 2020).

The harm is even more crushing for Indigenous learners. The colonial ethos of Platonist mathematics (Bishop, 1990) requires Indigenous students to park their culture and self-identities at the classroom door. Their culture's mathematics is known as "Indigenous mathematizing" (Meyer & Aikenhead, 2021), but is seldom acknowledged or taught as one of the pluralist ways of making quantitative, spatial, and relational sense of the world.

Systematic racism was mentioned by British Columbia's provincial Auditor-General report (Bellringer, 2019), "Our 2015 report highlighted the impact of the racism of low expectations [for Indigenous students]" (p. 13). Moreover, the high school graduation rates show an extraordinary inequity: Saskatchewan's "graduation rate for Indigenous students stands at 44.5 per cent for three years to complete high school versus 86.5 per cent for non-Indigenous students" (Canadian Press, 2019, website quote). These data, I submit, illustrate the darkest feature of the myth cycle.

Pais (2012) summarized a critical approach to equity as ameliorating "the antagonism between the societal demands for equity and the role of schools as exclusionary institutions" (p. 76), thereby creating an ambivalence within the myth cycle (Figure 1). Ernest (1991) identified a source that maintains the myth cycle's existence: "There are powerful forces at work [in society] keeping cultural domination and institutional racism in place, for it serves the interests of capital and the politically powerful" (p. 268).

Borovik (2016) gave voice to learners' reactions to privileges denied: "Mathematics education in the West is being destroyed by a quiet thought (or even a subconscious impulse): 'If they think they teach me something useful, let them think I am learning'" (p. 355). Learners have extraordinary skills at making teachers think that in-depth learning has occurred when, in fact, rote memorizing and skillful acting produced reasonable assessment results; a process science educators call "playing Fatima's Rules" (Aikenhead, 2011, pp. 114-115; 2021a). These rules apply equally to school mathematics.

More generally, the Platonist mathematics' ideology of quantification becomes harmful when it leads: (a) to dehumanizing people by viewing or treating them as objects" (Ernest, 2018, p. 208), and (b) to believing that mathematics is beyond ethical scrutiny: "Training in mathematical thinking, when misapplied beyond its own area of validity to the social domain is potentially damaging and harmful. …Ethical reasoning tends to be eroded whenever mathematics becomes engaged in standardization, routinization and dehumanization" (p. 198). Ernest advised, "The way we teach and how we use mathematics and its impact on our thinking are what can be harmful" (p. 212). These negative engagements tend to damage learners' mathematical growth mindsets Boaler (2015).

A savvy, articulate, and down-to-earth insider's account of school mathematics' dark side comes from a 16-year-old, Rachel Steinig (2016), whose title of her article in the *Journal of Humanistic Mathematics* serves as her article's Abstract: "Stop Ruining Math! Reasons and Remedies for the Maladies of Mathematics Education" (p. 128). Her reasons include mathematics teachers' propensity for the hypothetical: "In math classes where practical applications are introduced, they are usually NOT things that would actually come up in real life. …Other than the basics, most people do not end up using the math that they learned in school when they are adults" (p. 137). Her remedies are twofold: "[Assume that] everyone is intelligent in a different way. …If we know what's being done wrong, we can make it right" (p. 146).

Schooling in general (Anyon, 1980), but school mathematics in particular (Jorgensen, 2016; OECD, 2019), tend to offer variations in content, pedagogy, and expectations, depending on the social class of a school's neighbourhood or a leaner's family (Andrews, 2016). The end result is to reinforce and maintain the social status quo, expressed by the myth cycle (Figure 1). This often occurs when relationships of *equality* (doing the same thing for everyone) rather than relationships of *equity* (helping each individual to attain their own potential) are prominent in mathematics classrooms (Levitan, 2016).

In summary, harm is the usual outcome when mathematics is over valued. This results in some learners' failures, incomprehension, anxieties, physiological trauma and loss of self confidence; resulting in: (a) negative attitudes towards mathematics; (b) passing their negativity on to their children; and/or (c) dropping out of school (Belbase, 2013; Ernest, 2019).

Positive outcomes have normally favoured the math-interested, math-curious, and math-oriented learners. Figure 1 certainly captures what happens to most of these 30 percent of high school graduates.

Who benefits and who is harmed in
the school mathematics cycle of myths?

9. Conclusion

In her chapter, "Morality and Mathematics," Muntersbjorn (2016) asks:

> [W]hy should students be required to take mathematics courses? If so few pupils have a taste or talent for mathematics, why are we obligated to teach mathematics to as many of the next generation as possible? Universal mathematical literacy seems a noble (if naïve) goal of any culture… *But whence the normative force behind the claim*, "all pupils must learn mathematics"? (p. 387, emphasis added).

Above, Ernest (2020) blamed the normative force on "the ubiquity and importance of mathematics [causing] an over evaluation of mathematics that distorts education for all" (p. 21). A group of Canadian mathematics educators blamed it on the mainstream culture's "power of authority" to promote Platonist mathematics (Abtahi, 2020, p. 282; Nicol et al., 2020b, p. 197). These two reasons are more than academic viewpoints when foisted upon the total population of learners in Grade 7-12. An ethics audit is long overdue.

Another plausible answer to Muntersbjorn's "[W]hence the normative force behind the claim?" is to keep the cycle of myths running smoothly.

> *The myth cycle of inequities is energized by Platonists' false epistemic dogma, sponsored by a profit society, and indoctrinated into the public psyche, thereby gaining authoritative power to dictate educational policy regardless of Platonists' false claims and rhetorical abuses.*

More specific reasons include the desire of society's socio-politico-economic power groups to censor:

- Jorgensen's (2016) elephant in the room (i.e., schools' tendency to preserve the social class structure of society) by classifying this issue as *informal* mathematical discourse – "Platonese" for irrational or not rigorous, and thus taboo;
- Ravn and Skovsmose (2019) for for adding a social and ethical dimension to the nature of mathematics;
- Pais' (2012) notion that the role of schools is to be exclusionary;
- Skovsmose (2019) conclusion that Platonist mathematics can be used as a pacifier to control the masses;
- the impulse to teach in an authentically contextualized way (e.g., Aikenhead, 2021a; Barta et al., 2014; Leung et al, 2020); exemplified by: "Who needs word problems when the news is rich with real-life mathematics problems?" (Petti, 2009, website quote), or rich in humanistic contexts (e.g., *Journal of Humanistic Mathematics*); and
- the conclusions that school mathematics has reached a relevancy gap crisis (Borovik, 2017; Skovsmose, 2019);

The normative force targeted by Muntersbjorn (2016) resides in the hegemonic politics of Platonist mathematics education (Skovsmose & Greer, 2012), guided by influential university professors who seldom see in their university classes any of the 70 percent group of high school graduates.

This article explored complex and wide-ranging reasons for addressing what has contributed to a Grades 7 to 12 mathematics' content crisis; the increasing relevancy gap between the content of today's curriculum and the *majority* of learners' experiences with their everyday world. For countries with Indigenous citizens, it is crucial to include some Indigenous mathematizing for the sake of reconciliation (Aikenhead, 2017; 2021a; Barta et al., 2014; Lunney Borden, 2021; Meyer & Aikenhead, 2021; Nicol et al., 2020a), and for creating an excellent segue into learning accurate features of *the nature of* Western mathematics.

This conclusion ends with a note about what Section 1 called "the solution." Political educational agendas appropriate for each country are required to diminish the cycle of myths. Aikenhead (2021a) provided a long list of specific political actions effective in some jurisdictions. The following are more general.

1. To move away from absolutist Platonism, one must move towards its replacement, such as the curriculum goal to understand mathematics as a *pluralist* entity, only one of which is Platonist (Western) mathematics, and the others are various forms of non-Platonist mathematics (e.g.,

Aikenhead 2021d; Alrø & Johnsen-Høines, 2016; Andersson & Ravn, 2012; Sugimoto et al., 2017). Both Platonist and non-Platonist mathematics can exist as different emphases in two separate pathways in the curriculum for later grades. For example, in 2008 the Province of Saskatchewan's curriculum added "mathematics as a human endeavour" (p. 9) to its other three goals: "logical thinking, number sense, and spatial sense" (pp. 7-8); and "Indigenous ways of knowing" became a specified human endeavour topic (SaskMATH, 2021).

2. A long-term aim for school mathematics would be to *ameliorate* the following five common myths: (a) mathematics content represents the greatest intellectual achievement of humankind; (b) high proficiency in mathematics indicates the most intelligent humans; (c) mathematics is value-free, non-ideological, culture-free, purely objective and perfectly certain in its use; (d) meeting the most rigorous proficiency standards of Platonist mathematics should be the goal for all learners; and (e) a learner's low mathematics proficiency results in their becoming a poor citizen.
3. Worth repeating: "We know what is wrong. We can make it right" (Steinig, 2016, p. 146) through planned, concerted, political action to change the curriculum. The needs of savvy citizens are mostly about knowing *how to identify and critique* the uses/misuses of Platonist mathematics in political, commercial, economic, social and family situations (Nikolakaki, 2016). In other words, there is certainly academic rigor in the humanistic fabric of contextualized mathematics, labelled taboo by Platonist educators.

I suggest setting aside Muntersbjorn's (2016) question, "Whence the normative behind the claim?" (p. 387), and instead, focus on the question: Is there an *ethical* justification to continue the policy that dictates that *all* learners must pursue a Platonist pre-calculus-like program in school mathematics (Boylan, 2016)? Why not a rigorous culture-based or humanistic mathematics program for the 70 percent (Aikenhead, 2021a and 2021b, respectively)?

In the next few generations during society's transition to a sustainable society (Carney, 2020b), the high jacking of the 19th century mathematics curriculum can be rectified by an equitable settlement in which the 30 percent will prosper equitably along with the 70 percent of high school graduates. This will occur when each of the two Gilligan (1982) value clusters, humanistic and mathematical, harmonize with their respective curriculum for the 70 and 30 percent.

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